A criterion for determining air-flow separation from wind waves

By JIN WU, *Hydronautics Inc., Laurel, Maryland*

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ABSTRACT

Based on laboratory studies of wind-wave interactions, a criterion for determining air-flow separation from wind waves is proposed. The air separates from waves having a phase velocity less than the shear velocity, which characterizes the turbulent wind field. The successful application of this criterion to the analysis of oceanic data is herewith demonstrated.

Introduction

The wind stress acting at the air–sea interface locally governs the wind–wave interaction, and globally influences the geophysical state of the atmosphere. Extensive studies have been conducted to determine the wind-stress coefficient. However, the mechanism by which the transfer of momentum from the wind to waves takes place has not been completely explained, although great progress in the subject has been made by Phillips, Miles and Benjamin, for example; see Phillips (1966).

Based on compiled oceanic data, the air flow over the sea has been shown to be aerodynamically rough in most cases (Wu, 1968*a*). As a result, the momentum flux to, or the form drag of, the interfacial roughness accounts for a large fraction of the wind stress. However, the air-sea interface consists of surface waves of various lengths. Based on his study of the aerodynamic drag of a free water surface, Francis (1951) suggested that small ripples or wavelets compose the active roughness elements responsible for drag. This idea has been generally accepted on the ground that the height of the roughness element was found to be small. Corresponding to these small heights, waves of short lengths naturally result. However, the effective height of the aerodynamic roughness is different from the physical roughness height (Schlichting, 1960). The aerodynamic roughness depends not only on the height but also on the shape, distribution and density of the physical roughness. Consequently, more experimental verification of the foregoing suggestion would be very welcome.

The present results provide some indirect evidence that for a fully developed sea, the surface roughness is substantially due to 'short' waves. Going beyond to confirm the role of 'short' waves, a criterion for identifying the specific length of these 'short' waves is provided. It is hoped that the present paper provides oceanographers with some physical insight into problems involving the air–sea interface.

Wind-wave interactions

In turbulent flow over a solid rough surface, the mean velocity profile is logarithmic and follows the Karman–Prandtl distribution (Schlichting, 1960),

$$\frac{u}{u_*} = \frac{1}{\varkappa} \ln\left(\frac{y}{\eta}\right), \tag{1}$$

where u is the velocity at a distance y from the surface, u_* is the shear velocity, $\varkappa$ is the Karman constant, and η is the dynamic roughness length. The shear-stress coefficient, C_y, can thus be defined in terms of the mean velocity at a convenient height, y, as

$$C_y = \frac{\tau_0}{\varrho u^2} = \left[\frac{\varkappa}{\ln (y/\eta)}\right]^2, \tag{2}$$

where τ_0 is the shear stress and ϱ is the fluid density. The validity of the Karman–Prandtl distribution in air flow over a water surface has been experimentally confirmed by Wu (1968*b*); the turbulence structure and the consequent mean velocity profile not too close to the water surface seems to be retained, despite the water induced air motion.

In aerodynamically smooth flow, there is a viscous sublayer near the wall. This layer is a region of constant stress substantially supported by fluid molecular viscosity. The thickness of this layer, δ, has generally been considered to be (Rouse, 1953):

$$\delta = \frac{11.6\nu}{\sqrt{\tau_0/\varrho}} = \frac{11.6\nu}{u_*}, \tag{3}$$

where ν is the kinematic viscosity. In aerodynamically rough flow, the sublayer is disrupted by surface roughness. Here, flow separation from the roughness elements occurs. Consequently, it is the momentum flux to, or the form drag of, the roughness which accounts for the shear stress.

The roughness at the air-sea interface is composed of a great variety of moving waves, differing in height, length, shape and phase velocity. Moreover, the waves are subjected to continuous and irregular changes. The instantaneous picture of the complicated sea-surface structure shows short waves superimposed upon longer waves of various size. A crucial difference between roughness at a solid surface and roughness at the air-sea interface is that the latter is moving generally in the air-flow direction and the former is stationary. For interfacial roughness, the sublayer flow may not be disrupted if the roughness is moving at a velocity comparable with that of the sublayer, whose average velocity, V, can be determined from Equation (3)

$$V = \frac{1}{2}\frac{\tau_0}{\mu}\frac{11.6\nu}{\sqrt{\tau_0/\varrho}} = 5.8\,u_*, \tag{4}$$

where μ is the dynamic viscosity. The value of the proportionality constant is unimportant here, the important result being that the average velocity of the sublayer flow is proportional to the shear velocity. This result gives particular concrete meaning to the shear velocity which is used later in determining the separation criterion.

It is generally considered that shorter waves with steeper faces, rather than longer waves with flatter shapes, cause the drag to air flow. It is also known that for deep water waves of not overly short length, the wave phase velocity is proportional to the square root of the wave length. The question now is whether there exists a critical lower limit to wave length, below which the wave (the roughness element) cannot keep adequate pace with the sublayer flow so that the air flow starts to separate. No separation would occur from waves with greater length, as the relative velocity between sublayer and roughness is smaller. For a fully developed sea, waves shorter than this critical length have smaller heights, to which the momentum flux from air is negligible, because the air flow has already been separated from waves of greater heights.

As indicated, it is assumed that air-flow separation from waves depends on the comparison between the average velocity of the viscous sublayer and the phase velocity of waves which constitute the surface roughness. Therefore the criterion for determining separation would naturally occur in a form of the ratio between these two velocities. In the following two sections, laboratory results on wind-wave interactions and oceanic data on wind stress and surface roughness are presented. In the last section, a critical velocity ratio serving as a criterion for determining air-flow separation from waves is deduced from the laboratory studies; then a successful application of this velocity ratio to the analysis of oceanic data is demonstrated.

Laboratory studies of wind–wave interactions

Experiments have been conducted in a wind-wave tank with a 1.5 m by 1.5 m cross-section and 14 m in length. Mounted at the upstream end of the tank is an axial flow fan. The top of the tank is covered for the first 7 m up to the test section. The maximum velocity obtainable with the 0.3 m deep air passage above the normal 1.2 m water depth is 14 m/sec.

The wind velocity profile in the channel is determined by the vertical traverse of a pitot-static tube. The test station is located at nearly equal distances between two ends of the wave tank, where the mean water level is least af-

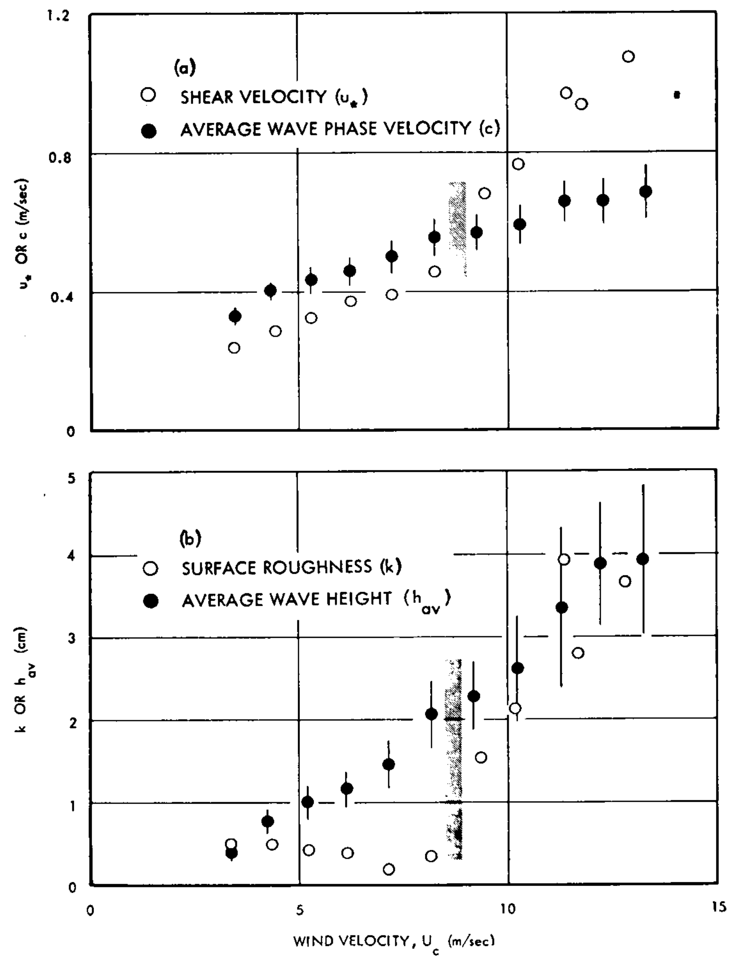

Fig. 1. Laboratory results of wind-wave interactions.

fected by the water surface setup. The waves are measured by a conductivity-type wave height gage, which is recorded continuously on a visicorder. The drift current is measured by timing floats of various sizes passing two stations along the tank. The velocity of the float is taken as the drift current at the centroid of the submerged, projected area of the float. The surface drift current is determined by extrapolating the current-distribution curve to the water surface.

A detailed description of the equipment and experimental techniques was given in an earlier article (Wu, 1968*b*).

Shear velocities and surface roughnesses

Previously (Wu, 1968*b*), confirmation of the application of the Karman–Prandtl velocity distribution, Equation (1), was shown for the turbulent air flow over a water surface in the aerodynamically rough flow regime; an alternate form of Equation (1) was used.

$$\frac{u_c}{u_*} = \frac{1}{\varkappa} \ln\left(\frac{y}{k}\right) + 8.5, \qquad (5)$$

where u_c is the wind velocity relative to the water surface, and $k = 30\,\eta$ is the physical length scale characterizing the roughness of the water surface (Schlichting, 1960).

The shear velocity and surface roughness, determined from the wind profiles and plotted in Fig. 1*a*, *b*, respectively, are seen to be separated into two groups. These two groups were shown to be separated physically by the occurrence of wave breaking (Wu, 1968*b*). The white caps started to appear at the crests of basic gravity waves at the wind velocity, $U_c = 8.5$ m/sec, where U_c is the free stream wind velocity relative to the water surface.

As shown in Fig. 1*b*, the roughness decreases before wave breaking and then increases as the wind velocity increases beyond breaking. It was shown that in the lower range of wind velocity, the surface roughness is governed by the ripples with steeper faces than the gravity waves. In the higher-wind-velocity range, it was deduced that the wind stress is supported by the form drag of gravity waves, indicating that flow separation very likely occurs along the basic wave profile.

Wind waves

The average wave period, p_{av}, and wave heights, h_{av}, at various wind conditions, can be determined from wave records. The phase velocity, c, and wavelength, λ, of the typical (average) waves are then calculated from the wave periods through several iterations between

$$c = \sqrt{\frac{g\lambda}{2\pi}\left(1 + \frac{\pi h_{av}}{\lambda}\right)^2 + \frac{2\pi\sigma}{\lambda\varrho_w}} \quad \text{and} \quad \lambda = c p_{av}, \tag{6}$$

where σ is surface tension and ϱ_w is density of water. The average phase velocities are plotted in Fig. 1*a*, short vertical lines drawn through the data points indicate the standard deviation of wave periods. The average wavelength is found to be less than one-third of the water depth; waves of the deep water type are thus assured in this experiment. In addition, the width of the wave tank is more than four times the wavelength; hence no side-wall effects on the development of wind profiles is expected.

The average wave heights are presented in Fig. 1*b*, the short vertical line also indicates the standard deviation of wave heights. It was also shown (Wu 1968*b*) that the waves are flatter than a sinusoidal shape in the low-wind range ($U_c < 8.5$ m/sec) and are more peaked than a sinusoidal shape in the high-wind range ($U_c > 8.5$ m/sec).

Wind stress and surface roughness at air–sea interface

The results of thirty independent oceanic wind-stress investigations were collected by Wu (1968*a*). For these data, the wind velocity, U_{10}, was measured at a height of 10 meters above the mean sea level. The wind-stress coefficients, C_{10}, defined in terms of U_{10}, were further sorted into bands of 1 m/sec, having their upper and lower bounds at integral wind velocities. The data falling into each band were first averaged and their standard deviation from the average value was subsequently determined. The average wind-stress coefficient of each band, as well as the standard deviation, indicated by a short vertical line drawn through each data point, is presented in Fig. 2*a*.

Based on the Karman–Prandtl velocity distribution, the surface roughness can be determined from:

$$k = \frac{30\,y}{\exp(\varkappa C_y^{-\frac{1}{2}})} = \frac{300}{\exp(0.4\,C_{10}^{-\frac{1}{2}})}, \tag{7}$$

and is plotted in Fig. 2*b*. Different flow regimes, determined from the roughness Reynolds number (Schlichting, 1960), are tabulated in Table 1.

Detailed discussions of wind stresses and surface roughnesses at the air–sea interface have been presented by Wu (1968*a*).

Criterion for determining air-flow separation from wind waves

Two different mechanisms governing the wind field over the water surface in the aerodynamically rough flow regime were suggested by Wu (1968*b*). An equation of state relating shear velocity to surface roughness involving either surface tension or gravity was shown to depend on whether capillary or gravity waves

Table 1. *Boundary-layer flow conditions at the air–sea interface*

Flow regimes	Regime limits	Wind-velocity ranges (m/sec)
Aerodynamically smooth	$ku_*/\nu < 5$	$U_{10} < 3$
Transition	$5 < ku_*/\nu < 70$	$3 < U_{10} < 7$
Aerodynamically rough	$ku_*/\nu > 70$	$U_{10} > 7$

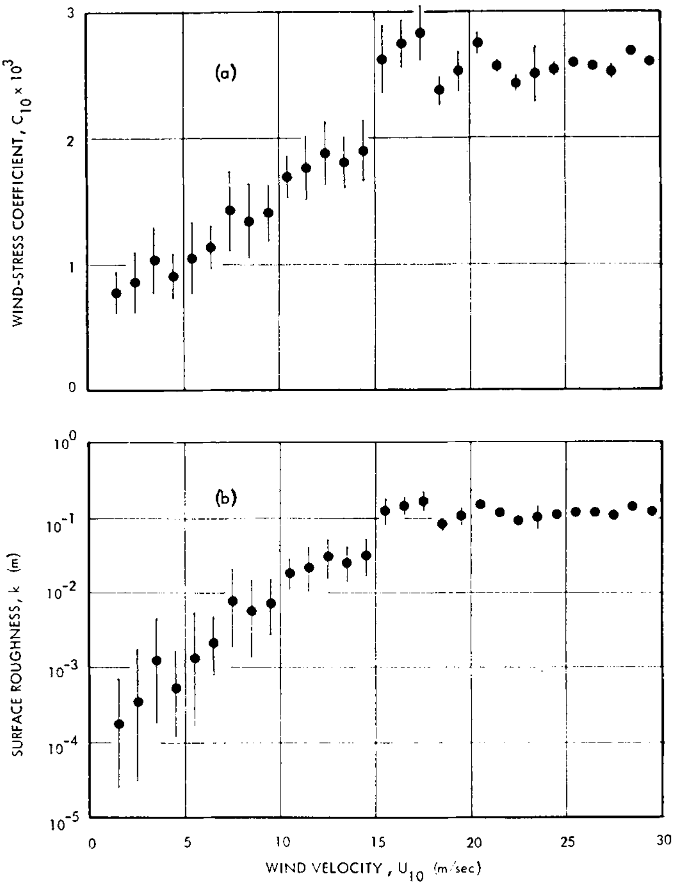

Fig. 2. Wind stress and surface roughness at air-sea interface.

act as the principal contributor to the surface roughness. The shift of the governing mechanism was related to the occurrence of the wave breaking phenomena in the wind-wave tank. Before the occurrence of the wave breaking (at low wind velocity), the surface roughness is constituted by ripples with steeper faces than the gravity waves. This is the Weber-number scaling region, in which the surface roughness decreases with increasing wind velocity, and the surface roughness has a much smaller value than the average wave height (the average of gravity-wave heights) see Fig. 1*b*. As noted earlier (Wu, 1968*b*), at the lowest wind velocity, where the heights of the ripples are comparable to those of gravity waves, the surface roughness has nearly the same magnitude as the average wave height, as might be expected. After the occurrence of wave breaking (at high wind velocities), the similarity (on both trend and magnitude) between the average wave height and the roughness indicates that the wind stress is supported by the form drag of basic gravity waves, and that flow separation is likely to occur along the basic wave profile. This is the Froude-number scaling region, in which the surface roughness increases with wind velocity.

The shifting of the surface-roughness governing mechanism is marked by the rapid change of surface roughness shown in Fig. 1*b*. This shifting, as discussed previously, is also marked by the observed appearance of whitecaps at the wave crests, as well as by the change of the gravity-wave profile from flat to cusped. One would expect all these occurrences when the air flow starts to separate from the gravity waves, and, consequently, when the latter constitute the surface roughness. In summary, the wind

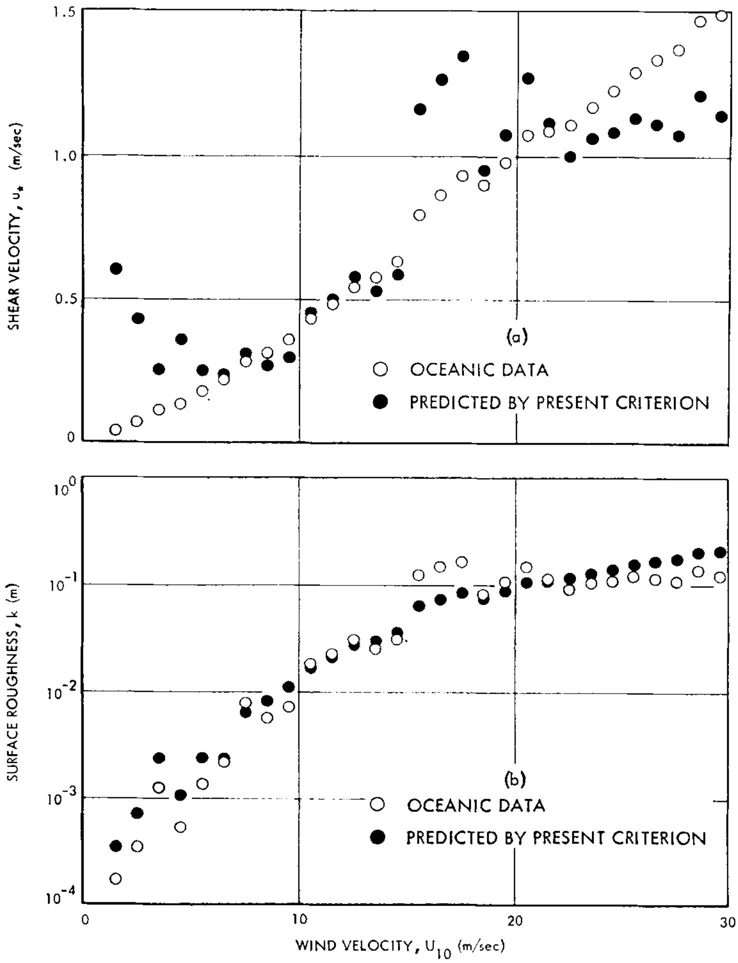

Fig. 3. A comparison of oceanic wind-stress and surface roughness data with those predicted by the present criterion.

and the wave conditions when this shift takes place are experimentally well-defined; these conditions thus provide the basis for determining the criterion of air-flow separation from waves.

It is shown in Fig. 1*a* that the shear velocity increases beyond the average wave phase velocity right at the wind velocity where the shift from one wind-stress governing mechanism to another occurs. This seems to suggest that the equality of shear velocity and average wave phase velocity should be adopted as the criterion for determining air-flow separation from waves: *from waves having a phase velocity less than the shear velocity, air-flow separation occurs.* As a consequence, the average wave phase velocity of critical waves (from which separation just occurs), is about 5.8 times as great as the average velocity in the viscous sublayer.

This proposed criterion is based only on general physical arguments and on indirect experimental evidence. Since no analytical prediction of boundary-layer flow separation from a solid, stationary surface with given geometric shapes can be made with confidence at the present time, it is quite unlikely that an analytical verification can be made of this criterion applying to moving wave profiles at the air-water interface with complicated and ever-changing shapes.

This criterion will next be applied to the well-correlated oceanic data presented in the previous section. If one considers the maximum wave-height-to-length ratio to be 0.14 and assumes that waves responsible for the surface roughness have been fully developed and have reached this maximum ratio, the following comparisons can be made:

(*a*) Taking the surface roughness as the wave height, the critical wavelength can be immediately estimated and then the wave phase velocity, *c* (the velocity of the roughness element), can be determined from Equation (6). This calculated phase velocity, which should be the same as the shear velocity according to the proposed separation criterion, is readily compared with the oceanic shear-velocity data; see Fig. 3*a*. Excellent agreement is shown for the most important portion of the comparison, that is for wind velocities ranging from 7 to 15 m/sec. For wind velocities lower than 7 m/sec, the air flow is still in an aerodynamically smooth regime or in the transition region (see Table 1), and the comparison is not valid. At very high wind velocities (storm at sea), the comparison is very difficult; nevertheless, good agreement is still shown in Fig. 3*a*.

(*b*) It is interesting to present the scomparison in a different way. Taking u_* as the phase velocity of critical wind waves we can find from Equation (6) the corresponding wave-length. Multiplying these calculated wavelengths by the maximum wave-height-to-length ratio, we find the corresponding maximum wave heights, which are now readily compared with the oceanic surface-roughness data; see Fig. 3*b*. This comparison has essentially the same feature as the previous one. It can also be deduced from Fig. 3*b* that waves of short lengths indeed constitute the surface roughness and, consequently, support the wind stress.

Conclusions

In the aerodynamically rough flow regime, the wind stress at the air–water interface is supported by roughness elements; from these elements, air flow separation occurs. Based on experimental evidence, it is deduced that the air flow separates from waves having a phase velocity smaller than the shear velocity. The ratio between the shear velocity (characterizing the turbulent wind field) and the critical wave phase velocity (characterizing the rough water surface) is thus an important criterion in determining the flow condition very close to the water surface. Setting this ratio to unity provides a means for determining which waves (waves of critical length) support the wind stress. This has been a controversial subject for years; it has been generally assumed (but without any analytical or experimental proof) that the momentum flux to shorter waves accounts for a substantial fraction of the wind stress. This assumption has been clearly verified here.

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КРИТЕРИЙ ДЛЯ ОПРЕДЕЛЕНИЯ ОТСТАВАНИЯ ВЕТРОВЫХ ВОЛН ОТ ПОТОКА ВОЗДУХА

На основе лабораторных экспериментов по взаимодействию ветра с волнами предлагается критерий для определения отставания ветровых волн от потока воздуха. Поток воздуха опережает волны, имеющие фазовую скорость меньшую, чем сдвиговая скорость, которая характеризует турбулентное поле ветра. Демонстрируется успешное применение этого критерия к анализу океанических данных.