Comparative measures of potential to kinetic energy conversion

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ABSTRACT

Presented for comparison are estimates of the conversion of available potential energy to kinetic energy as obtained from three sets of objective analyses of the large-scale atmospheric motions of the northern hemisphere for January 1964. The analyses of the geopotential heights made twice daily of the 85, 70, 50, 30, and 20 cb levels are used. Relatively large differences in the energy conversion estimates as a function of the zonal wave number are found which are due solely to differences in the analyses. An estimate is also presented of the seasonal variation of the release of kinetic energy for the 85–20 cb layer. A discussion of some of the deficiencies of the techniques used is given.

Introduction

The primary purpose of this investigation is to obtain estimates of the range in measurements of the available potential to kinetic conversion of atmospheric energy which can be expected to arise from differences in the analyses of the large-scale atmospheric motions. The rates of release of atmospheric kinetic energy, as measured with the quasi-geostrophic tools, are presented here for three sets of objective analyses for January 1964. Objective analyses made twice daily of the geopotential heights of the 85, 70, 50, 30, and 20 cb surfaces have been obtained from the following sources: (1) Global Weather Central, U.S. Air Force, (2) Fleet Numerical Weather Facility, U.S. Navy, and (3) National Meteorological Center, U.S. Weather Bureau. In addition, an estimate of the seasonal variation of the release of kinetic energy is given.

The conversion of available potential energy to kinetic energy, $C(A, K)$, is often thought of as consisting of the sum of two parts (e.g., see Lorenz, 1955). One is the zonal available potential to zonal kinetic component, $C(A_Z, K_Z)$, and the other is the eddy available to eddy kinetic component, $C(A_E, K_E)$. By definition for a given layer $\Delta p = p_2 - p_1$

$$C(A, K) = -(gS\Delta p)^{-1} \int_S \int_{p_1}^{p_2} \omega\alpha \, dS\, dp, \quad (1)$$

where g is the acceleration of gravity, S is the area, ω is the departure from the area average of the vertical velocity dp/dt, p is presure, and α is the specific volume area-averaged departures. Let

$$(\;) = (\;)_Z + (\;)_E, \quad (2)$$

where

$$(\;)_Z = L^{-1} \int_0^L (\;)\, dx. \quad (3)$$

Here x is the eastward directed coordinate and L is the distance. Thus (1) may be written

$$C(A, K) = -(gS\Delta p)^{-1} \int_S \int_{p_1}^{p_2} w_Z \alpha_Z \, dS\, dp - (gS\Delta p)^{-1} \int_S \int_{p_1}^{p_2} \omega_E \alpha_E \, dS\, dp, \quad (4)$$

where the terms on the right side are $C(A_Z, K_Z)$ and $C(A_E, K_E)$, respectively.

Method of estimate

The ω fields have been evaluated from the following approximate relationship:

$$f_0 \frac{\partial^2 \omega}{\partial p^2} + \frac{\sigma}{f_0} \nabla^2 \omega = \frac{\partial}{\partial p} (\mathbf{V} \cdot \nabla \eta) - \nabla^2 \left(\mathbf{V} \cdot \nabla \frac{\partial \psi}{\partial p} \right) - g \frac{\partial^2}{\partial p^2} \left(\frac{\partial \tau_y}{\partial \chi} - \frac{\partial \tau_x}{\partial y} \right), \quad (5)$$

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Table 1. *$C(A, K)$ for January 1964 north of 20° N from U.S. objective analyses from (1) Air Force, (2) Navy, and (3) Weather Bureau*

Units are $\times 10^{-7}$ kj m^{-2} sec^{-1} cb^{-1}

$n\ldots$ / P	0	1	2	3	4	5	6	7	8	9	10	11	12	$\sum_{n=1}^{20}$
85–70														
(1)	−141	55	64	86	45	50	52	52	48	42	22	17	10	580
(2)	−186	86	75	62	39	35	43	52	49	41	18	15	9	549
(3)	−229	73	77	85	48	49	56	59	54	45	22	18	11	632
70–50														
(1)	−197	67	65	103	62	52	65	68	62	46	18	16	8	649
(2)	−250	116	98	79	57	40	59	61	63	53	21	20	13	703
(3)	−347	108	112	117	72	66	75	83	75	65	24	21	12	863
50–30														
(1)	−288	77	55	123	56	47	48	50	33	27	13	11	11	557
(2)	−340	123	129	77	50	44	57	47	46	46	19	13	10	676
(3)	−526	112	113	121	73	44	49	44	37	27	11	3	2	639
30–20														
(1)	−185	52	−11	71	33	25	22	−1	5	−18	−1	−5	0	144
(2)	−227	10	109	34	23	29	40	25	−3	16	0	−4	3	279
(3)	−303	51	69	73	62	5	20	11	13	−16	−5	−4	−4	260
85–20														
(1)	−210	65	50	100	52	42	50	48	41	29	14	11	8	527
(2)	−259	95	104	68	45	38	52	49	44	42	17	13	10	594
(3)	−368	92	98	104	65	46	54	55	49	36	15	11	7	648

where

$$\mathbf{V} = \mathbf{k} x \nabla \psi \tag{6}$$

$$\sigma = \sigma(p) = -\alpha_s \frac{\partial \ln \theta_s}{\partial p} \tag{7}$$

and

$$\nabla^2 \psi = \nabla \cdot f^{-1} \nabla \Phi. \tag{8}$$

Here f_0 is the Coriolis parameter at 45° N, η is the absolute vorticity, φ is the geopotential, and the subscript s refer to a representative area average. The seasonal values of σ, the static stability, were those observed by Gates (1961). The stress, τ, due to the small-scale eddies was assumed to decrease linearly with pressure from a maximum value of $p = p_0 = 101.3$ cb to zero at $p = p_0 - \Delta p^*$, where Δp^* is the friction depth here taken to extend from p_0 to 90 cb. (The results using $\Delta p^* = 24$ cb produced little differences in the monthly averaged results.) Furthermore

$$\tau_x = -C_d V u / \alpha, \tag{9a}$$

$$\tau_y = -C_d V v / \alpha. \tag{9b}$$

C_d is the geostrophic drag coefficient as used by Cressman (1960), V is the wind magnitude and u and v are the eastward and northward wind components. The stream function ψ was obtained from (8) using a lateral boundary condition which produced nondivergent geostrophic winds throughout the octagonal grid.

At $p = 0$, ω was set to zero, and at $p = p_0$

$$\omega = -V \cdot \nabla p_g$$

where V was obtained by linear extrapolation in pressure to p_g, the pressure at the ground as given by Berkosky & Bertoni (1955). At the lateral boundaries of the octagonal grid the Dirichlet condition $\omega = 0$ was used. Centered differences were used in (5) and the resulting algebraic system of equations was solved through use of the implicit alternating direction iterative method (e.g., see Varga, 1962).

Results

The results averaged for January 1964 are presented in Table 1 in the domain of zonal wave number for each pressure layer (e.g.,

Table 2. *$C(A, K)$ for the region north of 20° N obtained from the Weather Bureau analyses*

Row *a*, *b*, *c*, and *d* refer to the 85–70, 70–50, 50–30, and 30–20 cb layers, respectively.
Units are $\times 10^{-7}$ kj m^{-2} sec^{-1} cb^{-1}

n... Date		0	1	2	3	4	5	6	7	8	9	10	11	12	$\sum_{n=1}^{20}$
Jan. 1962	*a*	−124	50	118	98	59	50	49	61	57	31	43	15	17	686
	b	−140	90	127	127	72	78	58	82	75	46	54	15	8	848
	c	−164	138	101	136	53	94	39	62	55	43	42	4	−1	764
	d	−164	98	35	60	−7	40	−14	−5	20	−8	11	−4	−7	213
Apr. 1962	*a*	−59	18	4	29	42	39	37	67	40	37	28	19	14	418
	b	−94	22	15	30	54	49	37	87	44	38	29	18	13	457
	c	−138	23	28	32	65	60	43	92	29	27	19	17	3	442
	d	−68	22	5	21	29	36	23	36	11	−3	5	−3	−3	161
July 1962	*a*	−17	14	−4	16	2	15	14	13	14	14	7	5	5	130
	b	−32	14	11	18	5	19	12	13	16	13	4	2	1	134
	c	−61	37	68	44	21	34	15	19	27	18	10	−1	1	302
	d	−41	6	25	74	13	14	12	24	33	5	7	2	2	220
Oct. 1962	*a*	−134	38	34	27	24	43	54	51	40	31	19	20	12	425
	b	−187	58	54	42	27	61	69	57	44	29	15	19	10	505
	c	−281	75	96	70	31	81	79	61	37	28	73	11	4	591
	d	−136	64	82	39	34	38	26	12	0	0	3	−4	1	287
Jan. 1963	*a*	−299	108	113	98	60	44	35	61	54	34	29	17	16	711
	b	−372	157	154	116	82	44	55	75	66	37	37	16	13	878
	c	−541	131	170	141	70	41	34	39	44	19	17	1	4	722
	d	−422	−65	97	120	43	18	20	8	−6	−22	0	−4	−8	199

Saltzman, 1957; Wiin-Nielsen, 1959). The grid data were interpolated to a latitude-longitude grid consisting of a 2.5 degree mesh. The results are valid for the region north of 20° N. Values for the zonal wave number $n = 0$ represent the $C(A_Z, K_Z)$ values and those for $n = 1$ to 20 are for $C(A_E, K_E)$. The 85–20 cb values are the pressure weighted averages. The differences in the conversion rates resulting from analysis differences are due primarily to analysis differences over the lower latitudes of the region. This is evident from an inspection of the latitudinal variations presented in Fig. 1. In the lower latitude regions the data coverage is relatively poor and the analyses reflect more strongly the techniques utilized in the various schemes.

Presented in Table 2 are the results for other months obtained from the Weather Bureau analyses. The seasonal trend in the energy conversions is apparent for this technique. Notice that the planetary waves, $n = 1$ to 4, contribute significant amounts to the total energy conversions, particularly in the winter months in the 50–30 cb layer.

When inspecting the results presented here, one should bear in mind that a major portion

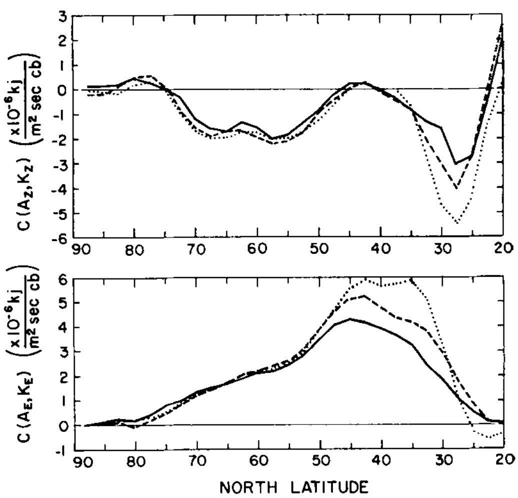

Fig. 1. Energy conversions as a function of latitude for the 85–20 cb layer as obtained from the objective analyses of the Air Force (solid lines), Navy (dashed lines), and Weather Bureau (dotted lines). Values contain cosine latitude weighting to account for variation of area with latitude.

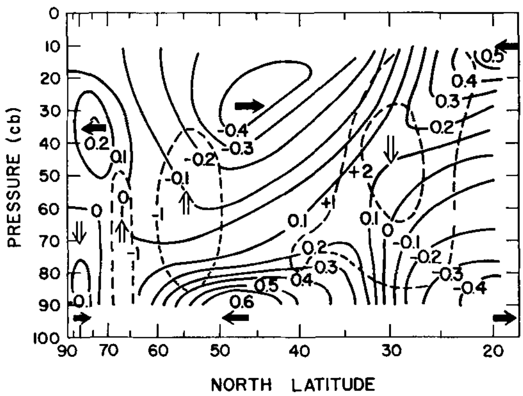

Fig. 2. Monthly averaged meridional wind field (solid lines) and ω field (dashed lines) for January 1964 as obtained from the Weather Bureau geopotential height analyses. Meridional winds are in units m sec^{-1} and vertical motion units are $\times 10^{-5}$ cb sec^{-1}.

of the Hadley cell is not contained in the analyses by virtue of the limited area which was considered. This is evident from an inspection of Fig. 2 which is the meridional wind field and the zonally averaged ω field for January 1964 obtained from the Weather Bureau geopotential height analyses. Thus the large negative values of $C(A_Z, K_Z)$ are primarily a reflection of the dominance of the indirect cell for this area under consideration. Furthermore, the results presented here exclude the contribution above the 20 cb level and below the 85 cb level.

Upon comparing the results presented here with those obtained by other investigators, one finds that the magnitudes here are in general larger. For example, Krueger, Winston & Haines (1965) obtained values of -109×10^{-7} and 537×10^{-7} kj m^{-2} sec^{-1} cb^{-1} for $C(A_Z, K_Z)$ and $C(A_E, K_E)$, respectively, for the 85–50 cb layer using the Weather Bureau analyses for January 1963. Their ω fields were obtained using only 85, 50, and 20 cb data. Furthermore, their ψ fields were obtained from the nonlinear balance equation (e.g., see Shuman, 1957).

Numerical calculations have been made using the procedure as presented here for the first five days (ten analysis sets) of January 1964 but using only 85, 50 and 20 cb data of the Weather Bureau. The averaged result for the 85–20 cb layer north of 20° N for $C(A_Z, K_Z)$ was -365×10^{-7} kj m^{-2} sec^{-1} cb^{-1} and that for $C(A_E, K_E)$ was 537×10^{-7} kj m^{-2} sec^{-1} cb^{-1}. (The comparable 5 level results for this period were -372×10^{-7} and 498×10^{-7} kj m^{-2} sec^{-1} cb^{-1}, respectively. Thus the results for this technique appear to be less sensitive to vertical resolution.) The calculations were repeated but the lateral boundary condition on ψ was $\psi = \varphi/f_0$, rather than the geostrophic wind condition mentioned earlier. Then $C(A_Z, K_Z) = -222 \times 10^{-7}$ kj m^{-2} sec^{-1} cb^{-1} and $C(A_E, K_E) = 492 \times 10^{-7}$ kj m^{-2} sec^{-1} cb^{-1}. Thus it appears that the zonal conversion is more sensitive to the lateral boundary condition on ψ than is the eddy conversion. Still another set of calculations were performed for these five days using the 85, 50, and 20 cb data but this time using the nonlinear balance equation ψ fields as obtained by the Weather Bureau, rather than the more simplified fields used earlier. These ψ's incorporate the condition $\psi = \varphi/f_0$ at the lateral boundary. There resulted $C(A_Z, K_Z) = -222 \times 10^{-7}$ and $C(A_E, K_E) = 423 \times 10^{-7}$ kj m^{-2} sec^{-1} cb^{-1}. The results obtained indicated that the use of the simplified stream function of (8) tend to produce values of $C(A_E, K_E)$ which are too large. This is undoubtedly due largely to the fact that the effect of curvature is ignored, thereby producing overestimates of the horizontal temperature advection in the regions of cyclonic activity.

The results which are presented here are intended to demonstrate the sensitiveness of the available potential to kinetic energy conversion on the analysis of the data and on the technique employed.

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СРАВНИТЕЛЬНЫЕ МЕРЫ ПОТЕНЦИАЛЬНОЙ И КИНЕТИЧЕСКОЙ ЭНЕРГИИ

В данной работе предлагаются для сравнения оценки преобразования доступной потенциальной энергии в кинетическую, полученные из трех примеров объективного анализа крупномасштабных движений в северном полушарии для января 1964 года. Используется анализ высот геопотенциала, взятых за два срока в день на уровнях 85, 70, 50, 30 и 20 *сб*. Обнаружены относительно большие различия в оценках преобразования энергии как функции зонального волнового числа, что является исключительно следствием различия в анализах. Представлены также оценки сезонных изменений генерации кинетической энергии для слоев, лежащих в пределах 85–20 *сб*. Приводится обсуждение некоторых недостатков использовавшейся техники анализа.