Evaporation rates based on tritium measurements for hurricane Betsy

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ABSTRACT

Assuming the same turbulent diffusion processes to be acting on both ordinary and tritiated water vapour, the evaporation rate of ordinary water can be computed from Ostlund's tritum measurements in hurricane Betsy. The hurricane evaporation rates, in the range of one to five cm per day, are subject to large uncertainties but are of the same order as found by Malkus and Riehl. The higher rates occur on a day when the hurricane is more intense, the lower rates when less vigorous.

Ostlund (1968) reported a remarkable series of observations showing that the tritium content of moisture decreased inwards towards the center of hurricane Betsy in 1965. In large part, the present interpretation of these data continues Ostlund's own analysis. Where the latter deduced the rate of inflow into the hurricane center based on certain assumptions, I now compute the rate of evaporation from measured radial wind speeds; the observed winds from the ESSA Research Flight Facility aircraft only recently became available.

The basis for a relationship between hurricane tritium and evaporation of ordinary water substance rests on two important assumptions. First, the flux-gradient relationships in a layer of air in contact with the sea surface are the same for both tritiated and ordinary water. Second, the radial changes in tritium from the storm center gives the flux of tritiated water into or out of the sea.

Derivation

The first assumption above states the following:

$$E = k(q_s - q_a);$$

$$E_\tau = k(q_s C_s - q_a C_a) = kb(q_s \tau_s - q_a \tau_a),$$

where E is the rate of evaporation of ordinary water and E_τ is the evaporation rate of tritiated water; k is a proportionality coefficient similar to a diffusion coefficient; q is the humidity mixing ratio; C, the mass ratio of tritiated to ordinary water; τ is the same ratio expressed in tritium units, T.U. (one T.U. equals 1 tritium atom per 10^{18} protium atoms); b converts T.U. to the mass ratio of tritiated to ordinary water, C, and will later cancel out; the subscripts "s" and "a" refer to values in the air directly in contact with and in the air at some height above the sea surface respectively. The one dimensional flux-gradient relationship for either ordinary or tritiated water vapor would divide the difference of mixing ratios at two heights by the altitude interval and the constant of proportionality will become the Austausch (the diffusion coefficient times air density). Thus, the k in the above equations would be the Austausch divided by the height interval. More often, however, one uses Jacobs' formula (1951) which is identical to the equations written above where k is a known empirically determined function of wind speed and state of the sea. Thus, as written, the simple formulae include the other two forms. The above equations can also be written as

$$E = kq_s(1 - h)$$

and

$$E_\tau = kq_s b(\tau_w \gamma - \tau_a h)$$

so that

$$E = E_\tau(1 - h)/b(\tau_w \gamma - \tau_a h)$$

where h equals q_a/q_s and is a rough approximation to atmospheric relative humidity; γ is a fractionation factor for tritiated water and depends slightly on temperature; and the sub-

script "w" refers to sea surface water. The rate of evaporation of ordinary water can thus be expressed in terms of the rate of evaporation of tritiated water without any coefficient of proportionality, k. Indeed it is the substitution of tritium information for the coefficient of proportionality, k, which lies at the heart of the present effort. It is understood that water vapor and tritium differences will be measured through the same height interval.

The net gain or loss of tritiated water at the sea surface will reflect itself by changes in the tritium concentration of the overlying spiralling rings of air. Taking into account the equation of mass continuity, the removal of tritiated water by precipitation, and assuming negligible flux of tritiated water through the tops of the columns, a relationship between E_τ and the change of tritiated water in a column or ring can be given as

$$E_\tau = g^{-1}\Delta\left(\int_{p_0}^{p_0-\Delta p} qC_a\,dp\right) \Big/ \Delta t = \bar{v}_r \frac{\Delta p}{g} \bar{q} \left(\frac{\overline{\Delta C_a}}{\Delta r}\right),$$

where g is the acceleration of gravity; p is pressure and Δp is the pressure thickness of the column whose tritium content is changing; $\bar{v}_r$ is the radial wind speed averaged around the storm center and through a depth, Δp; $\bar{q}$ is the mean humidity mixing ratio; $(\overline{\Delta C_a/\Delta r})$ is the mean rate of change of the tritium concentration with radius, r, from the storm center; and t is time. Thus the rate of evaporation can be written as:

$$E = \bar{v}_r \frac{\Delta p}{g} [\bar{q}/(\tau_w \gamma - \tau_a h)]\,(1-h)\left(\frac{\overline{\Delta \tau_a}}{\Delta r}\right).$$

Evaporation in Betsy

With the help of Ostlund's tritium measurements, information from the hurricane reconnaissance aircraft and additional approximations, numerical values for each of the right-hand terms of the above equation are given in Table 1. The radial wind speed can be observed but its reliability is poor. The thickness of the layer, Δp, through which air parcels have a decreasing tritium concentration while spiralling towards the storm center, can be estimated from radial wind data. On September 5th the air blew inward at 3240 feet (988 m at about 900 mbs) in Table 1, but by 6400 feet (1951 m at about 800 mbs) the net inflow became very weak or zero. On September 3rd the flight at 1770 feet (540 m at about 950 mbs) revealed the inflow of Table 1 but the next higher altitude flight at 8090 ft (2466 m at about 750 mbs) again showed little or no inflow or outflow. Thus, a value of Δp equal to about 150 mbs (about 5000 feet), while uncertain, has been used. The humidity mixing ratio at flight level represents the average of many aircraft readings. This value is used as q_a since the corresponding tritium concentration, τ_a, is derived from flights at the same altitude. The mean mixing ratio in the layer, $\bar{q}$, has been estimated from q_a and the vertical distribution of moisture in a near saturated atmosphere. The sea surface temperature, derived from the lowest values from ship observations near the storm, permits calculation of the saturated humidity mixing ratio in equilibrium with the sea surface water. The relative humidity, h, is taken as the ratio of the observed humidity mixing ratio, q_a, to this saturated humidity mixing ratio. All of the tritium information has been taken directly from Ostlund's work. On September 3rd, the best fit between the tritium measurements and radial distance required a square root relationship, $\tau_a = 6.11 + 3.172\ r^{\frac{1}{2}}$ while on the 5th a linear association is needed, $\tau_a = 22.80 + 0.135\ r$, where r in both formulae is in kilometers. Sepall and Mason (1960) assign γ equal to 0.92 at about 25°C.

Table 2 provides the values of the evaporation rates in both grams of water per unit area and time and the more conventional units of depth of water per day. The five values on September 5th reflect a variability probably due to uncertainties of the input data. The two estimates on Sept. 3 are higher than the five on September 5th. Hurricane Betsy was much more intense (minimum pressure 946 mbs, maximun wind speed 60 m/s) on September 3rd than on September 5th (minimum pressure 970 mbs, maximum wind speed 45 m/s). The larger evaporation rate coincides with the more intense storm.

Malkus & Riehl (1960) suggest that local evaporation of moisture adds about 2420 cal/cm²/day to a typical hurricane. This heat gain would result from the evaporation of about 4 cm H_2O per day, a value which closely corresponds to the numbers in Table 2.

Table. 1. *Input data to compute rate of evaporation, Hurricane Betsy 1965*

Element		Sept. 3rd	Sept. 5th	Discussion of uncertainty
Flight altitude, feet		1770	3420	
Δp, mb		150	150	Plausible range: 50–400.
Sea surface temperature, °C		26°	26°	Ship reports on each day within 300 n. miles of storm showed range: 26–28°.
q_s, g/kg		22	22	Depends mainly on uncertainty of sea surface temperatures.
q_a, g/kg		13.8	15.6	Extreme observed range: Sept. 3rd, 11.5–15; Sept. 5th, 12.5–18.
$\bar{q}$, g/kg		12	15	Extreme estimated range: Sept. 3rd, 10–15; Sept. 5th, 12–17.
h, per cent		63	71	Depends on uncertainty of q_a/q_s.
τ_w, T.U.		13.5	13.5	
τ_a, T.U.				
In 10 n. mi. rings centered at:				
n. mi.	km			
45	82		33.7	
55	100	38.2	36.7	
65	119	41.1	39.4	
75	137		41.9	
85	156		44.2	
$\overline{\Delta\tau_a}/\Delta r$, T.U./km				
In 10 n. mi. rings centered at:				
n. mi.	km			
45	82		0.135	
55	100	0.157	0.135	
65	119	0.144	0.135	
75	137		0.135	
85	156		0.135	
$\bar{v}_r$, m/s				
In 10 n. mi. rings centered at:				
n. mi.	km			
45	82		5.2	Uncertainty probably larger than variability between adjacent ranges.
55	100	9.6	4.4	
65	119	7.0	4.6	
75	137		7.0	
85	156		6.0	

Table 2. *Evaporation Rates, Hurricane Betsy, 1965*

		$\frac{\text{g } H_2O}{\text{cm}^2 \text{ sec}} \times 10^{-5}$	$\frac{\text{cm } H_2O}{\text{day}}$	$\frac{\text{g } H_2O}{\text{cm}^2 \text{ sec}} \times 10^{-5}$	$\frac{\text{cm } H_2O}{\text{day}}$
10 n. mi. rings centered at:		Sept. 3rd		Septh 5th	
n. mi.	km				
45	82			4	3
55	100	9	7	3	2
65	119	5	4	3	2
75	137			4	3
85	156			3	3

Critique and conclusions

Since the water vapor concentration of air parcels spiralling into the center of the storm essentially remains constant, the decrease of the ratio of tritiated to ordinary water has been implicitly attributed to molecular exchange of tritiated water vapor into the ocean. But another explanation is possible, viz., that during the evaporation process, both tritiated and ordinary water molecules may enter the atmosphere. If this evaporated water partially replaces the moisture already present, the atmosphere would then consist of water vapor with a ratio of tritiated to ordinary water closer to that of the ocean than existed before the evaporation. Since the tritiated to ordinary water ratio in the ocean is much lower than that in the atmosphere, evaporation of water of both kinds of molecules would produce the same qualitative change with distance as found by Ostlund. But neither Ostlund nor I now consider this interpretation to be correct. The molecular exchange processes at the air–water interface must be considered separately for the two kinds of water molecules; a net evaporation of water vapor into non-saturated air does not imply an upward flux of tritiated water. There is one exception: complete evaporation of droplets wherein the atmosphere is fed moisture with the same tritiated to ordinary water ratio as existed in the ocean. However, it is thought that only a small amount of moisture is added to the atmosphere by the complete evaporation of spray droplets.

The other sources of uncertainty in the computed rates of evapora tiondepend on the input data and the interpretation of the change in tritium content of the inward spiralling air. The observed radial wind speeds leave much to be desired; the variability of the radial wind speed component over short intervals at the same radial distance reveals more uncertainty than might be judged from the mean speeds in Table 1. Also, the sea surface temperature might be even lower than that obtained from the minimum of the ship reports because of the cooling which was found in the wake of the hurricane storm passage (Leipper 1966). Similarly, the tritium content of the surface water may be lower than estimated by Ostlund, again because of the upwelling of or mixing with low tritium in the deep water but this has never been confirmed.

The determination of the loss of tritiated water from changes in the atmospheric content is crucial. That tritiated water molecules are being lost to the sea surface (neglecting total evaporation of droplets) seems clear from Ostlund's findings of a decreasing tritium concentration inward towards the storm center. But a quantitative assessment of the loss is another matter. Ostlund's data indicate that the concentration decreases inward towards the storm center at altitudes above the lowest 150 mbs. This might suggest that the loss of tritium extends through a deeper layer than the lowest 150 mbs. But the tritium to ordinary water ratios of the moving air parcels at an altitude above about 850 mbs. cannot decrease since the parcels do not travel radially as do those at lower altitudes. However, at some early transient stage in the hurricane development, before a steady state was assumed to exist, the loss of tritium at the sea surface produced decreases through a layer deeper than the lowest 150 mbs. of the atmosphere. This assumption is needed to explain Ostlund's observations above 850 mbs. Later, when approximate equilibrium between lower and upper troposphere developed only the lower atmosphere loses tritium to the sea surface. Thus I have assumed approximate equilibrium conditions on both Sept. 3rd and 5th.

The flight on Sept. 3rd at 1770 feet took place both within and below the cloud base, but on Sept. 5th, at 3240 feet, entirely within saturated air. On both days, tritiated and ordinary water molecules may have been added to or removed from the atmosphere at aircraft altitudes through raindrop evaporation or molecular exchange and these have been ignored. But because of the presence of clouds at flight level, a further complication arises. The differences in humidity and tritium mixing ratios between the air at the sea surface and at flight altitude may be the consequence of processes other than turbulent diffusion, namely condensation. However, the condensation process does not appear to be a serious defect to the theory of turbulent exchange since the variation of staturated humidity mixing ratio and the psuedo-adiabats with altitude are similar at temperatures between 18 and 25°C and pressures above 900 mbs. That is, one may argue that the humidity mixing ratio might have been only slightly higher had not some water vapor been con-

densed, of the order of 1 gm/kg higher in this case.

The use of tritium measurements have bypassed the thorny problem of the proportionality ccnstant between the flux and gradient of moisture at the sea surface. The resulting evaporation rates appear reasonable in the light of our meagre knowledge of their values in hurricanes. But the uncertainties in the theory and input numbers suggest confidence to no better than a factor of two. Finally, the higher rate of evaporation during the more intense state of the hurricane while conforming to one's preconceived notion must be tempered by the assumption that the pressure thickness, sea surface water temperature and tritium content was the same even though the storm was of different intensity.

Acknowledgement

The idea for this paper arose from the reading of Taylor (1967) who developed a similar relationship between the evaporation of ordinary to tritiated water. I have extended Taylor's formula specifically to hurricane evaporation and included details on the assumptions inherent in its derivation. I wish to acknowledge the assistance of Mr. R. Sheets of the National Hurricane Research Project in Miami, Florida for providing the necessary information from the hurricane reconnaissance flights on September 3rd and 5th, 1965 and for assisting in the interpretation of that information. Finally, discussions with Prof. Ostlund have been most helpful. In fact, Prof. G. Ostlund corrected an error in the analysis which overestimated the rate of evaporation by a factor of two.

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СКОРОСТИ ИСПАРЕНИЯ, ОСНОВАННЫЕ НА ИЗМЕРЕНИЯХ ТРИТИЯ ДЛЯ УРАГАНА БЕТСИ

Предполагая, что турбулентная диффузия одинакова для обычного водяного пара и для пара, содержащего тритий, можно вычислить скорость испарения обычной воды по измерениям трития, проведенным Остлундом для урагана Бетси. Скорости испарения в урагане в интервале от немногих до десяти сантиметров в день испытывают большие изменения, однако, имеют тот же порядок величины, что и найденные Малкус и Рилем. Большие скорости испарения встречаются днем, когда ураган более интенсивен, меньшие скорости имеют место, когда интенсивность урагана ослабевает.