Turbulence spectra in stable and convective layers in the free atmosphere

By L. O. MYRUP,[1] *Atmospheric Research Group and Meteorology Research Inc.*

(Manuscript received May 15, 1968, revised version October 11, 1968)

ABSTRACT

The form of the spectrum of turbulence is studied in an evolving atmospheric system consisting of a growing convective layer surmounted by an inversion layer. Aircraft measurements of the one-dimensional spectrum functions for longitudinal and vertical velocity fluctuations were made at a hieight of 1000 feet above a desert dry lake. The spectra of both velocity components exhibit the inertial subrange $-5/3$ power law within the convective layer throughout the wavelength bandwidth of the aircraft system, 6.9 to 276 meters. Within the bounding stable layer, for wavelengths greater than 69 meters, the spectra of both velocity components appear to follow a power law approaching -3, the theoretical prediction for the buoyant subrange according to Shur and Lumley.

Examination of the time series of spectral density at specific frequencies indicates approximate isotropy for these data, although occasional large deviations from isotropy are found, particularly at long wavelengths within the stable layer. The turbulence kinetic energy budget is estimated and the results support the physical models behind the buoyant and inertial subrange concepts. The bounding stable layer was found to be a region into which energy was imported from below and where energy was suppressed by the buoyancy forces associated with a downward flux of heat into the convective layer. In the region near the top of the convective layer, buoyant production of turbulence energy was found to equal the dissipation rate, as is assumed in the inertial subrange theory.

Introduction

The central subject of this paper is the form of the spectrum of turbulence in neutral and stable layers in the free atmosphere. Considering that aircraft spectral measurements have been available for almost twenty years (Clementson, 1950), surprisingly little is known of the influence of hydrostatic stability. For neutral stratification the predictions of the inertial subrange theory, as formulated by Kolmogorov (1941) and Obukhov (1941), have now been well verified in the atmosphere (MacCready, 1962), ocean (Grant *et al.*, 1962) and in laboratory systems (Gibson, 1962). In the inertial subrange, the one dimensional spectrum function, $S(k)$, is supposed to have the form

$$S(k) = a\varepsilon^{\frac{2}{3}}k^{-\frac{5}{3}}, \qquad (1)$$

where k is wavenumber, ε is the rate at which turbulent energy is dissipated to heat and a is a constant whose value has been discussed by Panofsky & Pasquill (1963).

The associated prediction of isotropy has not been as well documented as the spectral form. There is some evidence that the atmosphere is commonly anisotropic in the boundary layer (Weiler & Burling, 1967). The data in the free atmosphere, however, are fragmentary and inconclusive due to the difficulty of the measurement. The measurements to be reported here appear to help clarify the situation.

Bolgiano (1959, 1962), Shur (1962), Monin (1962) and Lumley (1964) have made theoretical studies of the form of the spectrum of turbulence to be expected for stable statification and small mean shearing motion. Bolgiano proposed the term "buoyant subrange" be applied to that portion of the spectrum in which turbulence energy is suppressed through the action of the negative buoyancy forces associated with a hydrostatically stable environment. In the buoyant subrange, turbulent eddies are sup-

[1] Now at University of California, Davis.

posed to be small enough that whatever shear is present has no influence, but still large enough that buoyancy is important.

The predictions of the various theories are of the form

$$E(k) \propto k^b, \tag{2}$$

where $E(k)$ is the three-dimensional turbulent energy spectrum and b is a constant. If the turbulence in the buoyant subrange remains approximately isotropic, despite the anisotropic influence of the stable environment, the same form would be expected for the much more easily measured, one-dimensional spectrum function, $S(k)$. In the words of Lumley (1964), such a spectrum would be thought of as slightly "leaky" in that a relatively small percentage of the turbulent energy cascading from long to small wavelengths is being removed by buoyancy at a given wavelength in the buoyant subrange. Bolgiano's theory predicts a value of $-\frac{11}{5}$ for b, while both Shur and Lumley obtain -3. Monin's theory is an interpolation scheme between Bolgiano's result and the inertial subrange value of $-\frac{5}{3}$.

In general, none of the predictions of the buoyant subrange theories have been unambiguously verified in the atmosphere. In the boundary layer, the buoyant subrange has not been found, presumably because of the importance of shear produced turbulence. In the free atmosphere, the only published measurements until recently have been those of Shur (1962) who obtained turbulence spectra in the stable environment of the jet stream at an altitude of 8000 meters. It should be noted that Shur's measurements were made by the acceleration-transform method which is considered less reliable than the techniques in which the aircraft motions are explicitly measured and removed from the data. Shur's spectra showed a transition from a Kolmogorov $-\frac{5}{3}$ slope to a -3 slope at wavelengths of about 600 meters.

Shur's results, however, have not been accepted uncritically as examples of the buoyant subrange. According to Reiter (1965), the general opinion at the International Colloquium on the Fine Structure of the Atmosphere, held in Moscow in 1965, was that the buoyant subrange, if it existed at all, should be found at shorter wavelengths. In addition, it was agreed that the most likely place to find a buoyant subrange would be in an inversion layer surmounting an adiabatic mixed layer.

Lenschow & Johnson (1968) have recently made aircraft measurements similar to those presented here. One case is shown for stable conditions in which the logarithmic slope of the spectral densities for vertical and longitudinal velocity fluctuations is about -2.5 at wavelengths longer than 200 meters. It is stated that this behavior was typical of spectra measured in stable conditions. Busch & Panofsky (1968) have presented a collection of spectra from various sources. They note that under stable conditions relatively more turbulence energy is concentrated at high frequencies than for unstable stratification and speculate that a buoyant subrange may be the explanation. No logarithmic plots of the spectra are presented to test this hypothesis, however.

The discussion at the Moscow Colloquium underlines a number of points of great significance to the study of turbulence in the free atmosphere. In general, the observational attention paid to turbulence spectra in relation to the environment and its evolution in time has been inadequate. For instance, the spectral theories discussed above are for equilibrium conditions, yet this is rarely verified in observational experiments. The environmental stability and wind shear are obviously of primary importance but often are not measured in aircraft experiments. Finally, since the theory of the spectrum of turbulence is intimately connected with the production, redistribution and dissipation of turbulent energy, it would be highly useful to form a kinetic energy budget to accompany spectral measurements. For instance, in experimental studies of the buoyant subrange, it would be desirable to demonstrate, independently of the spectra, that energy is indeed being suppressed by buoyancy. In this study we have attempted to document the environment and the physical processes occurring there, as well as measuring turbulence spectra.

Instrumentation and observational procedures

The data reported here were gathered as a phase of a larger program of research into the properties of turbulent diffusion in the free atmosphere (MacCready, Niemann & Myrup,

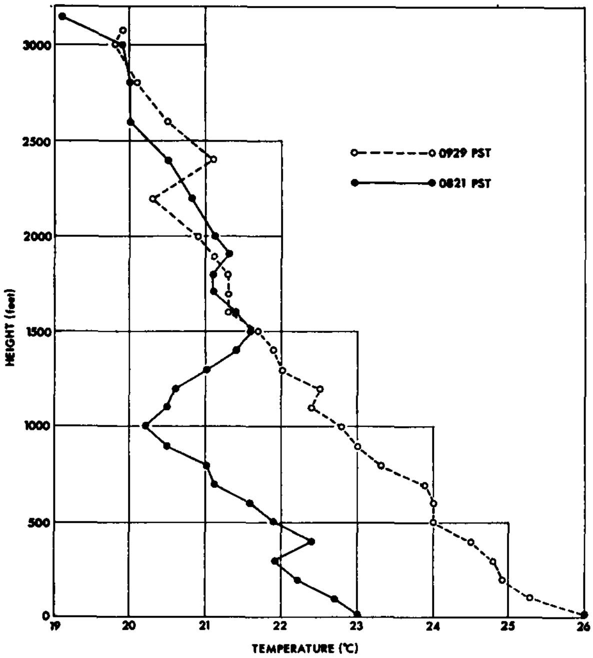

Fig. 1. Temperature soundings for 0821 PST and 0929 PST, June 25, 1966.

1967). A light aircraft was instrumented to measure, in addition to conventional meteorological parameters, longitudinal velocity fluctuations, by means of a pitot tube, and vertical velocity fluctuations by the vane-gyroscope-accelerometer technique. The latter measurement has been discussed by Telford & Warner (1962) and by Myrup (1965). The effective band width of the turbulence measurement is estimated to be from 10 to 0.25 cycles per second, which corresponds to turbulence wavelengths from 6.9 to 276 meters at the airspeed used in this study. The turbulence data were recorded on magnetic tape in frequency modulated form, and the spectral density of a velocity component was calculated in the laboratory by means of an analog filter technique. The band width of the filter was close to one octave. The instrumentation and data processing procedures are described in more detail in the first reference cited in this section and in Myrup (1968).

The aircraft, a Piper Apache, was flown at a constant altitude of 1000 feet above El Mirage dry lake in the Mojave Desert of Southern California. The horizontal traverses were approximately 5 kilometers long and were made alternately in the crosswind and downwind direction. Periodically, a temperature and turbulence sounding was made.

In the following sections, we present a quantitative analysis of the aircraft data for one observation period, the morning of 25 June, 1966. This period was selected for concentrated attention because the data were especially complete and, as we will describe below, the atmosphere above El Mirage on this day was found to be undergoing interesting physical processes of wide geophysical significance.

Meteorological conditions and observed evolution of the system

The meteorological conditions on the morning of 25 June, 1966 were typical of the El Mirage area during the summer months. It is probable that the processes described here are common to the deserts of western North America and other similar continental areas as well. Fig. 1 shows two temperature-height soundings for this period. The earlier sounding at 0821 PST shows a near-adiabatic layer, indicating effective mixing, capped by an inversion layer at 1000 feet above the lakebed. Earlier soundings (not shown) indicated a progressively shallower mixed layer. At 0929 PST, the adiabatic mixed layer had deepened to at least 2200 feet. At this time it is not clear whether the weak inversion layer at that height caps the mixing processes or whether mixing extends above the highest point of the sounding. The wind speeds at the traverse level for this period, measured by means of double phototheodolite triangulations of sounding balloons, were light, generally between one and three meters per second, and the flow was from the east. The wind field will be analyzed in more detail in a later paper, by other authors, dealing with the diffusion data which were gathered concurrently with the aircraft measurements. Dissipation soundings were also made and will be discussed below in connection with the kinetic energy budget.

The most important point established by the sounding information is that the hydrostatic stability of the traverse level (1000 feet above the lakebed) changed from stable to neutral sometime between 0800 PST and 0830 PST. Consequently, the turbulence data gathered before and after this time are ideal for the purpose of comparing the effect of neutral and stable environments. It should also be kept in mind that this is an evolving system.

Discussion of the system

Since the wind speed was small during the time period in question, the mechanism by which the adiabatic layer was mixed must have been turbulent convective motion, initiated at the rapidly warming desert surface. Thus, the physical system considered here consists of a convective mixed layer bounded by a hydrostatically stable layer. Such two-layer systems are common in nature and appear to differ significantly from the experimentally and theoretically convenient horizontal parallel plate convective system.

Two-layer or naturally bounded convective systems have attracted wide experimental and theoretical interest in recent years. Rouse and Dodu (1955), Cromwell (1960), Turner & Yang (1963), Townsend (1964), Turner (1965), and Turner & Kraus (1967) have studied laboratory models of this sytem. It has been found, in the case that turbulence is suppressed in the upper layer, that the depth of the convective layer increases continuously as fluid from the bounding layer is mixed into the convective layer. This process seems to be a special case of the general tendency of free turbulent flows to entrain bounding, non-turbulent fluid. Ball (1960), Kitaigorodsky (1960), Kraus & Rooth (1961), Veronis (1963) and Kraus & Turner (1967) have formulated theoretical models of the entrainment process. In the present absence of analytical solutions for the fully turbulent case or numerical integrations of the governing equations, heat and kinetic energy budget considerations have provided the chief theoretical guidance for all those studies, with the exception of Veronis' finite amplitude perturbation analysis. The mixing between the stable and turbulent layers, and the consequent increase of depth of the latter, has been found to involve a conversion of turbulence kinetic energy to potential energy. In the atmospheric case, there is a downward transfer of heat through the stable layer, perhaps extending well into the convective layer, which is associated with suppression of turbulence imported from below.

Processes within the bounding stable layer have been discussed by Townsend (1964, 1966), who treats a theoretical model in which a field of random internal gravity waves is formed as a result of "bombardment" by convective elements from below (in the atmospheric case). The mechanism by which internal waves transfer heat downwards, however, remains obscure.

Kinetic energy budget

As mentioned above, the turbulence kinetic energy budget provides an appropriate framework within which to present turbulence measurements. Consequently, we have attempted to evaluate the budget for this case. The budget equation is

$$\frac{\partial e}{\partial t} = \frac{gH}{\varrho C_P T} + \frac{\tau}{\varrho}\frac{\partial u}{\partial z} - \varepsilon - \frac{\partial}{\partial z}\overline{we}, \tag{3}$$

where e is the turbulence kinetic energy per unit mass, $H = \varrho C_P \overline{wT'}$ is the turbulent flux of sensible heat (positive upwards), T' is the temperature fluctuation about the mean, τ is the turbulent shear stress, u is the wind speed, ε is the dissipation rate, w is the vertical velocity, the overbar indicates an averaged quantity, and the remaining terms have their conventional meanings. This form of the budget equation presupposes horizontal homogeneity and neglects work by the pressure fluctuations and molecular transport.

The time-rate of change term in equation [3] would be expected to be maximum in the entrainment region at the interface between the two layers where we estimate $\partial e/\partial t \simeq 3$ ergs $gm^{-1}s^{-1}$ which is small relative to the dominant terms for this case. Hence, for some purposes, this sytem can still be considered in an equilibrium state. The shear production term, under these conditions could be significant only below the level of lowest aircraft measurement, 4.5 meters. In addition Hess and Panofsky (1965) and Record & Cramer (1966) have presented evidence that shear produced turbulence tends to be dissipated locally. Consequently, we shall neglect the shear production and time-dependent terms for the conditions of these measurements.

The budget equation then reduces to

$$\frac{gH}{\varrho C_P T} - \varepsilon = \frac{\partial}{\partial z}\overline{we} \tag{4}$$

which we have attempted to evaluate as a function of height for the time period of interest. An estimate of the dissipation term was available from the Universal Turbulence Indicator, manufactured by Meteorology Research Inc. (MacCready, 1964). This device essentially exploits the Kolmogorov intertial subrange prediction for the longitudinal velocity fluctuations (equation (1)). The present usefulness of the data depends on the validity of the inertial subrange formula for the band width of the instrument, which is 2 to 5 cps. We shall return to this point below when the turbulence spectra are presented and provisionally accept the dissipation data for the purposes of the current discussion.

Neither of the two remaining terms were measured directly. However, we have been able to estimate the buoyant production/suppression term from the sounding information. Assuming the mean vertical motion is zero and the radiative flux divergence is unimportant over the time period of interest, the divergence of the sensible heat flux is related to the local temperature change by

$$\frac{\partial H}{\partial z} = -\varrho C_P \frac{\partial T}{\partial t} \tag{5}$$

In finite difference form, this equation allows calculation of the heat flux divergence from any pair of soundings, thus yielding

$$\frac{\partial H}{\partial z} = f(z) \tag{6}$$

To obtain the heat flux itself we require either an upper or lower boundary condition or the mean value of the heat flux throughout the layer in question. Since we made no micrometeorological boundary layer measurements, we do not know the lower boundary condition. One approach to this problem would be to assume the heat flux is zero at the highest data level and integrate [6] downwards. Another possibility suggests itself if equation (4) is integrated from the surface to the highest data level, (h), which was 3100 feet for this case.

$$\overline{we}(h) = \frac{g}{\varrho C_P T}\int_0^h H\,dz - \int_0^h \varepsilon\,dz. \tag{7}$$

The assumption that we $(h) = 0$, i.e., that negligible turbulent energy arrives at the top of the system, allows the average heat flux throughout the system to be calculated since $\varepsilon(z)$ is assumed to be known,

$$\overline{H} = \frac{\bar{\varepsilon}\varrho C_P \overline{T}}{g}, \tag{8}$$

where the overbar indicates an average over the entire sounding. This mean heat flux provides a reference that allows integration of equation (6) to give $H(z)$ from which the buoyant production can be calculated. This procedure appears to give reasonable results in that the calculated surface heat flux is consistent with the available micrometeorological "climatology". Applied to the 0821 and 0929 PST soundings, the surface heat flux was found to be 25 m W cm^{-2}, taken as applying at about 0900 PST, which seems a reasonable value for this time of the day and year at a desert site according to the available measurements, such as those of Swinbank (1964). The assumption that the heat flux was zero at the top of the system gave a surface heat flux about 10 per cent higher. We prefer the previous approach on the grounds that it is based on more information.

With estimates available for both the dissipation and buoyant production, the remaining

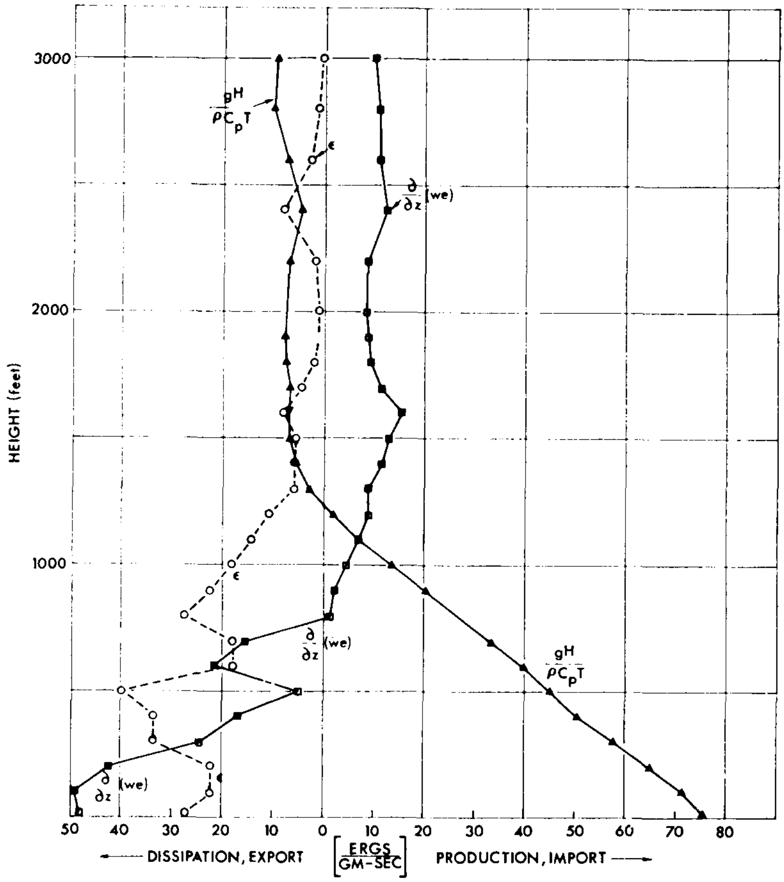

Fig. 2. Turbulence kinetic energy budget for 0821–0929 PST, June 25, 1966.

term in the budget equation (4), the turbulent export, can be computed as a residual. Fig. 2 shows the results of this analysis for the 0821 and 0929 PST soundings, taken as applying at 0855 PST. The dissipation profile is the average of the two dissipation soundings.

The kinetic energy budget calculation appears to be quite consistent with the physical models of the intertial and buoyant subranges. Below 800 feet, energy production exceeds dissipation and energy is exported aloft. At 800 feet, not far below the traverse level and at a height which was occupied by neutrally stratified air during the time period of this calculation, dissipation exactly equals production. This is one theoretical condition for the existence of a Kolmogorov equilibrium spectrum. Between 800 and 1200 feet, buoyant production plus energy import is balanced by dissipation. Above 1200 feet, where the inversion layer was being eroded away during this period, the heat flux becomes downward indicating suppression of turbulence by the buoyancy forces. This is, of course, the fundamental postulate of the buoyant subrange theories. It should be noted that neglect of the time-rate-of-change of energy does not effect this result. At most, the energy import residual would be increased by 3 ergs gm^{-1} s^{-1} above 1200 feet.

An identical analysis applied to the 0821 and 0734 PST soundings gave similar results. In particular, a downward heat flux was found in the inversion layer.

Turbulence spectra

The general nature of the turbulence spectra computed for the morning of 25 June 1966 is

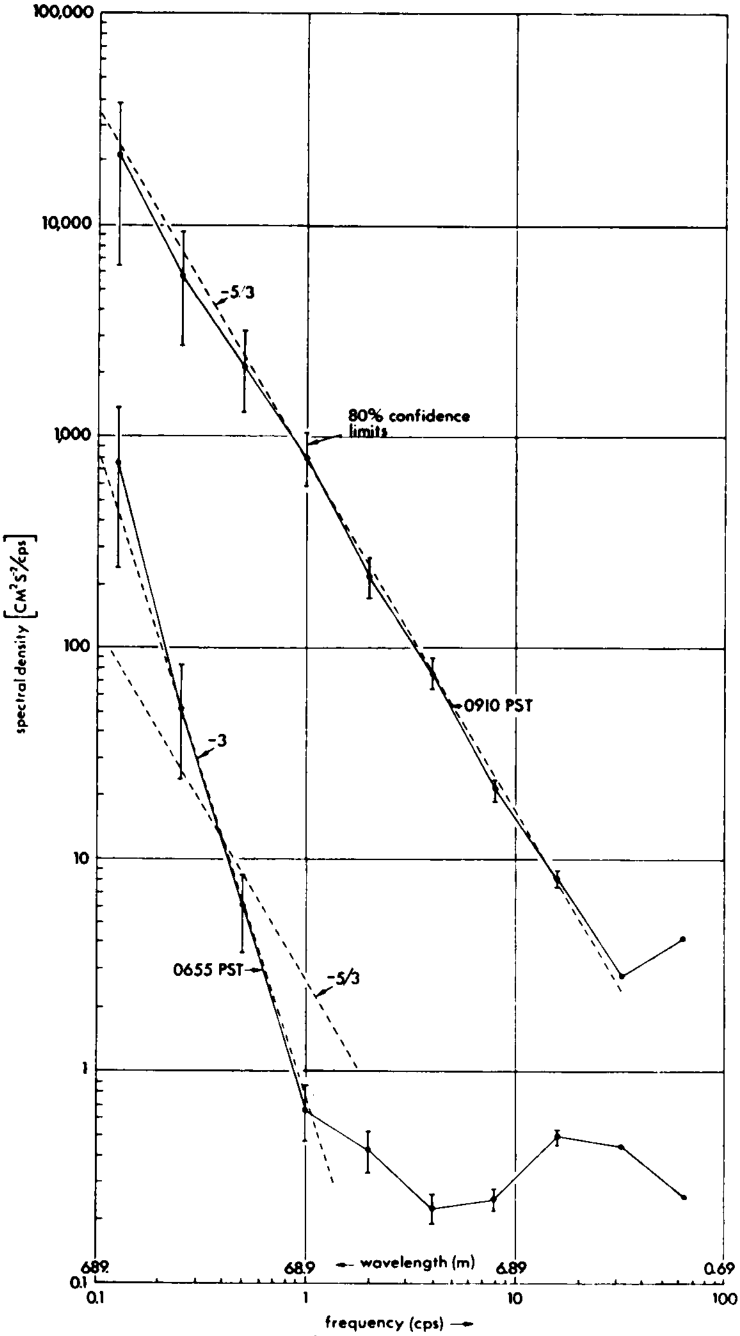

Fig. 3. Turbulence spectra for longitudinal velocity fluctuations computed for the 0655 PST and 0910 PST traverses. The aircraft was flown at an altitude of 1000 feet above a desert dry lake at a speed of 69 ms^{-1}.

illustrated by Fig. 3 which presents two longitudinal spectra for the 0655 PST and 0910 PST traverses, which represent stable and neutral conditions, respectively. It should be noted that "longitudinal" is used here to indicate the velocity component in the direction of aircraft motion and hence these longitudinal spectra include data from both downwind and cross-wind traverses. These spectra are given in terms of frequency. The wave number, k, which appears in the spectral theories is related to frequency, f, through the airspeed, V_a, by the formula,

$$k = \frac{f}{V_a}. \tag{9}$$

Hence, the logarithmic slope of spectra plotted as functions of frequency can be interpreted in terms of the exponent, b, in predictions for the spectral form in the buoyant subrange (equation (2)).

The turbulence level for the earlier of the two traverses was close to the lowest found in these data. For this reason, the system noise is unusually evident on this particular spectrum. The energy hump at 16 to 32 cps is due to boom vibration excited by the aircraft engine. The noise level of the recorder appears to have been about 0.1 spectral units and the 0655 PST spectrum is contaminated by system noise for frequencies greater than one cps. For lower frequencies, the spectral density appears to rise out of the noise and takes in a slope close to -3, the theoretical value for the buoyant subrange theories of Lumley and Shur. It is the significance of this slope with which we are chiefly concerned in the remainder of this paper.

The turbulence level for the 0910 PST traverse is about two orders of magnitude higher. Consequently, the spectrum is affected by engine noise only for frequencies above 32 cps. The spectrum can be seen to follow the inertial subrange $-\frac{5}{3}$ law strikingly well. We feel that this result, which was typical of all neutral spectra, constitutes evidence that the aircraft turbulence and data analysis systems were reliable and gave consistent results.

Also shown are 80 per cent confidence limits, calculated from a chi square distribution according to Blackman & Tukey (1958) for the filter bandwidth (one octave) and record length employed in the data analysis (one minute). It is apparent that the 80 per cent limits become rather wide for the lower frequencies of particular interest, and other slopes than those indicated are reasonably possible.

These statistical confidence calculations appear to decrease the significance one can attach to the low frequency logarithmic slopes on the experimental spectra. However, there is little reason to believe that the sampling theory applied in calculating the confidence limits is valid for the conditions of these data. The theory assumes that individual spectral estimates are distributed about the true spectra (i.e., one derived from an infinite time series) according to a Gaussian distribution and that the time series is stationary. The second assumption does not hold for our case. In addition, the basic reason for resorting to confidence calculations is to estimate the variability one might expect if a series of spectral estimates were to be made from various sections of a time series. In the usual case, this is not possible, so the confidence theory may be useful. However, our data essentially consists of a series of spectral estimates from a long time series, and we may judge our results directly. In the words of Blackman and Tukey (1958, pg. 5, Dover edition), "The precision of final overall values is ordinarily far more wisely

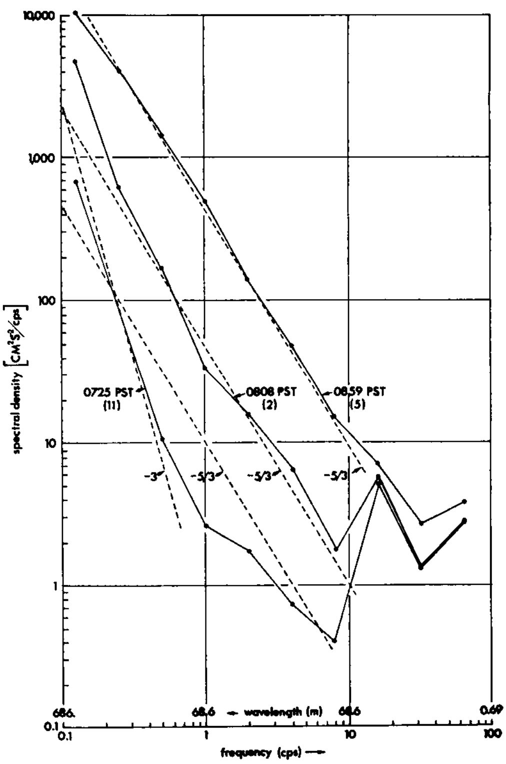

Fig. 4. Averaged aircraft turbulence spectra for longitudinal velocity fluctuations.

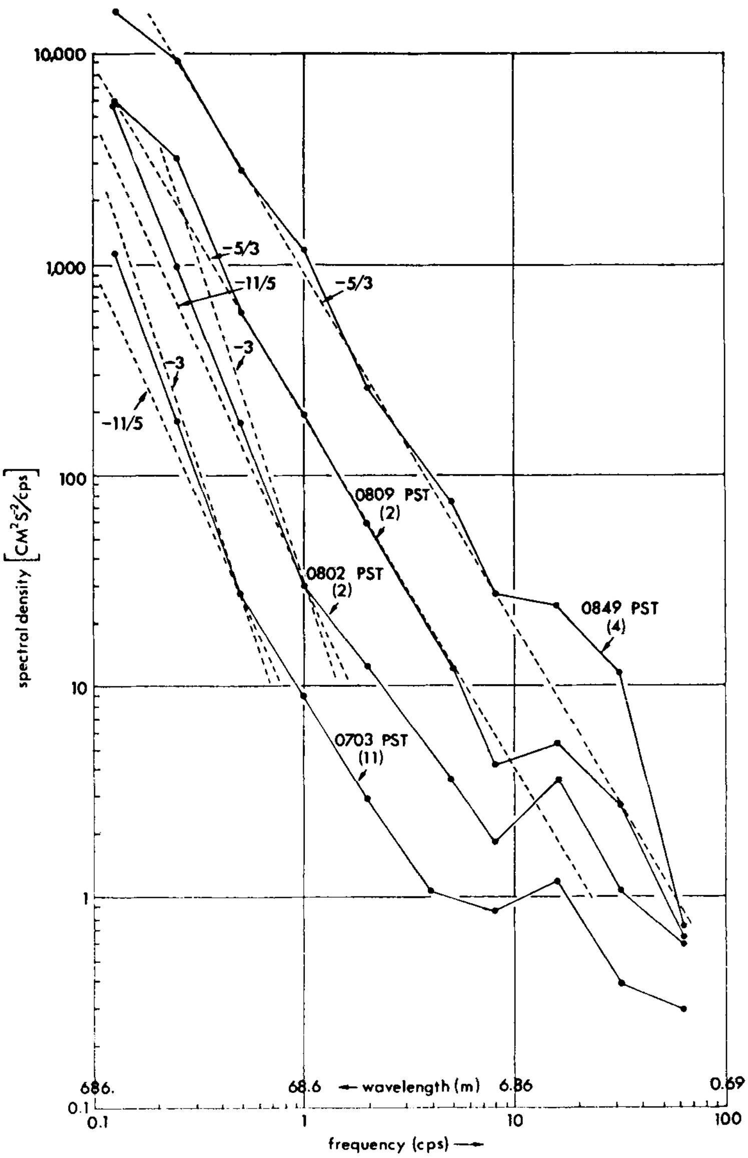

Fig. 5. Averaged aircraft turbulence spectra for vertical velocity fluctuations.

judged from the observed consistency of repeated measurements ... than from any theoretical variability based on a Gaussian assumption."

The two spectra displayed in Fig. 3 were found to be quite typical of all spectra computed for this period. The majority of all stable spectra (for traverses made before 0800 PST) showed logarithmic slopes considerably steeper than $-\frac{5}{3}$ for frequencies less than one cps (wavelengths greater than 69 meters). All netural spectra (for traverses after 0830 PST), on the other hand, showed slopes close to $-\frac{5}{3}$ for frequencies between 8 and 0.25 cps.

In order to clarify the sequence in time, we have arranged the individual spectra into time groups and calculated averaged spectra for each group. Fig. 4 shows the result for the longitudinal velocity fluctuations. Each curve is labeled with a time which is the average of the component observation times, and the number in parentheses is the number of individual spectra included in the group. The 0725 PST group includes all available stable cases and the 0859 PST group includes all available netural cases. The remaining 0808 PST group is made up of two transitional spectra.

It can be seen that the averaged longitudinal spectra are quite similar to the previously displayed individual spectra. The aircraft noise at 16 cps is more apparent on the averaged 0725 PST spectrum than on the 0655 PST case. This seems to be associated with the fact that the vibration of the nose boom, on which the basic sensors were mounted, tended to be intermittent in low turbulence situations (perhaps excited by the turbulence to some extent), and the 0655 PST spectrum was an example of small boom vibration. As before, the logarithmic slopes of the stable and neutral spectra are close to -3 and $-\frac{5}{3}$, respectively, for the same frequency ranges as found in the individual spectra. It should be particularly noted that the spectral region of steepened slope is far removed, *relative to the filter bandwidth of one octave*, from the aircraft vibration band. The transitional 0808 PST spectrum appears to be in the process of establishing the inertial subrange $-\frac{5}{3}$ slope.

In section 5 we used dissipation data based essentially on the assumption that the inertial subrange model was valid for frequencies of 2 cps and above. We cannot conclusively justify this assumption since the stable spectra do not clearly display a $-\frac{5}{3}$ region. However, the steepened buoyant subrange slope does not begin until one cps and it is physically reasonable that an inertial subrange would exist at higher frequencies. Consequently, we shall allow the dissipation calculations to stand as reasonable estimates.

Fig. 5 shows the result of the same analysis for the spectra of the independently measured vertical velocity fluctuations. The conventions are as before. The same general features are evident as were noted for the longitudinal spectra. The stable 0703 PST group and the earlier of the two transitional groups (0802 PST) both show a logarithmic slope between $-\frac{11}{5}$ (the Bolgiano prediction), and -3 for frequencies less than one cps. The 0849 PST neutral group, on the other hand, shows good agreement with the intertial subrange $-\frac{5}{3}$ slope. The transitional spectra are separated into two groups, 0802 PST and 0809 PST, in order to illustrate the rapidity with which the spectral regime changed. The 0809 PST spectrum already displays an extensive $-\frac{5}{3}$ region.

As described above, the aircraft traverses were alternately flown in the downwind and crosswind directions. Separation of the spectra into downwind and crosswind groups showed no significant dependency on the traverse direction. The energy levels were approximately the same and all of the features discussed above were unchanged.

The variation in time of these spectra is particularly significant to their interpretation. To investigate the time dependency further, we have plotted the spectral density at various frequencies as a function of time. Fig. 6 shows this analysis for 4 cps (wavelength of 17 meters). The spectral density of the longitudinal fluctuations, $S_u(f)$, and $\frac{3}{4}$ times the spectral density of the vertical velocity, $S_w(f)$, are shown. In isotropic turbulence these functions would be equal. Also shown are the 80 per cent confidence limits discussed above.

This figure illustrates the dramatic increase in the energy level of the turbulence which occurred as the convective layer rose through the traverse level. The increase was at least two orders of magnitude. In general, the spectral density of the vertical component appears to be more irregular than the longitudinal component at this frequency. The increase in the energy level for the longitudinal fluctuations occurs more smoothly.

Within the accuracy of the measurements, these curves indicate that the turbulence at this wavelength was approximately isotropic. The period between 0730 PST and 0800 PST, just before the arrival of the convective layer, in which the longitudinal energy is consistently larger than the vertical, may be a significant exception. It should also be noticed that the most rapid increase in turbulence energy occurs just after 0800 PST. This is precisely the time when a sudden change from a steepened slope to the $-\frac{5}{3}$ slope was noted in the spectra presented above.

Fig. 7 shows the same analysis for the spectral density at 0.25 cps (wavelength of 276 meters). The same large increase in turbulence energy can be seen. The spectral density of the vertical component is even more irregular and pulse-

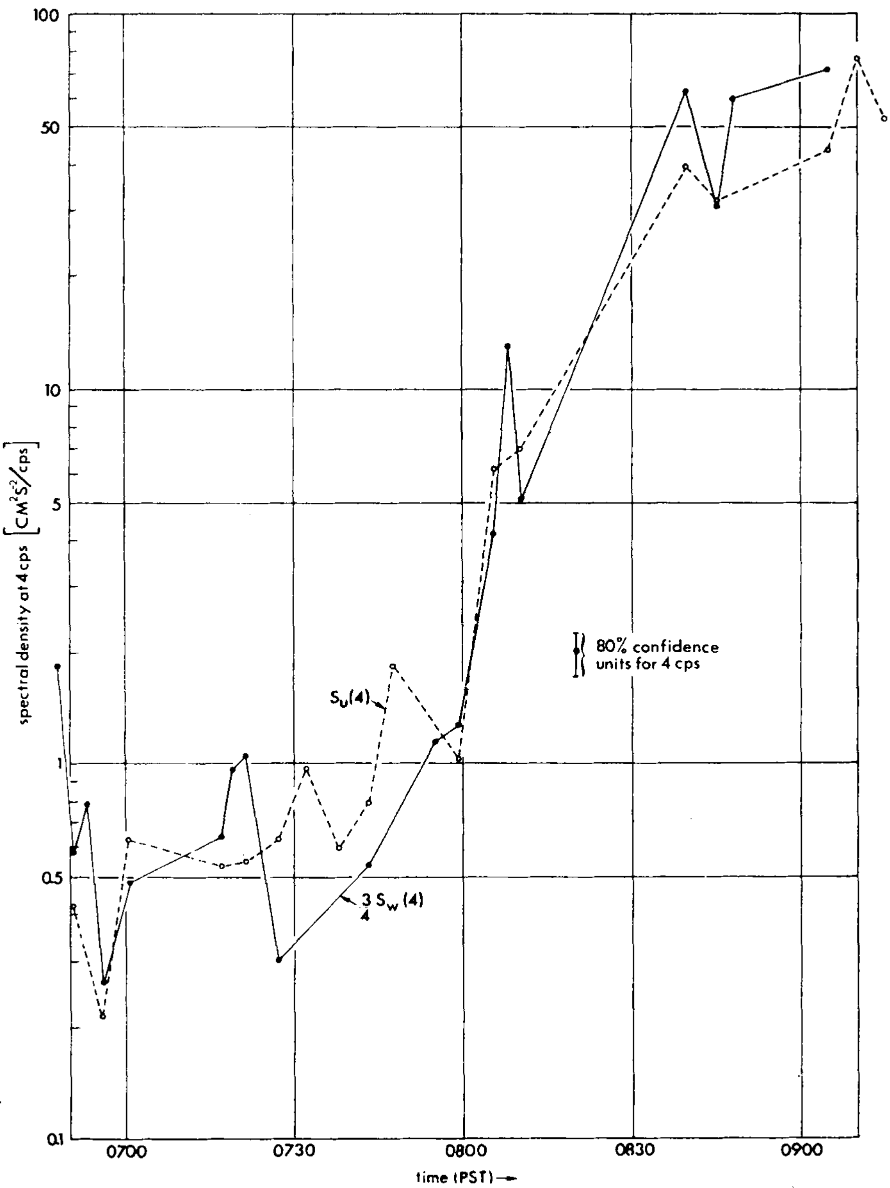

Fig. 6 .Time variation of spectral density of longitudinal and vertical velocity fluctuations at 4 cps, corresponding to a 17 meter turbulence wave length.

like than for 4 cps. Quite large, order of magnitude excursions above the level of the longitudinal energy occur. This phenomenon may be connected with the more rigorous sampling requirements for this lower frequency, as indicated by the wider 80 per cent confidence limits. However, these results are also consistent with a physical model of turbulence energy arriving from below in bursts of low frequency, vertically oriented energy which then gradually becomes more isotropic over a time period of the order of 15 minutes.

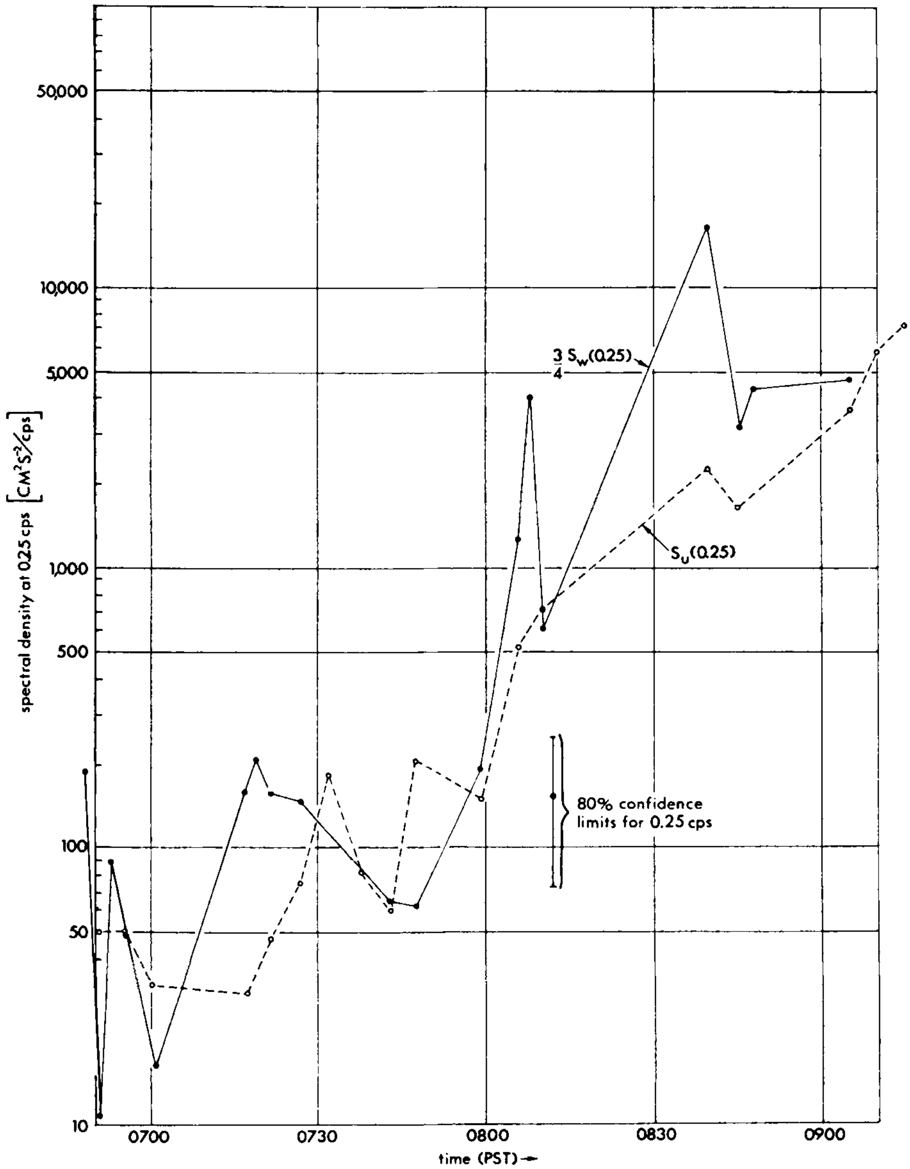

Fig. 7. Time variation of spectral density of longitudinal and vertical velocity fluctuations at 0.25 cps, corresponding to a 276 meter turbulence wave length.

Concluding remarks

In general, these results seem remarkably coherent considering the aircraft instrumentation was a relatively modest effort. Probably the turbulence spectra, considered alone, would not seem particularly strong evidence concerning the effect of stability on spectral form. However, the close correlation with the supporting meteorological data greatly strengthens the case. We feel this analysis illustrates the importance of giving careful consideration to meteorological information when making and analyzing aircraft turbulence measurements.

Acknowledgement

I wish to thank Dr. P. B. MacCready and B. L. Niemann for many helpful suggestions.

This program is supported by the Public Health Service, U.S. Department of Health, Education and Welfare, Grant AP 00359-031, to Atmospheric Research Group.

REFERENCES

Ball, F. K., 1960. Control of Inversion Height by Surface Heating. *Q. J. Roy. Met. Soc. 86*, 483–494.

Blackman, R. B. & J. W. Tukey, 1958. *The Measurement of Power Spectra.* Dover Pub. Inc., New York.

Bolgiano, R., 1959. Turbulence Spectra in a Stably Stratified Atmosphere. *J. Geophys. Res. 64*, 2226–2229.

Bolgiano, R., 1962. Structure of Turbulence in a Stratified Media. *J. Geophys. Res. 67*, 3015–3023.

Busch, N. E. & H. A. Panofsky, 1968. Recent Spectra of Atmospheric Turbulence. *Quart. Jour. Roy. Med. Soc. 94*, 132–148.

Clementson, G. C., 1950. *An Investigation of the Power Spectral Density of Atmospheric Turbulence.* D. Sc. Thesis, Mass. Inst. Tech., Rep. No. 6445-T-31 ATI-80319.

Cromwell, T., 1960. Pycnoclines Created by Mixing in an Aquarium Tank. *J. Mar. Res. 18*, 73–82.

Gibson, M. N., 1962. Spectra of Turbulence in a Round Jet. *J. Fluid Mech. 15*, pt 2, 161–173.

Grant, H. L., R. W. Stewart & A. Moilliet, 1962. Turbulence Spectra from a Tidal Channel. *J. Fluid Mech. 12*, 241–263.

Hess, G. D. & H. A. Panofsky, 1966. The Budget of Turbulent Energy Near the Ground. *Q. J. Roy, Met. Soc. 92*, 277–280.

Kitaigorodsky, S. A., 1960. On the Computation of the Thickness of the Wind-Mixing Layer in the Ocean. *Izv. Geophys. Ser. Acad. Sci.*, U.S.S.R., 3, 284–287.

Kolmogorov, A. N., 1941. The Local Structure of Turbulence in an Incompressible Viscous Fluid for Very Large Reynolds Numbers. *C. R. Acad. Sci.*, U.S.S.R. 30, 301–335.

Kraus, E. B. & C. Rooth, 1961. Temperature and Steady State Vertical Heat Flux in the Ocean Surface Layers. *Tellus 13*, 231–238.

Kraus, E. B. & J. S. Turner, 1967. A One-Dimensional Model of the Seasonal Thermocline. II. The General Theory and Its Consequences. *Tellus 19*, 98–106.

Lenschow, D. H. & W. B. Johnson, 1968. Concurrent Airplane and Balloon Measurements of Atmospheric Boundary-Layer Structure Over a Forest. *Jour. Appl. Met. 7*, 79–89.

Lumley, J. L., 1964. The Spectrum of Nearly Inertial Turbulence in a Stably Stratified Fluid. *J. Atm. Sci. 21*, 99–102.

MacCready, P. B., 1962. The Inertial Subrange of Atmospheric Turbulence. *J. Geophys. Res. 67*, 1051–1059.

MacCready, P. B., 1964. Standardization of Gustiness Values from Aircraft. *J. Appl. Meteor. 3*, 439–449.

MacCready, P.B., B. L. Niemann & L. O. Myrup 1967. *Atmospheric Diffusion in the Inertial Subrange.* Rep. ARGG71R-478, Atmospheric Research Group, Altadena, California.

Monin, A. S., 1962. Turbulence Spectrum in a Thermally Stratified Atmosphere. *Izv. Geophys., Ser. Acad. Sci.*, U.S.S.R., 3, 397–407.

Myrup, L. O., 1965. *The Structure of Thermal Convection in the Lower Atmosphere Under Conditions of Light Winds and Strong Surface Heating.* Ph. D. Dissertation, Dept. Meteorology, University of California, Los Angeles.

Myrup, L. O., 1968. Atmospheric Measurements of the Buoyant Subrange of Turbulence. *J. Atm. Sci.*, 25, 1160–1164.

Obukhov, A. M., 1941. On the Distribution of Energy in the Spectrum of Turbulent Flow. *C. R. Acad. Sci.*, U.S.S.R. 32, 19.

Panofsky, H. A. & F. Pasquill, 1963. The Constant of the Kolmogorov Law. *Q. J. Roy. Met. Soc. 89*, 550–51.

Record, F. A. & H. E. Cramer, 1966. Energy Dissipation Rates and the Budget of Turbulent Kinetic Energy in the Stmospheric Surface Layer. *Q. J. Roy. Met. Soc. 92*, 519–532.

Reiter, E. R., 1965. The Fine-Scale Structure of the Atmosphere. Colorado State University, *Atm. Sci. Misc. Rep.*

Rouse, H. & J. Dodu, 1955. Turbulent Diffusion Across a Density Discontinuity. *La Houille Blanche, 10*, No. 4, 522–532.

Shur, G. N., 1962. Experimental Investigations of the Energy Spectrum of Atmospheric Turbulence. *Trudy T. S.; A. O. 43*, 79–90.

Swinbank, W. C., 1964. The Exponential Wind Profile. *Q. J. Roy. Met. Soc. 90*, 119–135.

Telford, J. W. & J. Warner, 1962. On the Measurement From an Aircraft of Buoyancy and Vertical Air Velocity in Clouds. *J. Atm. Sci. 19*, 415–423.

Townsend, A. A., 1964. Natural Convection in Water Over an Ice Surface. *Q. J. Roy. Met. Soc.. 90*, 248–259.

Townsend A. A., 1966. Internal Waves Produced by a Convective Layer. *J. Fluid Mech. 24*, Pt. 2, 307–319.

Turner, J. S. & I. K. Yang, 1963. Turbulent Mixing at the Top of Stratocumulus Clouds. *J. Fluid Mech. 17*, Pt. 2, 212–224.

Turner, J. S., 1965. The Coupled Turbulent Transports of Salt and Heat Across a Sharp Density Interface. *Int. J. Heat, Mass Transfer 8*, 759–767.

Turner, J. S. & E. B. Kraus, 1967. A One-Dimensional Model of the Seasonal Thermocline. I. A Laboratory Model and Its Inrerpretation. *Tellus 19*, 88–97.
Veronis, G., 1963. Penetrative Convection. *Astrophys. J. 137*(*2*), 641–663.
Weiler, H. S. & R. W. Burling, 1967. Direct Measurements of Stress and Spectra of Turbulence in the Boundary Layer Over the Sea. *J. Atm. Sci. 24*, 653–664.

СПЕКТРЫ ТУРБУЛЕНТНОСТИ В УСТОЙЧИВЫХ И КОНВЕКТИВНЫХ СЛОЯХ В СВОБОДНОЙ АТМОСФЕРЕ

Исследуется форма спектров турбулентности в развивающейся атмосферной системе, состоящей из растущего конвективного слоя, над которым находится слой инверсии. Были проведены самолетные измерения одномерных спектров для продольной и вертикальной компонент скорости на высоте 300 *м* над сухим озером в пустыне. Спектры обеих компонент скорости имеют инерционный интервал, «закон степени — 5/3», внутри конвективного слоя в интервале масштабов от 6,9 до 276 *м*. Внутри устойчивого слоя для длин волн, больших 69 *м*, спектры обеих компонент приближаются к степенному закону с показателем −3, который был предсказан для интервала действия сил плавучести Шуром и Ламли.

Исследование временного ряда для спектральной плотности на определенных частотах указывает на существование приблизительной изотропии для этих данных, хотя встречаются иногда и большие отклонения от изотропии, особенно для больших масштабов внутри слоя устойчивости. Оценивается баланс кинетической энергии турбулентности, результаты которого свидетельствуют в пользу физических моделей интервалов инерционного и действия сил плавучести. Найдено, что верхний инверсионный слой является областью, куда энергия притекает снизу и где она подавляется силами плавучести, связанными с потоком тепла вниз в конвективный слой. В области вблизи верхней границы конвективного слоя генерация турбулентной энергии силами плавучести оказывается равной скорости ее диссипации, как это и предполагается в теории инерционного подинтервала.