Symmetrical formulation of the zonal kinetic energy equation

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ABSTRACT

An equation for the kinetic energy balance of the mean zonal flow in a symmetrical polar cap is derived under minimal assumptions, starting from basic concepts of mechanics. The form of the equation obtained differs from the comparable result of the traditional treatment, in that the rotation of the relative coordinate system used enters the volume integrals in a more symmetrical fashion, and appears also in the surface boundary terms. Various aspects of a still less restrictive possible formulation are enumerated.

Introduction

The equation for the kinetic energy balance of the zonally averaged zonal flow in the atmosphere has been written in slightly differing forms by several authors, mainly for the purpose of investigating the processes of the general circulation from observational material. As examples one may cite the discussions by Kuo (1951), van Mieghem (1956), Saltzman (1957) and Lorenz (1967). A generalized form including hydromagnetic effects is given by Starr and Gilman (1966). In the present treatment the equation appears in a form rather different from those given by these several investigators. As is shown later the new form is mathematically equivalent to the result obtained through the use of more customary methods of derivation. In a rather general sense, the procedures utilized in the present investigation follow along lines used mainly by Starr in the derivation of various integral properties of surface waves in a fluid. For a resume and references concerning this subject see Wehausen & Laitone (1960), and also Starr (1961) for later work.

Some effort is made in what follows to eliminate certain approximations commonly used in dealing with the present subject. On the other hand it is assumed, as in most other presentations, that the horizontal and temporal variations of the density can be neglected for the present purposes. The inclusion of these density variations entails a major complication of the treatment and produces a number of additional terms which for the most part are of secondary significance in the present connection. The writers therefore relegate their inclusion to possible later discussion at some future time. On the other hand, the terms retained are sufficient to exhibit the novel characteristics of the approach here developed.

Notation and basic equation

Use is made of a (rotating) spherical polar coordinate system (R, λ, ϕ), the symbols, in order, denoting radius, longitude and (geocentric) latitude. The absolute angular momentum of a unit mass of fluid about the polar axis is denoted by M, so that one may write

$$M = r^2(\Omega + d\lambda/dt) \equiv \Omega r^2 + ru \tag{1}$$

where r is the perpendicular distance to the polar axis, $R \cos \phi$. The coordinates are assumed to rotate about the polar axis at a constant rate Ω, so that, if t is time, $d\lambda/dt$ is the individual rate of increase of longitude for a fluid particle. The longitude is assumed to increase eastward in atmospheric applications. It follows then that $u = r\, d\lambda/dt$ is the linear eastward particle velocity component relative to the coordinate system.

The fundamental equation of motion expressing the inertia principle of Newtonian mechanics may, for present purposes, be written in terms of this symbolism in the form

$$\varrho \frac{dM}{dt} = -\frac{\partial p}{\partial \lambda} + rF \tag{2}$$

It expresses the fact that mass per unit volume ϱ when multiplied by the individual time rate of increase of angular momentum is equal to the sum of the torques per unit volume. Only two torques are included in (2), namely, the one due to the pressure p and the one due to friction-F per unit volume. These two effects are the major actions of this kind in the atmosphere of the earth. As was stated we assume that $\varrho = \varrho(R)$.

Since the objective is to study the kinetic energy of the zonally averaged zonal flow, it is necessary to define the averages involved in this quantity as well as in similar other quantities arising in the development. We define the square brackets as follows for this purpose

$$[(\ )] \equiv \frac{1}{2\pi} \oint (\ )\, d\lambda \tag{3}$$

where the cyclic integral is taken along a path having a constant radius and latitude. Henceforth the use of square braccets is reserved exclusively for purposes defined by (3). The integration with respect to λ is assumed to extend over a range of 2π, thus implying the tacit assumption that in applications to the earth's atmosphere we leave out of account all interruptions of the path of integration due to orographic barriers. It is thus assumed that the earth is smooth in this regard, although no restriction is made as to the absence of surface friction.

Equation (3) may be utilized to define the spatial deviation of a quantity $(\ )'$ from its zonal mean, so that

$$(\ ) \equiv [(\ )] + (\ )' \tag{4}$$

Preliminary manipulations

In virtue of the general mass continuity equation for a sourcefree region in a fluid, namely

$$\frac{\partial \varrho}{\partial t} + \nabla \cdot \varrho \mathbf{v} = 0 \tag{5}$$

where $\mathbf{v}$ is the vector velocity, one may rewrite the product of the density and the individual time rate of change of a quantity in the following fashion

$$\varrho \frac{d(\ )}{dt} = \frac{\partial}{\partial t}\{\varrho(\ )\} + \nabla \cdot \varrho(\ )\mathbf{v} \tag{6}$$

making use of Euler's relation between individual and partial time derivatives. By this means the basic equation (2) may be rewritten as

$$\frac{\partial \varrho M}{\partial t} + \nabla \cdot \varrho M \mathbf{v} = -\frac{\partial p}{\partial \lambda} + rF \tag{7}$$

This is the dynamic balance equation for absolute angular momentum.

The relative angular momentum may be introduced into the first term on the left of (7) by substitution from (1). Thus, noting that ϱ, Ω and r are independent of t, the result is gained that

$$\frac{\partial}{\partial t}\{\varrho r u\} + \nabla \cdot \varrho M \mathbf{v} = -\frac{\partial p}{\partial \lambda} + rF \tag{8}$$

Applying (4) to u in (8) it follows that

$$r\frac{\partial \varrho [u]}{\partial t} + r\frac{\partial \varrho u'}{\partial t} + \nabla \cdot \varrho M \mathbf{v} = -\frac{\partial p}{\partial \lambda} + rF \tag{9}$$

This is a balance equation giving the time rate of change of the relative angular momentum of the zonally averaged eastward motion, $r\varrho[u]$.

Formation of the energy equation

The differential form of the energy equation may be formed by multiplying (9) by $[u]$. Through further appeal to the assumptions made, this equation may be written as

$$\frac{\partial}{\partial t}\left\{\frac{\varrho [u]^2}{2}\right\} + [u]\frac{\partial \varrho u'}{\partial t} + \frac{[u]}{r}\nabla \cdot \varrho M \mathbf{v} + \frac{[u]}{r}\frac{\partial p}{\partial \lambda} - [u]F = 0 \tag{10}$$

It is to be noted that in (10) the quantity $\varrho[u]^2/2$ is the kinetic energy per unit volume of the zonally averaged zonal velocity component, while $[u]/r$ is the mean relative angular velocity or differential rotation with respect to the rotating coordinate system (R, ϱ, ϕ).

It is desirable to take the volume integral of (10) and choose the limits of integration as surfaces of revolution about the polar axis, so that the volume considered is bounded by the earth's surface, by a conical surface of constant latitude ϕ_1 and by a surface of constant $R = R_1$ at the top. The region thus consists of a polar cap extending down to latitude ϕ_1. One may note that no assumption has been made as to the precise sphericity of the (smooth) earth's surface which might even be pear-shaped so far as present considerations go. Of course, for most purposes the boundary at the surface may be taken as being essentially normal to the R direction.

Since the volume element to be used $d\tau = R^2 \cos\phi \, dR \, d\lambda \, d\phi$ involves a partial integration with respect to λ whose range is always 2π in the region chosen, the second and fourth terms in (10) will automatically integrate to zero for reasons which are obvious. We are thus left with the energy equation in the integral form

$$\frac{d}{dt}\int_\tau \varrho \frac{[u]^2}{2} d\tau + \int_\tau \frac{[u]}{r} \nabla \cdot \varrho M \mathbf{v} \, d\tau - \int_\tau [u] F \, d\tau = 0 \quad (11)$$

The middle term on the left of (11) may be rearranged by parts so as to yield an integrable portion by taking the angular velocity into the divergence operator. The result may then be expressed with the aid of Green's theorem in the form

$$\frac{d}{dt}\int_\tau \varrho \frac{[u]^2}{2} d\tau + \int_S \frac{[u]}{r} \varrho M C_n \, dS - \int_\tau \varrho M \mathbf{v} \cdot \nabla \left\{\frac{[u]}{r}\right\} d\tau - \int_\tau [u] F \, d\tau = 0 \quad (12)$$

Here dS is an element of the (closed) boundary surface S of the volume, and C_n is the outward normal velocity component measured at the boundary.

The two alternative statements of the energy equation expressed by (11) and (12) contain the essence of the results. The first of these contains, in the middle term, the concept of what may be called the *appearance* of zonal kinetic energy which is governed by the product of the relative angular velocity and the divergence of the transport of absolute angular momentum. It does not, however, furnish direct information as to what fraction of this action in an open region may be due to boundary transfer effects. The second, namely (12), incorporates such a separation, so that what may here for convenience be termed the *generation* consists of the product of the absolute angular momentum transport and the gradient of the mean angular velocity. The meaning of the remaining terms in (11) and (12) is fairly clear.

Further expansion of the energy equation

In order to render it more readily suited for such purposes as the interpretation of observational data, it is useful to expand the compact equation (12) to make it explicitly applicable to the polar cap region extending from the north pole to the arbitrary latitude ϕ_1. In this expansion it is useful, on the one hand, to separate horizontal and vertical processes, and on the other, to separate angular momentum transports into three categories, namely, (*a*) those associated with Ω, (*b*) those associated with the relative zonal motion $[u]$, and (*c*) those associated with the eddy action $[u'v']$. Here v denotes northward particle velocity. In this manner the generation integral will be broken up into six separate integrals, as will also the boundary integral when the southern boundary and top boundary are considered.

From what has just been said and from the several relations and definitions already given, the generation term may be rearranged progressively as follows, the radial velocity component being indicated by w.

$$\int_\tau \varrho M \mathbf{v} \cdot \nabla \left\{\frac{[u]}{r}\right\} d\tau = \int_\tau \varrho M v \frac{\partial}{R\partial\phi}\left\{\frac{[u]}{R\cos\phi}\right\} d\tau + \int_\tau \varrho M w \frac{\partial}{\partial R}\left\{\frac{[u]}{R\cos\phi}\right\} d\tau$$

$$= \iint \varrho \frac{\partial}{R\partial\phi}\left\{\frac{[u]}{R\cos\phi}\right\} R^2 \cos\phi \left(\oint M v \, d\lambda\right) d\phi \, dR + \iint \varrho \frac{\partial}{\partial R}\left\{\frac{[u]}{R\cos\phi}\right\} R^2 \cos\phi \left(\oint M w \, d\lambda\right) d\phi \, dR \quad (13)$$

The first inner cyclic integral may be expanded as follows

$$\oint Mv\, d\lambda = \oint (\Omega r^2 + ur) v\, d\lambda$$

$$= \oint r(\Omega r + [u] + u')([v] + v')\, d\lambda$$

$$= 2\pi\Omega r^2[v] + 2\pi r[u][v] + 2\pi r[u'v'] \quad (14)$$

In exactly similar fashion we may write that the second cyclic integral in (13)

$$\oint Mw\, d\lambda = 2\pi\Omega r^2[w] + 2\pi r[u][w] + 2\pi r[u'w'] \quad (15)$$

Substitution from (14) and (15) into (13) and finally into (12) leads to the final result, but before doing so the boundary integral in (12) will be similarly expanded.

It is assumed that the normal velocity at the bottom boundary at $R = R_0(\phi)$ of the polar cap is zero, so that the remaining surfaces are (in order) the southern latitude wall and the top at a fixed value of $R = R_1$. Accordingly, one may write that

$$\int_S \frac{[u]}{r} \varrho M C_n\, dS =$$

$$- \int_{R_0(\phi)}^{R_1} \varrho \frac{[u]}{r} R \cos \phi_1 \left(\oint Mv\, d\lambda \right) dR$$

$$+ \int_{\phi_1}^{\pi/2} \varrho \frac{[u]}{r} R_1^2 \cos \phi \left(\oint Mw\, d\lambda \right) d\phi \quad (16)$$

The cyclic integrals with respect to λ may again be expanded by using (14) and (15). With the further aid of the definitions and notations already used equation (12) may now be rewritten in the form

$$\frac{d}{dt} \int_\tau \varrho \frac{[u]^2}{2}\, d\tau = + \iint 2\pi [u][F] R^2 \cos \phi\, d\phi\, dR$$

Internal horizontal integrals

$$+ \iint 2\pi\varrho R^2 \cos^2 \phi (\Omega R \cos \phi)[v] \frac{\partial}{R\partial\phi} \left\{ \frac{[u]}{R \cos \phi} \right\} R\, d\phi\, dR \quad \{1\}$$

$$+ \iint 2\pi\varrho R^2 \cos^2 \phi [u][v] \frac{\partial}{R\partial\phi} \left\{ \frac{[u]}{R \cos \phi} \right\} R\, d\phi\, dR \quad \{2\}$$

$$+ \iint 2\pi\varrho R^2 \cos^2 \phi [u'v'] \frac{\partial}{R\partial\phi} \left\{ \frac{[u]}{R \cos \phi} \right\} R\, d\phi\, dR \quad \{3\}$$

Internal vertical integrals

$$+ \iint 2\pi\varrho R^2 \cos^2 \phi (\Omega R \cos \phi)[w] \frac{\partial}{\partial R} \left\{ \frac{[u]}{R \cos \phi} \right\} R\, d\phi\, dR \quad \{4\}$$

$$+ \iint 2\pi\varrho R^2 \cos^2 \phi [u][w] \frac{\partial}{\partial R} \left\{ \frac{[u]}{R \cos \phi} \right\} R\, d\phi\, dR \quad \{5\}$$

$$+ \iint 2\pi\varrho R^2 \cos^2 \phi [u'w'] \frac{\partial}{\partial R} \left\{ \frac{[u]}{R \cos \phi} \right\} R\, d\phi\, dR \quad \{6\}$$

Vertical boundary integrals at $\phi = \phi_1$

$$+ \int 2\pi\varrho R^2 \cos^2 \phi_1 (\Omega R \cos \phi_1)[v] \frac{[u]}{R \cos \phi_1}\, dR \quad \{7\}$$

$$+ \int 2\pi\varrho R^2 \cos^2 \phi_1 [u][v] \frac{[u]}{R \cos \phi_1}\, dR \quad \{8\}$$

$$+ \int 2\pi\varrho R^2 \cos^2 \phi_1 [u'v'] \frac{[u]}{R \cos \phi_1}\, dR \quad \{9\}$$

Horizontal boundary integrals at $R = R_1$

$$\left\{\begin{array}{ll} -\int 2\pi\varrho R_1^2 \cos^2\phi(\Omega R_1 \cos\phi)[w]\frac{[u]}{R_1\cos\phi}R_1\,d\phi & \{10\} \\ -\int 2\pi\varrho R_1^2 \cos^2\phi[u][w]\frac{[u]}{R_1\cos\phi}R_1\,d\phi & \{11\} \\ -\int 2\pi\varrho R_1^2 \cos^2\phi[u'w']\frac{[u]}{R_1\cos\phi}R_1\,d\phi & \{12\} \end{array}\right. \tag{17}$$

This is the new formulation of the zonal kinetic energy equation.

Recapitulation of traditional approach

Before discussing equation (17), it is helpful to compare it with the more traditional form of the kinetic energy equation for the mean zonal flow, written in the present symbolism and derived under assumptions and approximations identical with those used in formulating (17). In effect it becomes practically necessary to perform the manipulations of deriving the more familiar equation, utilizing the present methods. This will be done in outline only, however.

The absolute angular momentum M per unit mass may be eliminated from (2) by means of (1). The total derivative may then be expanded in the resulting equation with the aid of (6), so that the outcome is

$$\frac{\partial\varrho u}{\partial t} = -\nabla\cdot\varrho u\mathbf{V} - \frac{\varrho uw}{R} + \frac{\varrho uv}{R}\tan\phi - 2\Omega\varrho w\cos\phi + 2\Omega\varrho v\sin\phi - \frac{1}{R\cos\phi}\frac{\partial p}{\partial\lambda} + F \tag{18}$$

Here $\mathbf{V}$ is the vector velocity relative to the rotating coordinates (R, λ, ϕ), while the partial time derivative applies to a fixed point in this reference system.

As before we multiply (18) by $[u]$ and take a volume integral of the region of the polar cap retaining all the assumptions which were made, so that

$$\frac{d}{dt}\int_\tau \frac{\varrho[u]^2}{2}\,d\tau = +\int_\tau [u][F]\,d\tau - \int_\tau [u]\nabla\cdot\varrho u\mathbf{V}\,d\tau + \int_\tau 2\Omega\varrho[u][v]\sin\phi\,d\tau - \int_\tau 2\Omega\varrho[u][w]\cos\phi\,d\tau - \int_\tau [u]\frac{\varrho uw}{R}\,d\tau + \int_\tau [u]\frac{\varrho uv}{R}\tan\phi\,d\tau \tag{19}$$

The partial reductions effected in writing (19) stem from relations and definitions already used in writing (17). Further systematic application of these relations and definitions leads to a counterpart of (17) which may be written in various forms. The following form is desirable in order to demonstrate the points to be made presently.

$$\frac{d}{dt}\int_\tau \frac{\varrho[u]^2}{2}\,d\tau = +\iint 2\pi[u][F]R^2\cos\phi\,d\phi\,dR$$

Internal horizontal integrals

$$\left\{\begin{array}{ll} +\iint 4\pi\varrho\Omega R^2\cos^2\phi\sin\phi[v]\frac{[u]}{R\cos\phi}R\,d\phi\,dR & \{1'\} \\ +\iint 2\pi\varrho R^2\cos^2\phi[u][v]\frac{\partial}{R\partial\phi}\left\{\frac{[u]}{R\cos\phi}\right\}R\,d\phi\,dR & \{2\} \\ +\iint 2\pi\varrho R^2\cos^2\phi[u'v']\frac{\partial}{R\partial\phi}\left\{\frac{[u]}{R\cos\phi}\right\}R\,d\phi\,dR & \{3\} \end{array}\right.$$

$$\text{Internal vertical integrals}\left\{\begin{aligned} &-\iint 4\pi\varrho\Omega R^2\cos^2\phi[w]\frac{[u]}{R\cos\phi}R\,d\phi\,dR && \{4'\}\\ &+\iint 2\pi\varrho R^2\cos^2\phi[u][w]\frac{\partial}{\partial R}\left\{\frac{[u]}{R\cos\phi}\right\}R\,d\phi\,dR && \{5\}\\ &+\iint 2\pi\varrho R^2\cos^2\phi[u'w']\frac{\partial}{\partial R}\left\{\frac{[u]}{R\cos\phi}\right\}R\,d\phi\,dR && \{6\}\end{aligned}\right.$$

$$\begin{array}{l}\text{Vertical boundary integral}\\ \text{at } \phi=\phi_1\end{array}\left\{\begin{aligned} &+\int 2\pi\varrho R^2\cos^2\phi_1[u][v]\frac{[u]}{R\cos\phi_1}dR && \{8\}\\ &+\int 2\pi\varrho R^2\cos^2\phi_1[u'v']\frac{[u]}{R\cos\phi_1}dR && \{9\}\end{aligned}\right.$$

$$\begin{array}{l}\text{Horizontal boundary inte-}\\ \text{grals at } R=R_1\end{array}\left\{\begin{aligned} &-\int 2\pi\varrho R_1^2\cos^2\phi[u][w]\frac{[u]}{R_1\cos\phi}R_1\,d\phi && \{11\}\\ &-\int 2\pi\varrho R_1^2\cos^2\phi[u'v']\frac{[u]}{R_1\cos\phi}R_1\,d\phi && \{12\}\end{aligned}\right.$$

(20)

Inspection reveals that (17) and (20) differ solely in the terms containing Ω. Terms containing this parameter are present in the boundary integrals of the new equation, but are absent from the traditional boundary integrals in (20). It is not too difficult to show that the four integrals containing Ω in (17) may be reduced to the two corresponding ones in (20), so that these two equations are mathematically equivalent, as should be expected on a priori grounds. For this purpose it is advantageous to use the mass continuity equation specialized as follows under our general assumptions.

$$\frac{\partial\varrho[v]}{R\partial\phi}+\frac{\partial\varrho[w]}{\partial R}-\frac{\varrho[v]}{R}\tan\phi+\frac{2\varrho[w]}{R}=0 \qquad (21)$$

Discussion

There are several points to be noted concerning the form (17) of the zonal kinetic energy equation and its relation to the more familiar form (18). These are:

(a) Each of the equations is derivable from the other through the use of the definitions and relationships already utilized. The direct derivation of (17) as given above is no doubt simpler, however.

(b) Each internal integral in (17) involves one component of angular momentum transport multiplied by the spatial gradient of the relative mean angular velocity. (This, in vector form, was already expressed by eq. 12). It is thus shown that, for the present physical system, a generation of kinetic energy of the mean flow $[u]$ can result only from an up-gradient transport of angular momentum in one or another of the three components present. These three components are: that associated with the basic rotation Ω, that associated with the differential or relative rotation $[u]$, and that associated with the eddy motions u'.

(c) The boundary integrals in (17) at ϕ_1 and at R_1 likewise involve the three components of angular momentum transport multiplied by the value of the relative angular velocity at the boundaries.

(d) Since equations (17) and (20) are alike, save for the terms containing Ω, and since Ω is an arbitrary parameter in both formulations, an interesting special case arises upon assuming Ω to be zero. The remaining identical equations may then be interpreted as comprising a (single) kinetic energy equation for the absolute mean zonal flow.

(e) In (20) the terms enumerated as $\{1'\}$ and $\{4'\}$ may be simplified through the cancellation of the cosines in the denominators, etc. It is then seen that these terms are volume integrals of the work done by the zonal components of the Coriolis forces in virtue of $[u]$, associated respectively with $[v]$ and $[w]$. In (17) this action is recast into the action of a countergradient flux of angular momentum due to Ω, and also it is partially reduced to a boundary effect in the polar cap region along the free boundaries.

(f) As already has been stated the essential difference between the two schemes resides in the formulation of the terms containing Ω. It follows that in a more general problem other torques in addition to those due to pressure gradients and friction may be added. Thus magnetic effects might be included for application to hydromagnetic flows such as those in the sun and in the earth's core, in either scheme.

(g) Since the boundary terms arise from the integration of a vector divergence, no difficulty results for the case of a smooth earth at the surface boundary from the fact that this surface may not be one at which R is exactly constant. For the formal vanishing of this effect all that is required is that the velocity component normal to this surface be zero. In actuality, among other things, evaporation and precipitation processes interfere with this simplified conception.

(h) One of the major limitations of the discussion presented is that the earth's surface is not smooth, but has nonzonal inequalities due to orography. As far as the application of Green's theorem to the volume integrals is concerned there still is no formal difficulty as explained under (g) above, if in the case of absolute coordinates proper provision is made for the fact that the value of the surface elevation is a function of time as a consequence of the earth's rotation. On the other hand a difficulty which is perhaps more serious results from the fact that interruptions of cyclic continuity in the integration paths with respect to λ are introduced. The classic illustration of this are the mountain torques due to pressure differentials. But these are not the only such complications, and the entire subject is in need of a more precise treatment.

(i) As was stated, temporal and horizontal space variations in density have been neglected. Doubtless this is justified for most ordinary purposes. In principle, however, there is no difficulty in proceeding with either derivation making no such assumption, if one is reconciled to the tedium of sorting out the added terms which arise. This should be done if only for the sake of formal completeness.

(j) Evaluation of the various new integrals from extended data samples is desirable. We have already studied those integrals which are common to both expansions, over various time periods up to 5 years using the collection of world wide weather data contained in the MIT General Circulation Data Library. Similar applications of the same data to the new terms are entirely feasible but require considerable study. Such results have therefore been deferred for future publication.

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СИММЕТРИЧНАЯ ФОРМА УРАВНЕНИЯ ДЛЯ ЗОНАЛЬНОЙ КИНЕТИЧЕСКОЙ ЭНЕРГИИ

Основываясь на фундаментальных концепциях механики, при минимальных предположениях выводится уравнение для баланса кинетической энергии среднего зонального потока на симметричной полярной шапке. Полученное уравнение отличается от аналогичного уравнения, получаемого при традиционном подходе в том, что вращение используемой относительной системы координат входит в объемные интегралы более симметричным образом и появляется также в граничных поверхностных членах. Перечисляются различные аспекты возможных еще менее ограничительных формулировок.