On the observed large-scale atmospheric wave motions

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ABSTRACT

On the basis of data from the whole Earth during different periods of the International Geophysical Year the large-scale wave motions at the 500 mb level, and to some extent the 1000 mb level, are studied. The waves are represented by spherical harmonic components of the height field for the whole Earth as well as for each of the two hemispheres. The relative importance of the different components in the expansion of the height field for the whole Earth is illustrared, and the seasonal variations at the two hemispheres are considered.

The motion of the different wave components is investigated by computing mean values of the 24-hours phase angle changes. Except for the most large-scale, quasi-stationary components the mean velocities are found to be quite well in agreement with the Rossby–Haurwitz formula for the wave motions in a barotropic, non-divergent model with a solid rotating basic flow.

The fluctuations of the quasi-stationary waves are studied by means of quadrature spectrum analysis and by application of different time filters. A certain part of the regular fluctuations seems to be attributed to various westward wave motions with different velocities of propagation. Each of these westward wave motions seems to be composed of more spherical harmonic components with different amplitudes. The quantitative results concerning the velocities of propagation and the amplitudes are generally supporting the idea that these wave motions may be explained essentially by the Rossby effect, for the most large-scale components combined with a divergence effect.

1. Introduction

In a previous paper (Eliasen & Machenhauer, 1965) we have considered the planetary-scale waves represented by the spherical harmonic components of the horizontal atmospheric fields. Due to the poor state of the observations at the Southern Hemisphere, the considerations were restricted to the Northern Hemisphere. The spherical harmonic representation was based on the special condition of orthogonality for one hemisphere and it was connected with a definite condition of symmetry with respect to the Equator. For a complete treatment of the planetary-scale waves it should be necessary, however, to include the whole sphere, and a purpose of the present work has therefore been to consider the possibility of extending the previous investigations by using a harmonic representation of the horizontal fields over the whole Earth and by including both symmetric and antisymmetric components.

It is known that the most large-scale waves (i.e. mainly the waves with wave numbers 1, 2, and 3) are generally quasi-stationary, whereas the waves with somewhat smaller scales (i.e. mainly the waves with wave numbers 5 to 9) are moving more or less regularly towards the east. Concerning the fluctuations of the very long waves it seems possible to interpret a regular part of these as the result of wave motions of the Rossby-wave type (Eliasen & Machenhauer, 1965, and Deland, 1965). In the present work it is intended to investigate systematically the motions of the different spherical harmonic components of the height field, and especially to study more in detail the fluctuations of the long, quasi-stationary waves.

In our previous work we have considered the spherical harmonic components of the stream function obtained from the height field by using the linear balance equation. A more thorough investigation of this relation between the height field and the stream function seems to show, however, that especially for the most large-scale symmetric components of the stream function, the values obtained are doubtful, as a consequence of the fact that the linear balance equation is not a useful approximation in the equatorial area. For that reason we have decided in the present investigation to consider the actual wave motions only in terms of the components of the height field.

2. The spherical harmonic representation

As the fundamental horizontal field we consider the height z of a surface of constant pressure at a definite time as a function of the latitude ϕ and the longitude λ. For the whole Earth this function may be expressed by the following spherical harmonic representation

$$z(\phi,\lambda) = \sum_{m=0}^{\infty}\sum_{n=m}^{\infty}\{a_{m,n}\cos m\lambda + b_{m,n}\sin m\lambda\} \times P_{m,n}(\phi) = \sum_{m=0}^{\infty}\sum_{n=m}^{\infty} A_{m,n}\cos\{m \times (\lambda - \delta_{m,n})\}P_{m,n}(\phi), \quad (1)$$

where $P_{m,n}(\phi)$ are the associated Legendre functions of the first kind; m is the longitudinal wave number, and n is the degree of the harmonic. Each of the components in the expansion is determined by the cosine coefficient $a_{m,n}$ and the sine coefficient $b_{m,n}$ or by the amplitude $A_{m,n}$ and the phase angle $\delta_{m,n}$.

For two reasons we may be interested in considering a representation of the height field only for one hemisphere. The one is simply that the data are definitely better for the Northern Hemisphere than for the Southern Hemisphere, which means that results only based on the Northern Hemisphere data will be more reliable than the results based on the data for the whole Earth. But also even if we want to use the data from the Southern Hemisphere, a separate representation for each hemisphere will be useful, as it will make up a basis for a comparison of the conditions at the two hemispheres. The expansion for one hemisphere may be written in the same form, (1), as for the whole sphere, with the only difference that we take only half of the terms in the series, namely either the terms with $n-m$ even, or the terms with $n-m$ odd, i.e. terms either symmetric or antisymmetric with respect to the Equator. In the antisymmetric case only the deviations from the mean value at the Equator are described. For each hemisphere the expansion coefficients are computed for the symmetric as well as for the antisymmetric representation, and the computation of the expansion coefficients follows, with the same normalization of the Legendre functions, the procedure used earlier for the Northern Hemisphere (Eliasen & Machenhauer, 1965) with only obvious changes due to the use of actual data from the equatorial zone. The coefficients for the whole sphere are then given simply as the mean values of the corresponding coefficients for the two hemispheres.

The attempt to consider the large-scale fields over the whole Earth is based on the data for the height of the 500 mb surface and to some extent the 1000 mb surface from the World Weather Maps for the International Geophysical Year (1957–58), Part I (Northern Hemisphere, Weather Bureau, Washington), Part II (Tropical Zone, Seewetteramt, Hamburg), Part III (Southern Hemisphere, Weather Bureau, Republic of South Africa). For the computations we have used the values of z for each 5 degrees of latitude, and at the Northern Hemisphere for each 5 degrees of longitude, whereas for the Southern Hemisphere only for each 10 degrees of longitude. For the whole sphere the expansion coefficients were computed for wave numbers up to 9, and for the Northern Hemisphere for wave numbers up to 18. For the meridional descriptions values of $n-m$ up to 24 were used.

In order to illustrate the relative importance of the different components, the mean values for October 1957 of the amplitudes at the 500 mb level over the whole Earth are shown in Table 1 for values of m up to 9 and n up to 18. The values are given in meters, and it is seen that the largest values of the amplitudes of the wave components are of the order of 20 meters and occur for the smaller values of m and n. For each wave number the value of the amplitude is rather small for the first component, i.e. $n-m=0$, and increases to a maximum value for $n-m$ about 4. The zonal mean field described by $m=0$ shows up a pronounced symmetry with respect to the Equator.

3. The spectral distribution of the variance of ∇z

As another and with reference to the dynamics of the motion perhaps more relevant measure

Table 1

n	m = 0	1	2	3	4	5	6	7	8	9
0	5665									
1	64	18								
2	254	19	7							
3	19	20	10	6						
4	47	19	22	11	7					
5	13	17	14	13	10	4				
6	27	16	25	10	14	11	4			
7	3	13	15	17	15	12	8	3		
8	3	20	13	11	14	15	9	7	3	
9	6	14	11	11	12	14	11	7	7	2
10	3	10	9	9	12	12	10	9	6	3
11	6	11	7	9	7	9	8	7	8	4
12	4	8	9	6	9	6	7	7	7	4
13	2	5	4	5	5	5	6	5	6	5
14	3	5	5	4	5	4	5	5	5	4
15	2	3	4	4	3	4	4	4	4	4
16	2	3	3	3	3	3	3	3	3	3
17	1	2	3	3	3	3	3	3	3	3
18	1	2	2	2	2	2	3	3	2	2

Table 2

(a) 500 mb $E = 1333$

$n-m$	m = 0	1–4	5–8	9–12
0– 7	469	307	139	41
8–15	47	146	59	20
16–23	5	24	12	6
ZHA	523	490	219	72

(b) 1000 mb $E = 500$

$n-m$	m = 0	1–4	5–8	9–12
0– 7	21	182	67	22
8–15	11	66	32	17
16–23	3	16	10	6
ZHA	38	272	116	49

of the relative importance of the different components, we may consider the spectral distribution of the variance of the horizontal gradient of z. Introducing

$$E = \frac{1}{4\pi}\int_0^{2\pi}\int_{-\pi/2}^{\pi/2} (R\nabla z)^2 \cos\phi \, d\phi \, d\lambda, \qquad (2)$$

where R is the radius of the Earth and ∇ the spherical del-operator, we have

$$E = \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} E_{m,n} \qquad (3)$$

with

$$E_{0,n} = n(n+1)A_{0,n}^2$$

$$E_{m,n} = \tfrac{1}{2}n(n+1)A_{m,n}^2$$

The spectral distribution of the variance of gradient z will be the same, of course, as the spectral distribution of the kinetic energy of the geostrophic wind field determined with a constant value of the Coriolis parameter.

In order to have a better view of the spectral contributions to the total variance, we may add up the contributions from components in different groups. The result of such a grouping of the mean values for January 1958 is shown in Table 2 for the representation with $n-m$ even of the field over the Northern Hemisphere. Almost the same values are obtained, if the representation with $n-m$ odd is used. The values in part a of the table are for the 500 mb level, in part b for the 1000 mb level, and the values are given in the unit 10^3 m^2. The total variance, E, is computed from the zonal harmonic representation with wave numbers up to 18 by a numerical integration in the meridional direction. In the tables we have the contribution to the variance of the gradient of z from the zonal mean state, and from groups of wave components each containing four wave numbers. Concerning the grouping of the meridional description, each group in the table includes four values of $n-m$. The values in the last row of the tables are the values for the total variance of the gradient of z for the different wave numbers, obtained from the zonal harmonic analysis (ZHA). It is seen that the components with wave number 0, which are describing the zonal mean state, contribute as much as 39.2 per cent to the total variance at the 500 mb level, but only 7.5 per cent at the 1000 mb level, and furthermore that the most large-scale components in the meridional description give a very large contribution. The components with wave numbers 1 to 4 contribute 36.7 per cent at the 500 mb level and 54.0 per cent at the 1000 mb level; and the components with wave numbers 5 to 12 contribute 21.8 per cent at

Table 3

		July			January		
	$m \ldots$	0	1–4	5–8	0	1–4	5–8
	$n-m$						
NHS	0– 7	112	61	72	469	307	139
	8–15	29	110	44	47	146	59
SHS	0– 7	665	166	142	577	131	123
	8–15	31	146	60	46	92	53

the 500 mb level and 32.9 per cent at the 1000 mb level. Taking into account wave numbers up to 8 and using a meridional representation with 8 components, we obtain a description including 88 per cent of the total variance of the gradient of z at the 500 mb level, but only 75 per cent at the 1000 mb level. Similarly we obtain for wave numbers up to 12 and a meridional description with 12 components 95 per cent at the 500 mb level and 90 per cent at the 1000 mb level.

In order to compare the conditions at the two hemispheres and the conditions in the summer and winter season we may construct tables of the kind shown in Table 3. The values are from July 1957 and January 1958, and for the symmetric representation at each hemisphere. The unit and the grouping are the same as used in Table 2. Quite generally it is seen that the difference between January and July is essentially larger at the Northern Hemisphere (NHS) than at the Southern Hemisphere (SHS), at both hemispheres, of course, with the largest values in winter. For the zonal mean state we find the largest contribution at the Southern Hemisphere in July, but only a slightly smaller value in January, and then further a slightly smaller value at the Northern Hemisphere in January, and finally a quite small value in July at the Northern Hemisphere. For the components with wave numbers from 1 to 4 we find the largest contribution at the Northern Hemisphere in January and the second largest at the Southern Hemisphere in July. The contributions from the components with $n-m$ from 0 to 7 are decreasing considerably more from the winter month to the summer month at the Northern than at the Southern Hemisphere, while the changes for the components with $n-m$ from 8 to 15 are approximately equal and rather small at the two hemispheres. It is remarkable that for wave numbers from 1 to 4 at the Northern Hemisphere the contribution from the meridional components with $n-m$ from 8 to 15 in July becomes larger than the contribution from components with $n-m$ from 0 to 7.

4. The mean motions of the wave components

In order to study the mean motions of the different wave components, we have computed mean values of the 24-hours phase angle change for the components with $m \leq 8$ and $n-m \leq 7$. As a 24-hours phase angle change, $\Delta(\delta_{m,n})$, may be determined only apart from an integer multiple of the wave length, one of the possible values must be selected in each case, before the mean values can be computed. For the present computations we have assumed that no 24-hours phase angle changes are numerically larger than one wave length and that for each component all values are inside an interval I of one wave length. The problem is then to determine this interval in a reasonable way. For most of the components considered we generally find the majority of phase angle changes inside an interval of less than half a wave length around an absolute maximum. The frequencies, however, are often too far from a normal distribution to justify the use of the absolute maximum as the mid-point of I. Instead we have decided to determine the mid-point of I as what we may call, the vector mean value, $[\Delta(\delta_{m,n})]$, given by

$$\left.\begin{aligned} \cos\{m[\Delta(\delta_{m,n})]\} &= \overline{\cos\{m\Delta(\delta_{m,n})\}} \\ \sin\{m[\Delta(\delta_{m,n})]\} &= \overline{\sin\{m\Delta(\delta_{m,n})\}}, \end{aligned}\right\} \tag{4}$$

where a bar denotes the mean value for the considered period. If all the 24-hours phase angle changes, $\Delta(\delta_{m,n})$, are represented in a plane Cartesian coordinate system as unit vectors forming an angle to the absciss axis equal to $m\Delta(\delta_{m,n})$, it is seen from the definition that $[\Delta(\delta_{m,n})]$ is just the phase angle change corresponding to the mean vector of these unit vectors. Computing the mean value of the phase angle changes, by using the interval I fixed from the vector mean value, we obtain a mean value placed near the mid-point of the interval.

In Table 4 such mean values of the 24-hours phase angle changes at the 500 mb level, γ_{obs} are given for the period from 1st October, 1957 to 31st January, 1958 in degrees of longitude per day. It is seen that for the wave components with $m \geqslant 4$, the mean values for each m are generally increasing with increasing n, and that we find at each hemisphere for the same n velocities of almost the same magnitude. At the Southern Hemisphere the eastward velocities are seen to be larger than at the Northern Hemisphere. For the components with $m \leqslant 3$ we find, as expected, that the waves are almost stationary. It is seen, however, that also for these components there is a tendency to increasing eastward motion with increasing n, and that for several of the components with smaller meridional scales the mean velocities are of the same magnitude as for the components with $m \geqslant 4$, and with the same n. Concerning the standard deviations corresponding to the mean values in Table 4, they have values in per cent of the wave length ranging from 5 for some of the components with $m = 1$ to about 20 for the larger values of m and n, and somewhat larger values at the Southern than at the Northern Hemisphere.

In our previous paper (Eliasen & Machenhauer, 1965) the results of tendency computations, based on the non-divergent barotropic vorticity equation represented in the spectral domain, were compared with the observed changes for some spherical harmonics of the stream function at the 500 mb level. These computations indicate that the motion of moderately long waves to a large extent is determined by effects included in the simple barotropic equation. Moreover the computations indicate that the effect of the non-linear interactions between different wave components, in the mean, is small, and that the mean effect of the zonal mean flow with a good approximation may be accounted for by a solid rotating flow with a constant angular velocity ω. In order to see, to what extent the variation with n of the mean velocities in Table 4 may be explained by the Rossby-effect, we have listed in the same table also theoretical values γ_n, of the phase velocities, determined by the Rossby–Haurwitz formula

$$\gamma_n = \omega - \frac{2(\omega + \Omega)}{n(n+1)} \tag{5}$$

where Ω is the angular velocity of the Earth. As the theoretical values are obtained for the stream function components, whereas the observed values are for the components of the height field, we must, in order to make a comparison, take into account the relation between the height field and the stream function. Generally we have that a stream function antisymmetric with respect to the Equator is connected with a height field being symmetric, and vice versa. In the linear, non-divergent approximation we especially have that the

Table 4

Northern Hemisphere

	γ_{obs} m								
n	1	2	3	4	5	6	7	8	γ_n ($\omega = 14$)
1	0								−360
2	0	−9							−111
3	0	−5	−1						−48
4	−3	0	−3	1					−23
5	−3	−2	−3	1	0				−11
6	−4	−1	−1	3	1	2			−4
7	1	3	1	2	2	3	2		1
8	1	5	1	3	6	4	4	3	4
9		6	3	7	5	5	4	4	6
10			6	6	8	8	5	5	7
11				9	7	10	7	6	8
12					7	9	9	7	9
13						9	11	10	10
14							11	11	10
15								11	11

Southern Hemisphere

	γ_{obs} m								
n	1	2	3	4	5	6	7	8	γ_n ($\omega = 19$)
1	0								−360
2	3	−2							−108
3	3	−5	1						−44
4	−3	−4	3	1					−19
5	2	3	1	1	3				−6
6	−4	3	5	5	4	3			1
7	−3	1	7	7	6	6	6		5
8	0	−2	6	8	9	7	8	6	8
9		4	5	9	11	9	9	9	11
10			5	11	13	11	12	11	12
11				13	12	13	13	11	13
12					12	16	13	13	14
13						16	15	14	15
14							16	16	15
15								15	16

Table 5

	m							
n	1	2	3	4	5	6	7	8
1	−21							
2	−27	−13						
3	−27	−20	−11					
4	−16	−20	−13	−9				
5	−5	−20	−14	−15	−22			
6	−8	−18	−12	−12	−19	−12		
7	−9	−13	−9	−7	−15	−14	−10	
8	−8	−7	−10	−6	−10	−12	−14	1
9		−8	−5	−13	−8	−11	−14	−6
10			0	−21	−11	−11	−14	−8
11				−20	−16	−11	−15	−13
12					−16	−8	−18	−13
13						−9	−16	−14
14							−13	−13
15								−12

component of the height field $z_{m,m}$ is proportional to the stream function component $\psi_{m,m+1}$, and that a component of the height field with degree $n>m$ is a linear combination of the stream function components $\psi_{m,n-1}$ and $\psi_{m,n+1}$. If the stream function components in the mean are moving with the velocities γ_n, we should expect that the components $z_{m,m}$ should move with the mean velocity γ_{m+1}, and that the components $z_{m,n}$ with $n>m$ should move with a velocity between γ_{n-1} and γ_{n+1}. It therefore seems reasonable, in this rough estimate, simply to compare the mean velocities of the height components $z_{m,n}$ with the Rossby–Haurwitz velocities γ_n, except for the components with $n=m$, for which the velocities should be compared with the values γ_{m+1}. When the values 14 and 19 degrees of longitude per day are used for ω at the Northern and Southern Hemisphere, respectively, a quite good agreement is found between the observed mean velocities and the Rossby–Haurwitz velocities for components with $n \geq 7$ (for the Southern Hemisphere with the exception of some components with $m \leq 3$). The values used for ω seem reasonable when compared with the mean values over the considered period of the geostrophic, zonal mean velocities.

Concerning the motions of the waves at the 1000 mb level, it turns out that in most cases each wave component at that level follows the same component at the 500 mb level rather closely with a certain difference in phase angle. In general the components are displaced somewhat to the east at the 1000 mb level in relation to the 500 mb level. The magnitude of this displacement for the different components is illustrated in Table 5, giving the mean values for the period December 1957 and January 1958 of the phase differences in per cent of the wave length.

The motion and the amplitude of the zonal harmonic components at the different latitudes may, of course, be considered as determined by the motion and the amplitude of the different spherical harmonic components, taking into account the meridional shape of the corresponding Legendre functions. For wave numbers 1 and 2 and to some extent 3, the motion has at most latitudes a fluctuating character with mean velocities equal to zero or relatively small negative values. For larger values of the wave number ($m=4$ to 9) the mean velocities are still very small in the equatorial area, but at the middle latitudes we have an eastward motion with mean velocities up to

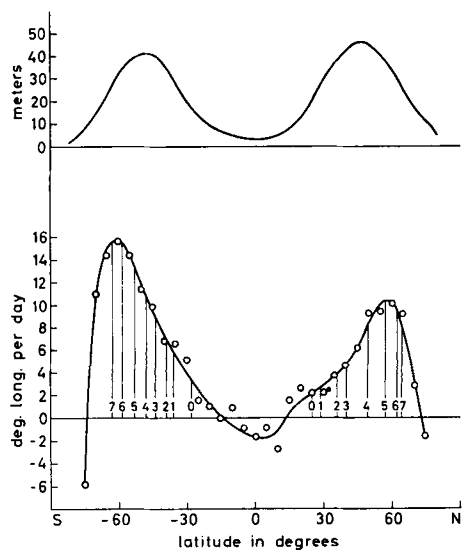

Fig. 1. Variation with latitude of the mean values for the period from 1st October, 1957 to 31st January, 1958 of the amplitude (upper part) and phase velocity (lower part) for the zonal harmonic component $m=6$ at the 500 mb level. The phase velocity is represented by a smoothed curve drawn on the basis of the actual values at each five degrees. The latitudes, at which the velocity is equal to the velocity of a spherical harmonic component, are indicated by vertical lines, marked with the respective values of $n-m$ from 0 to 7.

about 15 degrees of longitude per day. For these wave numbers we find a simple connection between the motion of the spherical harmonic components and the motion of the zonal harmonic components at the different latitudes. This is illustrated in Fig. 1 for the wave number 6 for the period 1st October 1957 to 31st January 1958. The curve in the upper part of the figure shows the variation with latitude of the mean value of the amplitude in meters. The mean velocities in degrees of longitude per day at the different latitudes are shown by the curve in the lower part of the figure. The vertical lines indicate the mean velocities for the different spherical harmonic components for each hemisphere. It is seen that increasing degree of the component and increasing latitude are connected with the mean velocities in a similar way. Roughly this may be considered as a consequence of the fact that the maximum values of the Legendre functions are displaced polewards with increasing degree. The correspondence between the degree of the harmonic and the latitude is not restricred to the mean velocity, but in many cases the day to day variations of the phase of a spherical harmonic component are very similar to the variations at a definite latitude.

5. Quadrature spectrum analysis applied to the fluctuations of the long quasi-stationary waves

From previous investigations of the flow over the Northern Hemisphere (Eliasen & Machenhauer (1965), & Deland (1965)), it is known that the most large-scale waves often fluctuate in a rather regular way about a mean position or a slowly changing position, and that their amplitudes are generally relatively large during the westward motions and relatively small during the eastward motions. This behaviour was interpreted as wave motions towards the west, superimposed upon stationary or slowly moving waves. As an ideal model we may consider a free wave with a constant amplitude, which is moving with a constant velocity towards the west, superimposed upon a stationary wave with a constant amplitude. For this model the variations of the cosine and sine coefficients of the resulting wave will be purely harmonic with that of the sine coefficient a quarter period ahead of that of the cosine coefficient. The variation of the amplitude and the phase angle will also be periodic, and if the amplitude of the stationary wave is larger than that of a free wave, we will have a relation between the amplitude and the phase velocity, as mentioned above.

In order to investigate to what extent the observed fluctuations of the most large-scale waves may be explained by this simple model, we have computed the quadrature spectra for the sine and cosine coefficients (cf. Panofsky & Brier, 1958). This method has previously been applied to the fluctuations of zonal harmonics by Deland (1964). For the spherical harmonic components describing the most large-scale waves, we have computed the quadrature spectra, using the following procedure. A correlation function, $R(h)$, of the two time series $b_{m,n}(t)$ and $a_{m,n}(t)$, each consisting of N observations, spaced in time by 1 day, is computed as

$$R(h) = \frac{1}{N-|h|}\sum_{t=1}^{N} b_{m,n}(t)a_{m,n}(t+h) - \overline{b_{m,n}}\,\overline{a_{m,n}} \quad (6)$$

for $-M \leqslant h \leqslant M$, where M and h are integers, $a_{m,n}(t+h)=0$ for $t+h \leqslant 0$ and $t+h > N$, and a bar indicates a mean value over the N days. The quadrature $q(i)$, is then computed for the frequencies $i/2M$, $M \geqslant i > 0$, where i is an integer, by the following expression

$$q(i) = \frac{2}{M}\sum_{h=-(M-1)}^{m-1} D(h)R(h)\sin\left(i\frac{\pi}{M}h\right) \quad (7)$$

with the lag window

$$D(h) = \tfrac{1}{2}\left(1+\cos\left(\frac{\pi}{M}h\right)\right)$$

used in order to obtain a smoothing of the spectral values. As a measure of the quadrature we shall use

$$Q(i) = \frac{q(i)\sqrt{|q(i)|}}{|q(i)|}. \quad (8)$$

In the case of superimposed wave motions each with constant amplitude, $A(i)$, and constant frequency, $i/2M$, the values of $|Q(i)|$ will be approximately equal to the amplitudes $A(i)$, when computed by using $D(h)=1$. The approximation being the better the heigher value of N, in proportion to M, we are using. The value of $Q(i)$ will be positive for wave motions towards the west and negative for wave motions towards the east. When we are using

$$D(h) = \frac{1}{2}\left(1 - \cos\left(\frac{\pi}{M}h\right)\right) \text{ instead of } D(h) = 1,$$

we obtain a smoothing of the values of $q(i)$ given by

$$q(i) \simeq 0.25\, q'(i-1) + 0.50\, q'(i) + 0.25\, q'(i+1),$$

where quantities marked with an apostrophe are the values, which would have been obtained by using $D(h) = 1$.

Although the number of observations at our disposal is rather small, $N = 123$, we shall in the following present quadrature spectra with $M = 30$, computed for the most large-scale components of the 500 mb height field from the period considered in the preceding section. If for a spherical harmonic component a relative maximum of $|Q(i)|$ is obtained for a certain frequency, this will indicate the existence in the 123-days period of a superimposed wave motion, towards the west if the value of $Q(i)$ is positive at the relative maximum, and towards the east if the value is negative. As an indication of the magnitude of the mean amplitude of the wave we shall use the maximum value of $|Q(i)|$, although this value must be considered as an underestimate of the real mean amplitude, because the real wave motion must be considered to be represented also by the neighbouring frequencies. Furthermore, the velocity corresponding to the frequency for which the relative maximum is obtained, will be considered as the mean velocity of the wave.

In Fig. 2. the quadrature spectra for 5 components with the zonal wave number one are given for the two hemispheres and for the whole sphere. Instead of frequencies, the values at the bottom line are numerical values of the corresponding phase velocities in degrees of longitude per day. The values of $Q(i)$ are given in meters. At the Northern Hemispheres we find for the components $(m, n) = (1, 1)$, $(1, 2)$, and $(1, 3)$ relative maxima, indicating a common wave motion towards the west with a velocity about $-70°$ day^{-1}, and with the largest amplitude for the component $(m, n) = (1, 2)$. For each of the 5 components we find an additional maximum, indicating an additional westward wave motion with a velocity about $-24°$ day^{-1}. For this wave motion the amplitude is increasing with n from about 3 meters for the component $(m, n) = (1, 1)$ to about 9 meters for $(1, 4)$, and is then decreasing with increasing n. Thus these spectra seem to indicate that two westward wave motions with different velocities are superimposed. In addition an eastward moving wave is indicated for the components $(m, n) = (1, 4)$ and $(1, 5)$. For the components with $n > 5$ the values of $Q(i)$ are negative for most of the frequencies, but the numerical values are rather small, except for the very low frequencies.

From Fig. 2 it is seen that there are some similarities between the spectra for the Northern and the Southern Hemisphere. Also at the Southern Hemisphere two westward wave motions are indicated by the spectra. The mean velocities of these wave motions and the variation of the mean amplitudes with n are almost the same as for the Northern Hemisphere. However, the amplitudes of these wave motions are smaller than at the Northern Hemisphere. For the components $(m, n) = (1, 4)$, $(1, 5)$, and $(1, 6)$ at the Southern Hemisphere, an additional westward wave motion, with a velocity about $-48°$ day^{-1}, is indicated. At the Northern Hemisphere a corresponding wave motion is indicated for the component $(m, n) = (1, 6)$, and to some extent for $(m, n) = (1, 5)$. In contradistinction to the conditions at the Northern Hemisphere minima with negative values of $Q(i)$, indicating a wave motion with eastward velocities about $42°$ day^{-1} are found for the first four components at the Southern Hemisphere.

Considering for the whole sphere at first the symmetric components, a westward moving wave with a velocity of $-72°$ day^{-1} is indicated in the quadrature spectra for $(m, n) = (1, 1)$ and $(1, 3)$ with the largest amplitude for the former. In addition a westward moving wave, with a velocity of $-24°$ day^{-1}, is indicated for $(m, n) = (1, 1)$, $(1, 3)$, and $(1, 5)$ with the largest amplitude for $(m, n) = (1, 3)$. The spectra for the antisymmetric components $(m, n) = (1, 4)$, $(1, 6)$, and to some extent $(1, 2)$, seem to indicate wave motions different from the wave motions indicated in the symmetric height field. So for the second and third antisymmetric component, $(m, n) = (1, 4)$ and $(1, 6)$, only one westward wave motion, with a velocity about $-45°$ day^{-1}, is indicated. For the first antisymmetric component $(m, n) = (1, 2)$ two westward wave motions are indicated. The corresponding velocities, however, are displaced from the

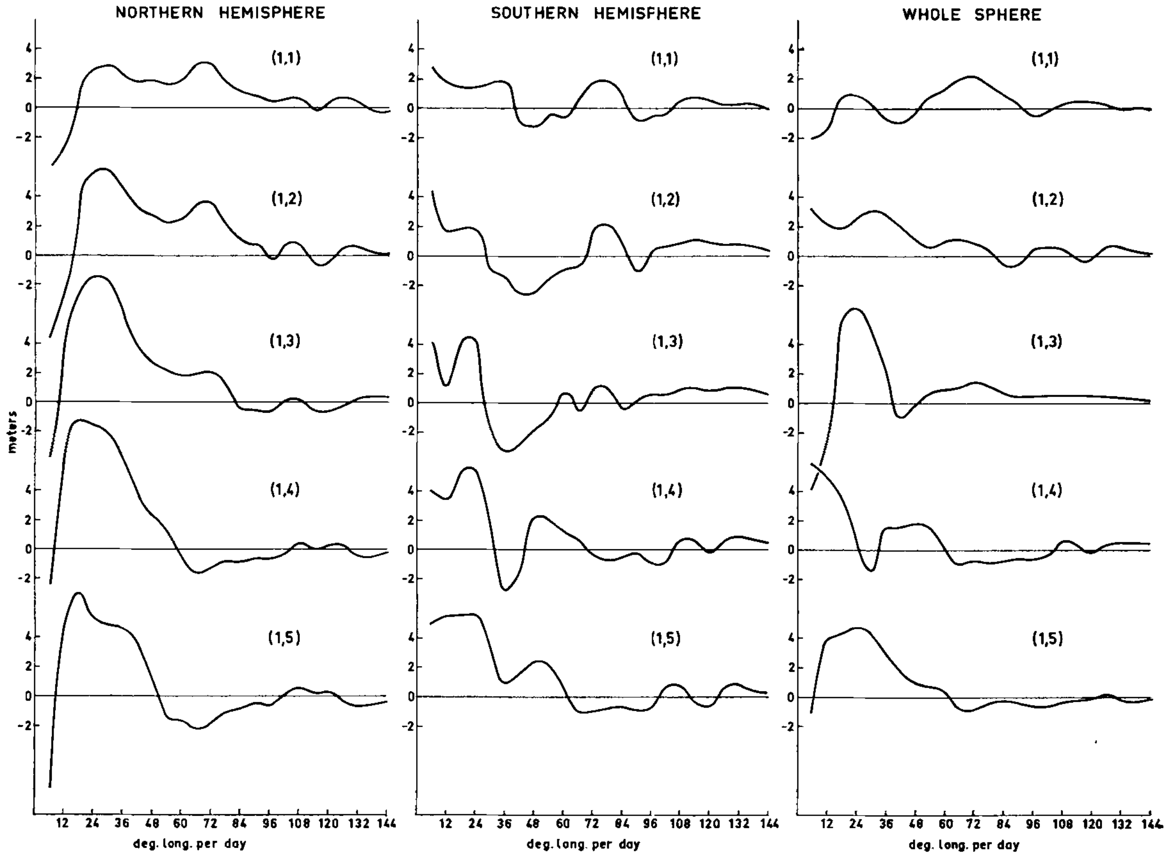

Fig. 2. Quadrature spectra for the spherical harmonic components with zonal wave number $m = 1$, and degree n from 1 to 5, of the 500 mb height field during the period from 1st October, 1957 to 31st January, 1958.

values indicated for the symmetric components towards the velocity indicated for the components $(m, n) = (1, 4)$ and $(1, 6)$. Concerning the components with $n > 5$ we find at the Southern Hemisphere and at the whole sphere, just as at the Northern Hemisphere, negative values of $Q(i)$ over an increasing part of the spectrum, when n is increasing.

In the same way as for $m = 1$ we have considered the quadrature spectra for components with $m = 2$ and 3. Also for these wave numbers the spectra indicate common wave motions for different meridional components. For all three wave numbers the velocities of the different common wave motions are listed as γ_{obs} in Table 6 in degrees of longitude per day. For each of the values of γ_{obs} the corresponding amplitudes (the maximum values of $|Q(i)|$) for the different meridional components are given in the table in meters. In cases when a wave motion is indicated, not by a relative maximum, but only by the form of the $Q(i)$-curve the amplitude is marked with a star. For each velocity the largest amplitude is emphasized. For each m various westward wave motions are indicated, generally with almost the same velocities at the two hemispheres and at the whole sphere. As for $m = 1$ the fastest wave motion for $m = 2$ and 3 is indicated only for the most large-scale meridional components. For the wave motions with smaller westward velocities the largest amplitude is generally displaced towards the higher values of n. As for $m = 1$ there are for $m = 2$, and especially for $m = 3$, some differences between the quadrature spectra for the symmetric and antisym-

Table 6

		$m=1$			$m=2$				$m=3$		
NHS	γ_{obs}	−69	−42	−24	−45	−26	−17	−9	−24	−11	—
	$n-m$										
	0	2.9	—	2.8	1.0	2.0*	3.6	—	1.6	2.8	—
	1	_3.6_	—	5.8	_1.1_	3.4*	6.0	—	_2.2_	4.7	—
	2	2.0	—	8.6	—	_3.8_	_6.1_*	7.6	1.6	_5.9_	—
	3	—	—	_8.7_	—	2.1	—	_9.6_	—	5.2	—
	4	—	_3.6_*	6.8	—	—	—	8.2	—	—	—
	5	—	2.9	1.6	—	—	—	—	—	—	—
SHS	γ_{obs}	−76	−48	−24	−49	−27	—	−11	—	−12	−6
	$n-m$										
	0	1.8	—	1.8	0.7	1.6	—	—	—	1.0	1.6
	1	_2.1_	—	1.8	_1.1_	_2.5_	—	—	—	_1.3_	3.5
	2	1.1	—	4.3	0.4	2.1	—	0.9	—	1.0	_5.4_
	3	—	2.3	_5.6_	—	1.6	—	0.6	—	1.0	5.0
	4	—	_2.4_	5.5	—	1.5	—	1.5	—	—	—
	5	—	1.6	4.8	—	1.9	—	_1.9_	—	—	—
WS	γ_{obs}	−72	−45	−24	−48	−27	−18	−10	−24	−11	−7
	$n-m$										
	0	_2.1_	—	0.8	_0.7_	—	—	—	_0.8_	1.3	1.2*
	1	1.1	—	3.1	—	_2.8_	_3.6_	—	0.7	_2.5_	—
	2	1.4	—	_6.4_	0.5	1.6	—	_4.5_	0.4	—	_4.0_
	3	—	1.7	—	—	1.2	—	3.7*	—	2.2	—
	4	—	—	4.7	—	1.3	—	4.3	—	—	—
	5	—	_2.0_	—	—	—	—	—	—	0.6	—

metric components for the whole sphere, indicating different wave motions in the symmetric and antisymmetric height field.

By considering the results concerning the wave motions at the Southern Hemisphere and the whole sphere, we must take into account the poor observational state at the Southern Hemisphere. However, the general picture obtained for the westward wave motions seems to be supported by the agreement with the Northern Hemisphere.

6. Application of time filters

As another way to study the fluctuations of the long, quasi-stationary waves, we may apply different time filters to the sine and cosine coefficients. So we have in our previous work (Eliasen & Machenhauer, 1965) considered the motion of the 24-hours tendency field, and the deviations from various running means of the wave components. For each component the most regularly moving field was selected as representative for a free part of the wave.

In the following we shall consider three different fields obtained by applying to

$$z_{m,n}(t) = \{a_{m,n}(t)\cos m\lambda + b_{m,n}(t)\sin m\lambda\}P_{m,n}(\phi)$$

three different time filters, namely the 24-hours tendency field

$$z^*_{m,n}(t) = z_{m,n}(t) - z_{m,n}(t-1), \tag{9}$$

the running 3-days mean of the 48-hours tendency field

$$z^{**}_{m,n}(t) = \frac{1}{3}\sum_{i=-1}^{1}\{z_{m,n}(t+i) - z_{m,n}(t+i-2)\}, \tag{10}$$

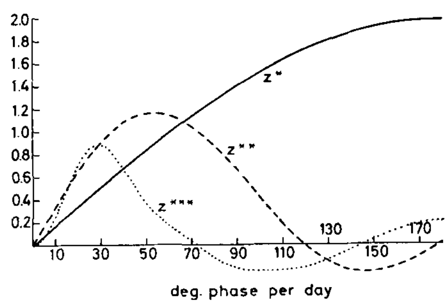

Fig. 3. The frequency response functions corresponding to the three time filtered fields z^*, z^{**}, and z^{***}.

Table 7

		z^*			z^{**}			z^{***}		
	m...	1	2	3	1	2	3	1	2	3
	n									
NHS	1	−69			−40			−21		
	2	−59	−26		−37	−13		−24	−16	
	3	−24	−23	−18	−25	−19	−9	−17	−16	−4
	4	−9	−20	−14	−22	−15	−8	−20	−9	−5
	5	6	−1	−6	−7	−5	−4	−5	−4	−4
	6	16	2	4	2	1	2	9	3	1
	7	25	11	9	18	7	3	6	6	5
	8	43	19	12	12	9	5	16	3	7
	9		28	15		8	8		4	9
	10			19			9			7
SHS	1	−68			−8			3		
	2	−100	−55		0	−10		0	4	
	3	24	−49	43	0	−1	−2	−11	4	−2
	4	26	33	27	−1	3	0	3	5	0
	5	−49	29	23	−14	1	6	−4	5	0
	6	−129	36	28	1	2	11	−14	4	2
	7	−86	28	33	16	3	13	14	4	7
	8	61	53	28	1	−1	10	21	−2	5
	9		55	26		1	9		4	9
	10			36			7			6
WS	1	−68			−31			17		
	2	−29	−41		−12	−1		−3	3	
	3	−35	−23	−8	−16	−19	0	−24	−6	−3
	4	28	7	0	−7	−4	−3	−1	−5	−1
	5	−1	12	15	−5	−1	3	−10	6	−4
	6	4	−1	18	2	−4	3	3	−4	2
	7	42	12	19	14	5	8	12	3	5
	8	23	35	22	13	5	6	17	5	4
	9		25	23		11	12		7	9
	10			23			4			5

and the running 5-days mean of the deviations from running 15-days means

$$z^{***}_{m,n}(t) = \frac{1}{5}\sum_{i=-2}^{2}\left\{z_{m,n}(t+i) - \frac{1}{15}\sum_{j=-7}^{7} z_{m,n}(t+i+j)\right\}, \tag{11}$$

where the unit for the time t is one day. The frequency response function (the ratio of the amplitude of an oscillation after filtering to the original amplitude before filtering as a function of the frequency) for each of the three fields are given in Fig. 3. Instead of frequencies the numerical values of phase velocities in degrees phase per day are used. It is seen that fluctuations of a relatively broad frequency interval are retained in each of the fields, so that we for each of the fields may speak of an effective interval.

For each of the three fields the mean velocities, computed in the same way as for the unfiltered field (section 4), are listed in Table 7 for the first 8 components of each of the wave numbers $m = 1$, 2, and 3, for both hemispheres as well as for the whole sphere. The corresponding standard deviations in per cent of the wave length have for the Northern Hemisphere the following values: between 19 and 24 for the z^*-field, between 11 and 15 for the z^{**}-field, and between 6 and 13 for the z^{***}-field. For the Southern Hemisphere and for the whole sphere the values are somewhat larger. When the frequency response functions are taken into account, we find that the velocities in Table 7 generally agree well with what should be expected from the quadrature spectra. It should be mentioned that the motion of a certain field can be expected to coincide with a wave motion indicated by the quadrature spectra, only when the corresponding extremum in the quadrature

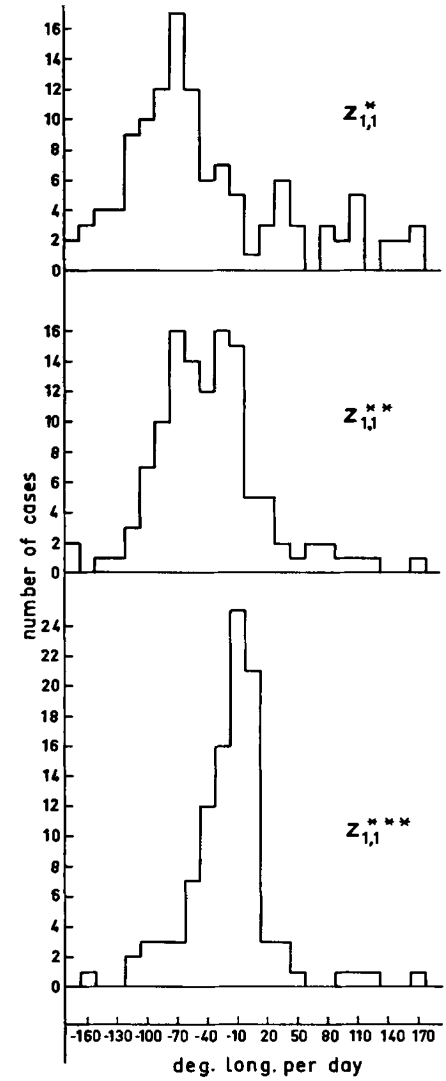

Fig. 4. The frequency distributions during the period from 1st October, 1957 to 31st January, 1958 of the 24-hours phase angle changes of the three time filtered fields z^*, z^{**}, and z^{***} for the component $(m, n) = (1, 1)$ of the 500 mb height field.

spectra is dominating in the effective frequency interval of the field. This may be illustrated by considering the motion of the three fields for the component $(m, n) = (1, 1)$ at the Northern Hemisphere. For this component the mean velocities of the z^*-field and of the z^{***}-field are seen to agree rather well with the velocity of respectively the fast and slow westward wave motion indicated by the quadrature spectrum, whereas the mean velocity of the z^{**}-field is between the two velocities indicated by the quadrature spectrum. This agrees with the fact that one of the two maxima in the quadrature spectrum is dominating within the effective frequency interval of the z^*-field and the other within that of the z^{***}-field, while both maxima are inside the effective frequency interval of the z^{**}-field. The histograms for the 24-hours displacements of the three fields are shown in Fig. 4. It is seen that for each of the fields z^* and z^{***} there is a distinct maximum corresponding nearly to the mean velocity of the field, whereas for the z^{**}-field there are two relative maxima, one at $-25°$ day^{-1} and another at $-70°$ day^{-1}. Also for the rest of the most large-scale components with wave numbers $m = 1$, 2, and 3, at the Northern Hemisphere, we find, as for the component $(m, n) = (1, 1)$, time filtered fields which are moving with mean velocities corresponding nearly to the westward wave motions indicated by the quadrature spectra.

From Table 7 it is seen that for all three zonal wave numbers there is for all three time filtered fields a gradual change from westward to eastward motion with increasing values of n. This increase of the velocity with increasing n, corresponds to an increase of the velocity of the zonal harmonics with increasing latitude, just as it was seen in section 4 for the unfiltered harmonics with $m \geqslant 4$.

As a support to the above results concerning the wave motions for $m = 1$, 2, and 3, at the Northern Hemisphere we shall mention the agreement with the wave motions found in our previous study based on data from the 500 mb level during three winter months in 1956–57 (Eliasen & Machenhauer, 1965). Only at one point there is an essential deviation, namely concerning the velocity of the z^*-field for $(m, n) = (2, 2)$, which was equal to $-40°$ day^{-1} for the winter months 1956–57 but equal to $-26°$ day^{-1} according to Table 7. This difference may be considered as the result of a relatively larger amplitude of the fastest westward motion with wave number $m = 2$ during the winter months of 1956–57 than during the period considered in the present study.

For the Southern Hemisphere the mean velocities in Table 7 are seen to vary from component to component in an irregular way without the gradual change from westward to eastward velocities with increasing n. The corresponding standard deviations are also, as mentioned above, larger for the Southern than for the Northern Hemisphere (cf. also Merilees (1966)). For the whole sphere the velocities stated in Table 7 are seen to vary with n in a more regular way than at the Southern Hemisphere, although the variation is less regular than at the Northern Hemisphere. For the Southern Hemisphere and for the whole sphere the westward wave motions indicated by the quadrature spectra are only for few of the most large-scale components clearly expressed in the motion of time filtered fields. This could also be expected, partly because of the smaller amplitudes of the moving waves (as indicated by the spectra) and partly because of the fact that both eastward and westward moving waves are indicated within the effective intervals of the time filtered fields.

7. On the theory of the Rossby waves

As a possible explanation of the observed short-periodic fluctuations of the planetary-scale components having the character of westward wave propagations it seems natural to consider the Rossby effect. Especially for the most large-scale components the wave motion connected with the Rossby effect will be influenced significantly by the effect of the horizontal divergence. As an attempt to investigate this influence a little closer, we may consider the solutions of the linear equations describing the wave motion superimposed upon a motion-free basic state. In order to simplify the treatment, we shall avoid the problems connected with the vertical structure by using an autobarotropic model of the atmosphere. From a theoretical point of view, the relevance of such a simple vertical model for the Rossby waves has been discussed recently by Diky (1965), and empirically its use may be supported by the fact that the moving parts of the planetary waves at the 1000 mb level are most of the time nearly in the same phase as at the 500 mb level. So we find for the phase differences between the 24-hours tendency field at the 500 mb and the 1000 mb level during the period December 1957 and January 1958 small negative mean values for the most large-scale components, for which the application of the 24-hours tendency field seems relevant. It should be remarked that for the unfiltered fields of the same components there are according to Table 5 considerable vertical tilts.

In the case of a homogeneous, incompressible atmosphere the horizontal motion is determined by the following three basic equations: the vorticity equation, the divergence equation, and the continuity equation combined with the boundary conditions in the vertical direction. Expressing the components u and v of the horizontal velocity vector by the stream function ψ and the velocity potential α, and introducing the height z of a pressure surface, we have the three linear equations

$$\left.\begin{aligned} &\frac{\partial}{\partial t}\nabla^2\psi+\beta v+f\nabla^2\alpha=0\\ &\frac{\partial}{\partial t}\nabla^2\alpha+\beta u-f\nabla^2\psi=-g\nabla^2 z\\ &\frac{\partial z}{\partial t}+H\nabla^2\alpha=0\end{aligned}\right\}\qquad(12)$$

determining the three fundamental variables ψ, α and z. As usual g denotes the accelaration of gravity, $f=2\Omega\sin\phi$ $\beta=(2\Omega\cos\phi)/R$, and H is the height of the homogeneous atmosphere. Assuming a wave motion of the form

$$\left.\begin{aligned} &\psi=\psi_m(\phi)\,e^{im(\lambda-\gamma t)}\\ &\alpha=i\alpha'=i\alpha_m(\phi)\,e^{im(\lambda-\gamma t)}\\ &z=\frac{2\Omega}{g}\eta=\frac{2\Omega}{g}\eta_m(\phi)\,e^{im(\lambda-\gamma t)}\end{aligned}\right\}\qquad(13)$$

the three equations may be written

$$\left.\begin{aligned} &q\nabla_S^2\psi+m\psi+\cos\phi\frac{\partial\alpha'}{\partial\phi}+\sin\phi\nabla_S^2\alpha'=0\\ &q\nabla_S^2\alpha'+m\alpha'+\cos\phi\frac{\partial\psi}{\partial\phi}+\sin\phi\nabla_S^2\psi=\nabla_S^2\eta\\ &q\eta+Q\nabla_S^2\alpha'=0\end{aligned}\right\}\qquad(14)$$

with $\nabla_S^2=R^2\nabla^2$, $q=-\dfrac{m}{2\Omega}\gamma$, and $Q=\dfrac{gH}{4\Omega^2R^2}$.

The velocity of propagation is expressed by q, and as fundamental parameter of the model we have the quantity Q. From the equations it is seen that if the stream function is symmetric with respect to the Equator, the velocity potential and the height field will be antisymmetric and vice versa. The meridional variation of the variables is now expressed in terms of unnormalized Legendre functions for wave number m. That is, in the case of a symmetric height field by the following series

$$\left.\begin{aligned} \eta_m(\phi) &= \sum_{r=0}^{\infty} \eta_{m,m+2r} P_{m,m+2r}(\phi) \\ \alpha_m(\phi) &= \sum_{r=0}^{\infty} \alpha_{m,m+2r} P_{m,m+2r}(\phi) \\ \psi_m(\phi) &= \sum_{r=0}^{\infty} \psi_{m,m+1+2r} P_{m,m+2r}(\phi) \end{aligned}\right\} \quad (15)$$

Inserting these series into the equations and collecting the terms containing the same Legendre function, we get the equations in the form

$$\left.\begin{aligned} &\sum_{r=0}^{\infty} (-q\eta_{m,m+2r} + (m+2r)(m+2r+1) \\ &\qquad \times Q\alpha_{m,m+2r}) P_{m,m+2r} = 0 \\ &\sum_{r=0}^{\infty} \left(\eta_{m,m+2r} + \left\{ \frac{m}{(m+2r)(m+2r+1)} - q \right\} \right. \\ &\qquad \times \alpha_{m,m+2r} - \frac{2r(m+2r-1)}{(m+2r)(2m+4r-1)} \\ &\qquad \times \psi_{m,m+2r-1} \\ &\qquad - \frac{(m+2r+2)(2m+2r+1)}{(m+2r+1)(2m+4r+3)} \\ &\qquad \left. \times \psi_{m,m+2r+1} \right) P_{m,m+2r} = 0 \\ &\sum_{r=0}^{\infty} \left(- \frac{(2r+1)(m+2r)}{(m+2r+1)(2m+4r+1)} \alpha_{m,m+2r} \right. \\ &\qquad + \left\{ \frac{m}{(m+2r+1)(m+2r+2)} - q \right\} \\ &\qquad \times \psi_{m,m+2r+1} \\ &\qquad - \frac{(m+2r+3)(2m+2r+2)}{(m+2r+2)(2m+4r+5)} \\ &\qquad \left. \times \alpha_{m,m+2r+2} \right) P_{m,m+2r+1} = 0 \end{aligned}\right\} \quad (16)$$

If these equations shall be satisfied the coefficients to the Legendre functions must all be zero, which leads to an infinite system of linear, algebraic equations. We shall consider especially the case with wave number 1 and the value of Q equal to 0.1, corresponding to the height 8800 m of the homogeneous atmosphere (or a surface temperature of nearly 300°K in an isentropic model). For this case the first six equations become

$$\left.\begin{aligned} &-q\eta_{1,1} + \frac{1}{5}\alpha_{1,1} = 0 \\ &\eta_{1,1} + \left(\frac{1}{2} - q\right)\alpha_{1,1} - \frac{9}{10}\psi_{1,2} = 0 \\ &-\frac{1}{6}\alpha_{1,1} + \left(\frac{1}{6} - q\right)\psi_{1,2} - \frac{16}{21}\alpha_{1,3} = 0 \\ &-q\eta_{1,3} + \frac{6}{5}\alpha_{1,3} = 0 \\ &\frac{4}{15}\psi_{1,2} + \eta_{1,3} + \left(\frac{1}{12} - q\right)\alpha_{1,3} - \frac{25}{36}\psi_{1,4} = 0 \\ &-\frac{9}{28}\alpha_{1,3} + \left(\frac{1}{20} - q\right)\psi_{1,4} - \frac{36}{55}\alpha_{1,5} = 0 \end{aligned}\right\} \quad (17)$$

It is seen that we have an eigen-value system with the velocity of propagation or more precisely q as eigenvalue. Considering the first three equations, and neglecting $\alpha_{1,3}$, we have a closed system of equations with three unknowns. The three possible values of γ in this case are listed in column I of Table 8 in degrees of longitude per day. In the same way we may use the first six equations by neglecting the quantity $\alpha_{1,5}$, and then we find the six γ-values shown in column II of the table, and so we may continue. It is seen that the values of γ obtained in the first approximation are still present with

Table 8

	I	II	III	IV	PW
R				− 9.6	− 10.0
			− 15.8	− 15.3	− 17.1
		− 30.0	− 28.5	− 28.4	− 36.0
	− 74.0	− 68.5	− 68.4	− 68.4	− 120.0
G	255	248	248	248	± 322
	− 661	− 636	− 635	− 635	
		883	876	876	± 789
		− 973	− 965	− 965	
			1332	1328	± 1247
			− 1361	− 1357	
				1772	± 1704
				− 1786	

small changes in the next approximation, and so on, and it seems that we have a quite rapid convergence (cf. also Diky & Golitzyn, 1966). The solutions corresponding to the large values in the lower part (G) of the table may be assumed to describe wave motions of the gravitional kind, whereas the essentially smaller velocities of propagation in the upper part of the table (R) seem to represent wave motions of the Rossby type. In the last column PW we have for a comparison the values of γ for the pure gravitational waves,

$$\gamma^2 = \frac{n(n+1)}{m^2R^2}\,gH,$$

and for the non-divergent Rossby waves,

$$\gamma = -\frac{2\Omega}{n(n+1)}.$$

Having obtained the characteristic values q, we may for each value find the corresponding characteristic solutions for the stream function, the velocity potential, and the height field, apart from an arbitrary factor. In quite a similar way, we may obtain the solutions for wave motions with an antisymmetric height field. Using the approximation with 12 equations, we get, still with $Q=0, 1$, for wave numbers $m=1, 2$, and 3 the velocities of propagation and the corresponding amplitudes for the components $\psi_{m,n}$, $\alpha_{m,n}$, and $\eta_{m,n}$, listed in Table 9. For each solution the arbitrary factor is chosen in the way that the largest amplitude in the height field becomes equal to 1. It should be noted that the expressions (16) and (17) are based on the unnormalized Legendre functions, whereas the values in Table 9 are based on the same normalization as used for the observed waves.

8. Discussion of the observed fluctuations in terms of Rossby waves

If we shall compare the observed wave motions, as stated in Table 6 with the theoretically obtained wave motions represented in the corresponding Table 9, it is obvious that the theoretical wave motions may be compared immediately with the observed wave motions for the whole sphere. Concerning the observed wave motions for each of the two hemispheres, however, we must take into account that each spherical harmonic component represents part of the symmetric as well as part of the antisymmetric height field. If the expansion coefficients for a hemisphere $a^{(H)}_{m,n}$ are expressed by the expansion coefficients for the whole sphere, $a_{m,n}$ we have, for the first components with wave number 1 the relations

$$a^{(H)}_{1,1} = a_{1,1} \pm 0.840\,a_{1,2} \mp 0.342\,a_{1,4} \pm 0.226\,a_{1,6} \mp 0.171\,a_{1,8} \pm \cdots$$

$$a^{(H)}_{1,2} = a_{1,2} \pm 0.840\,a_{1,1} \pm 0.523\,a_{1,3} \mp 0.130\,a_{1,5} \pm 0.062\,a_{1,7} \mp \cdots$$

$$a^{(H)}_{1,3} = a_{1,3} \pm 0.523\,a_{1,2} \pm 0.722\,a_{1,4} \mp 0.281\,a_{1,6} \pm 0.186\,a_{1,8} \mp \cdots$$

$$a^{(H)}_{1,4} = a_{1,4} \mp 0.342\,a_{1,1} \pm 0.722\,a_{1,3} \pm 0.571\,a_{1,5} \mp 0.158\,a_{1,7} \pm \cdots$$

For the Northern Hemisphere the upper signs shall be used and for the Southern Hemisphere the lower signs. From the theoretical results we should expect different velocities of propagation for the wave motions with the symmetric and the antisymmetric height field. As mentioned in section 5 this is also to some extent the case for the observed wave motions for the whole sphere, but not for the wave motions at one hemisphere. This fact may be explained from the above relations, according to which there will be for each component at one hemisphere a certain coupling between the symmetric and antisymmetric components for the whole sphere.

For wave number $m=1$ we have in the case of a symmetric height field a first solution corresponding to a Rossby wave with a velocity of propagation equal to about -70 degrees of longitude per day. In the height field of this wave motion the component with (m, n) equal to (1, 1) is dominating but also the component (1, 3) is present to some extent. For the next symmetric solution, corresponding to a Rossby wave, the velocity of propagation is about -28 degrees per day. In this solution the height field is dominated by the component (m, n) equal to (1, 3), but also the components (1, 5) and (1, 1) are present. In the following solutions the amplitude of the component (m, n) equal to (1, 1) is very small, and also the amplitude of (1, 3) is relatively small. As seen from Table 6 these solutions agree well with two of the westward wave motions indicated by the quadrature spectra for the whole sphere, both concerning the mean velocities and concerning the compo-

Table 9

		$m = 1$						$m = 2$					
	$\gamma \ldots$	-298.1	-68.4	-41.8	-28.5	-20.4	-15.3	-110.0	-47.0	-29.7	-20.8	-15.4	-11.8
	$n - m$												
Height	0		1.00		0.13		0.01		1.00		0.08		0.00
	1	1.00		1.00		0.12		1.00		1.00		0.09	
	2		0.42		1.00		0.11		0.57		1.00		0.08
	3	−0.21		0.45		1.00		−0.12		0.57		1.00	
	4		−0.09		0.51		1.00		−0.07		0.60		1.00
	5	0.02		−0.07		0.57		0.01		−0.06		0.63	
	6		0.01		−0.06		0.61		0.00		−0.06		0.66
	7	0.00		0.00		−0.06		0.00		0.00		−0.06	
Stream function	0	5.38		0.15		0.01		4.63		0.14		0.00	
	1		1.78		0.21		0.01		2.07		0.16		0.01
	2	0.59		1.53		0.19		−0.37		1.78		0.16	
	3		−0.24		1.53		0.17		−0.20		1.72		0.14
	4	0.04		−0.19		1.57		0.02		−0.16		1.71	
	5		0.01		−0.16		1.62		0.01		−0.14		1.72
	6	0.00		0.01		−0.14		0.00		0.01		−0.13	
	7		0.00		0.01		−0.14		0.00		0.01		−0.12
Velocity potential	0		0.47		0.03		0.00		0.22		0.01		0.00
	1	0.69		0.10		0.01		0.25		0.07		0.00	
	2		0.03		0.03		0.00		0.04		0.03		0.00
	3	−0.04		0.01		0.01		−0.01		0.02		0.01	
	4		0.00		0.01		0.01		0.00		0.01		0.01
	5	0.00		0.00		0.00		0.00		0.00		0.00	
	6		0.00		0.00		0.00		0.00		0.00		0.00
	7	0.00		0.00		0.00		0.00		0.00		0.00	

nents which are dominating in the wave motions. Also for the two hemispheres these wave motions are indicated with almost the same velocities. As the odd components at one hemisphere are describing the symmetric as well as the antisymmetric part of the height field, the wave motions mentioned are indicated also for odd components at each hemisphere, and it is even seen that the largest amplitude, in each case, occurs for an odd component.

Concerning the solutions with the antisymmetric height field, we have for the first solution, corresponding to a Rossby wave, a velocity of propagation equal to approximately -300 degrees of longitude per day. In the height field of this solution the component (m, n) equal to $(1, 2)$ is clearly dominating. If we have, however, the observation of the position of the wave only once a day, a motion of 300 degrees towards the west will be understood as a motion of 60 degrees towards the east. For the next solution the velocity of propagation is about -42 degrees of longitude per day. Also for the height field corresponding to this solution the component $(1, 2)$ is dominating, even if the component $(1, 4)$ is also present to some extent. So from these solutions it should be expected that the components $(m, n) = (1, 2)$, $(1, 4)$, and perhaps $(1, 6)$ of the height field will contain a wave motion with a mean velocity of about $-42°$ day^{-1}, but the component $(m, n) = (1, 2)$ should also contain a wave motion which will be considered as a motion of $+60°$ day^{-1}. In agreement with this it is seen in Table 6 that a wave motion of approximately $-42°$ day^{-1} is indicated for the components $(m, n) = (1, 4)$, and $(1, 6)$ for the whole sphere. No indication of this wave motion is however found for the component $(m, n) = (1, 2)$, a fact which very well may be the result of the interaction be-

$m=3$					
−57.3	−31.8	−21.4	−15.6	−11.9	−9.4
	1.00		0.05		−0.01
1.00		1.00		0.06	
	0.70		1.00		0.05
−0.09		0.66		1.00	
	−0.06		0.67		1.00
0.01		−0.06		0.69	
	0.00		−0.05		0.71
0.00		0.00		−0.05	
4.46		0.12		0.00	
	2.43		0.13		−0.03
−0.26		2.02		0.13	
	−0.17		1.89		0.12
0.01		−0.14		1.84	
	0.01		−0.13		1.82
0.00		0.00		−0.11	
	0.00		0.00		−0.11
	0.11		0.00		−0.01
0.12		0.04		0.00	
	0.03		0.02		0.00
−0.01		0.01		0.01	
	0.00		0.01		0.01
0.00		0.00		0.00	
	0.00		0.00		0.00
0.00		0.00		0.00	

tween the two wave motions, which this component should contain. Also at the two hemispheres the wave motion with the velocity about $-42°$ day^{-1} is seen to be present, again for both even and odd components.

Also for the wave numbers 2 and 3 there is a rather good agreement between the theoretical wave motions represented in Table 9 and the observed wave motions in Table 6. Except for the very fast wave motions, the theoretical velocities, especially for $m=3$, are seen to be somewhat larger, numerically, than the observed velocities, a discrepancy which may possibly be ascribed to the neglect of the zonal flow in the theory. As for $m=1$ we have that for each of the wave numbers 2 and 3, the solutions for the antisymmetric height field, connected with the fastest wave motions, are dominated by the most large-scale components, i.e. $(m, n)=(2, 3)$ and $(3, 4)$ respectively. These fastest wave motions are moving too fast, however, to be indicated in the quadrature spectrum, but not so fast that they should prevent the observations of the wave motions with the second fastest velocities. So we have for $m=2$ a wave motion with the velocity $-27°$ day^{-1}, dominated by the component $(m, n)=(2, 3)$, and for $m=3$ a wave motion with a velocity $-11°$ day^{-1}, dominated by the component $(m, n)=(3, 4)$.

The general agreement between Table 6 and Table 9 seems strongly to support the conclusion that an essential part of the short-periodic fluctuations of the very long wave components may be explained in terms of wave motions of the Rossby type.

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О НАБЛЮДАЕМЫХ ДВИЖЕНИЯХ КРУПНОМАСШТАБНЫХ АТМОСФЕРНЫХ ВОЛН

На основе данных со всей Земли за различные периоды Международного Геофизического Года изучаются движения крупномасштабных волн на уровне 500 *мб* и, до некоторой степени, на уровне 1000 *мб*. Поле высоты изобарической поверхности разлагается по сферическим гармоникам как для всей Земли, так и для каждого полушария. Прослеживается относительная важность различных компонент разложения поля высоты для всей Земли и рассматриваются сезонные изменения в двух полушариях.

Исследуется движение различных волновых компонент путем вычисления средних 24-х часовых изменений фазового угла. Найдено, что за исключением наиболее крупномасштабных, квазистационарных компонент, средние скорости хорошо согласуются с формулой Россби–Гаурвица для волновых движений в баротропной, бездивергентной модели с твердым вращением в качестве основного потока.

Флуктуации квазистационарных волн изучаются с помощью квадратурного спектрального анализа и путем использования различных временных фильтров. Некоторая часть регулярных флуктуаций, по-видимому, объясняется различными волновыми движениями на запад с разными скоростями. Каждое из этих волновых движений в западном направлении, по-видимому, состоит из нескольких сферических гармоник с различными амплитудами. Количественные результаты, касающиеся скоростей распространения и амплитуд в общем подтверждают идею, что эти волновые движения могут быть объяснены эффектом Россби, который для наиболее крупномасштабных компонент комбинируется с эффектом сжимаемости.