Thermal convection in vertical tubes with application to geysers

By T. S. MURTY,[1] *Oceanographic Research, Marine Sciences Branch, Department of Energy, Mines and Resources, Ottawa, Canada*

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ABSTRACT

Thermal convection in vertical tubes has been studied theoretically as an initial value problem using numerical techniques. The thermal and viscous boundary layers at the wall are taken into account approximately. Special emphasis is placed on the Geophysical application of this problem, particularly in examining the convective theories of geyser eruptions from a hydrodynamic point of view and determining the forms of convection during the preliminary stage before boiling starts. The proper time scales and other associated properties are explained through determining the appropriate eddy coefficients.

1. Introduction and formulation of the physical problem

The phenomena of turbulent thermal convection are of interest in problems arising in Hydrodynamics, Geology and Engineering. The purpose of the present work is to study convection in vertical tubes as an initial value problem with turbulence parameterized through eddy coefficients, using numerical techniques and applying it in determining the possible forms of convection in geysers during the early stages of preparation for a new eruption after a previous eruption.

There are essentially two schools of thought on the mechanism of geyser eruptions. The first one is due to Bunsen (1847) and is based on the effect of pressure on the boiling point of water. The second theory is due to Thorkelsson (1928) and is based on the idea that dissolved gases play an important role in the eruption phenomena. We are not going to say anymore about the gas theories till the very end and all the subsequent discussions in this paper are mainly confined to the boiling theories, notably those of Bunsen (1847) and Hales (1937).

The explanation given by Bunsen for the eruption phenomena essentially depends on the formation of a "critical zone" (a Zone in which the local temperature approaches the local boiling temperature at that pressure). By implication, Bunsen and later authors like Tyndall (1902), Sherzer (1933) and Hales (1937) took the critical zone to extend fully across the geyser vent, probably because observations were made at only one position—namely, the axis—in each horizontal plane in the vent. The essential question is whether such an extended critical zone is possible, if active convection is present during the pre-boiling stage. As will be seen in the results of the present study, what will be called the "critical region" of close approach to the local boiling temperature is far from being the extended zone suggested by Bunsen and most of the later authors.

Bunsen visualized that the strong effect of the hydrostatic pressure on the boiling temperature of water is important in the geyser problem. Initially local temperature everywhere is smaller than the local boiling temperature by varying amounts. Since the temperatures at the top and bottom are observed in a number of geysers to be more or less constant and less than the corresponding boiling temperatures,

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boiling does not occur in these regions. If it occurs, it has to occur somewhere in between, depending upon the following two factors: (1) proximity to the bottom (highest local temperature), (2) proximity to the top (lowest boiling temperature).

There are two possibilities in the pre-boiling stage after a previous eruption. The first one is active average convective motion throughout this pre-boiling stage and this idea is tested in this present study. The other possibility is that in which, for a major portion of the pre-boiling stage, the temperature gradient is sub-critical and thus mean convective motion is suppressed, and due to thermal conduction the temperature gradient slowly increases to the critical value.[1] Hales (1937) investigated this later idea treating the vent of a geyser as a cylindrical tube and using linearised mathematical approach similar to that of the classical convection problem. His results showed that the most unstable mode is the fundamental asymmetric mode and the appropriate value of the thermal diffusivity is 130 $cm^2 \cdot sec^{-1}$.

At the outset there appears to be some evidence against Hales' theory as a picture of the early phase up to the critical zone formation. If mean convective velocites are absent and the temperature profile (see Bunsen, 1847, diagram 1) changes only by eddy conductive effects, then the only likely position for the critical zone is at the bottom. Another possible significant discrepancy is that Hales' theory predicts the fundamental asymmetric mode to be the most unstable mode whereas the published descriptions of the circulation patterns (hot water raising at the axis with a cold downward current near the walls) in geysers definitely point to the fundamental symmetric mode as the observed one.

One of the important problems that arises in any theory of geyser eruptions is to obtain the correct time scales for the formation of the critical zone (or region). Most of the present computations with ν (eddy Kinematic viscosity) and $\varkappa$ (eddy thermal diffusivity), 10^2, 10^3 and 10^4 times the molecular values (molecular values of ν and $\varkappa$ are 0.0028 and 0.0018 $cm^2 \cdot sec^{-1}$ at the initial mean temperature of the Great Geyser of Iceland) showed that exchange coefficients 10^5 times bigger than molecular values are necessary to explain the proper time scales. It is interesting to note that this order of magnitude for ν and $\varkappa$ agree with that of Hales.

As in Hales' study, the present work treats the Great Geyser as a circular vertical tube, but unlike Hales' work the present study is time dependent and nonlinear. Starting with a basic state in which the vent is full with water but no motion, with prescribed top and bottom temperatures and wall gradient, the temperature and velocity fields are computed with increasing time. The computation is terminated when a critical region has formed. To determine the formation of this critical region, at every time step the difference between the boiling temperature and the fluid temperature at each grid point is calculated. When this difference became zero or negative it was assumed that a critical region has formed. The boiling temperature was calculated as a function only of the Hydrostatic pressure though it could also be a function of the chemical composition, degree of nucleation and other factors. Since there is no easy way to incorporate them into the present problem and since these factors may effect the boiling temperature at every depth uniformly, these are ignored.

2. Mathematical analysis

Free convection in vertical tubes with prescribed boundary temperatures is studied as an initial value problem under the Boussinesq approximation using numerical techniques with turbulence parameterised through eddy coefficients. Two geometries are used in the study: (*a*) Rectangular slot (two-dimensional cartesian coordinates $x-z$, (*b*) circular tube (equal area cylindrical polar coordinates r^2-z). In each case the length of the tube is taken as 22.5 meters and the radius of the tube (half the width for the slot) as 1.5 meters which are appropriate dimensions for the Great Geyser of Iceland. For the rectangular slot cases uniform grid intervals and uniform exchange coefficients were used. For the circular tube, the grid interval is uniform in the vertical direction but decreases with the inverse of the radial distance. In this manner the grid gets finer and finer as the wall is approached. In addition to this, the coefficients

[1] The idea of retarded convection due to the narrow and twisted nature of the geyser vent and the consequent importance of conduction in forming the critical zone is popularly attributed to Bunsen (see, for example, Marler, 1963).

were made to decrease from eddy values at the axis and in the bulk of the liquid to molecular values at the wall. These two things together provide for an approximate inclusion of the viscous and thermal boundary layers at the wall.

The initial temperature distribution is taken as follows (which is again appropriate for the Great Geyser). The bottom is at a uniform and constant temperature of 125°C, the top at 85°C and the walls have a linear gradient from 125°C to 85°C. In addition, to start the motion, a small amplitude (10^{-5} °C) sinusodal temperature perturbation in the horizontal direction is imposed such that there is a sine wave from wall to wall with the maximum at the axis and the minimum at the walls. The reason for selecting the minimum at the walls is that as some descriptions show, after an eruption, some of the ejected water cools before it flows back into the vent along the sides.

All the computations with the circular tube assume symmetry about the axis of the tube. In order to avoid the assumption of symmetry, but still stay within the limits of the computer capacity (CDC 6600 computer at the National Centre for Atmospheric Research, Boulder, Colorado, U.S.A. with a storage capacity of 64,000), asymmetric perturbations were used for a few rectangular slot cases. "Symmetric perturbation" refers to one whose amplitude and phase is the same on either side of the axis. The asymmetric perturbation used consisted of a symmetric perturbation plus half a sine wave of amplitude one-tenth of the symmetric perturbation, but with phase such that the maximum is on the left wall and the minimum is on the right wall. A big asymmetric perturbation will make the critical Rayleigh number irrelevant.

Here we will describe briefly the equations and the symbolic finite-difference forms for the primitive equation model in $r^2 - z$ coordinate system. The momentum equations, continuity equation and thermal energy equation with space dependent coefficients are:

$$\frac{\partial U}{\partial t} = LU - 2\sqrt{X}\frac{\partial P}{\partial X} + 2\nu_r\left(4X\frac{\partial^2 U}{\partial X^2} + 4\frac{\partial U}{\partial X} - \frac{U}{X}\right)$$

$$+ 4\frac{d\nu_r}{dr}\sqrt{X}\frac{\partial U}{\partial X} + \nu_z\left(\frac{\partial^2 U}{\partial z^2} + 2\sqrt{X}\frac{\partial^2 W}{\partial z \partial X}\right),$$

$$\frac{\partial W}{\partial t} = LW - \frac{\partial P}{\partial z} + \alpha g(T - \widetilde{T}) + \frac{\partial}{\partial z}\widetilde{W^2}$$

$$+ \nu_r\left(4X\frac{\partial^2 W}{\partial X^2} + 4\frac{\partial W}{\partial X} + 2\sqrt{X}\frac{\partial^2 U}{\partial z \partial X} + \frac{1}{\sqrt{X}}\frac{\partial U}{\partial z}\right)$$

$$+ \frac{d\nu_r}{dr}\left(\frac{\partial U}{\partial z} + 2\sqrt{X}\frac{\partial W}{\partial X}\right) + 2\nu_z\frac{\partial^2 W}{\partial z^2}. \tag{2.1}$$

$$2\sqrt{X}\frac{\partial U}{\partial X} + \frac{U}{\sqrt{X}} + \frac{\partial W}{\partial z} = 0,$$

$$\frac{\partial T}{\partial t} = LT + \varkappa_r\left(4X\frac{\partial^2 T}{\partial X^2} + 4\frac{\partial T}{\partial X}\right) + 2\frac{d\varkappa_r}{dr}\sqrt{X}\frac{\partial T}{\partial X}$$

$$+ \varkappa_z\frac{\partial^2 T}{\partial z^2},$$

where

$$L\varphi \equiv -\left\{2\frac{\partial}{\partial X}(\sqrt{X}U\varphi) + \frac{\partial}{\partial z}(X\varphi)\right\},$$

$$\varphi \equiv U, W, T,$$

$$X \equiv r^2. \tag{2.2}$$

Here U and W are the radial and vertical velocities, P is the pressure, T is the temperature and α is the coefficient of thermal expansion. ν_r and ν_z are the eddy Kinematic viscosities in the r and z directions; $\varkappa_r$ and $\varkappa_z$ are the eddy thermal diffusivities in the r and z directions and upper curved dash on T and W^2 in the vertical momentum equation denotes horizontal area averaging.

The variables U, W, P and T are represented on a staggered grid with fictitious grid levels exterior to the regular grid for convenience in satisfying the boundary conditions (Deardorff 1966, Murty 1967). The finite difference forms used are similar to Lilly's (1965) and Arakawa's (1966) schemes. However, some modification was necessary for the equal area cylindrical polar coordinate system. The symbolic finite-difference forms for the momentum equations, continuity equation and thermal energy equations are:

$$\delta_t U = -2\delta_X(D_{2i}\bar{U}^X\bar{U}^X) - \delta_z(\overline{W}^X\bar{U}^z) - 2D_{2i}\delta_X P$$

$$+ 2\nu_r\left(4D_{2i}^2\delta_X^2 U + 4\delta_X U - \frac{U}{D_{2i}^2}\right)$$

$$+ 4\frac{dv_r}{dr} D_{2i}\,\delta_X U$$

$$+ v_z(\delta_z^2 U + 2D_{2i}\,\delta_X\delta_z W),$$

$$\delta_t W = -2\delta_X(D_{2i+1}\,\bar{U}^z\overline{W}^X) - \delta_z(\overline{W}^z\overline{W}^z)$$

$$-\delta_z P + \delta_z \overset{\wedge\wedge\wedge}{W^2}$$

$$+\alpha g(\overline{T-\bar{T}})^z + v_r\Bigg(4D_{2i+1}^2\,\delta_X^2 W + 4\delta_X W$$

$$+ 2D_{2i}\,\delta_X\delta_z U + \frac{1}{D_{2i}}\,\delta_z U\Bigg)$$

$$+\frac{dv_r}{dr}(\delta_z U + 2D_{2i}\,\delta_X W) + 2v_z\delta_z^2 W, \qquad (2.3)$$

$$2\cdot\delta_X(D_{2i+1}\,U) + \delta_z W = 0,$$

$$\delta_t T = -2\cdot\delta_X(D_{2i+1}\,U\bar{T}^X) - \delta_z(W\bar{T}^z)$$

$$+\varkappa_r(4D_{2i+1}^2\,\delta_X^2 T + 4\delta_X T)$$

$$+2\frac{d\varkappa_r}{dr}D_{2i}\,\delta_X T + \varkappa_z\delta_z^2 T,$$

where

$$\delta_X\varphi \equiv \frac{\varphi_{i+\frac{1}{2}} - \varphi_{i-\frac{1}{2}}}{\Delta X}, \qquad (2.4)$$

$$\bar{\varphi}^X \equiv \frac{\varphi_{i+\frac{1}{2}} + \varphi_{i-\frac{1}{2}}}{2},$$

with φ being any dependent variable with similar definitions for other independent variables. Also we have:

$$D_{2i} \equiv (\sqrt{X})_i, \qquad (2.5)$$

where the subscript i denotes the radial direction.

From these equations the radial and vertical velocity fields and the temperature field can be computed prognostically. However, the pressure field has to be computed simultaneously at all the grid points from the following divergence equation formed from the momentum and continuity equations:

$$\nabla^2 P = Q - \overline{Q}, \qquad (2.6)$$

where ∇^2 denotes the Laplacian operator in the $r^2 - z$ system and Q is the forcing function which has the following general form:

$$Q \equiv \frac{\partial}{\partial X}(LU) + \frac{\partial}{\partial z}(LW) + \alpha g(T - \overset{\wedge\wedge\wedge}{T}). \qquad (2.7)$$

The traditional nondimensionalisation can be performed on these equations to bring out the relevant non-dimensional parameters.

$$\text{Rayleigh number} = Ra \equiv \alpha g a^4 (T_b - T_t)/\nu\cdot\varkappa\cdot H, \qquad (2.8)$$

$$\text{Prandtl number} = P_a \equiv \nu/\varkappa. \qquad (2.9)$$

Here a is the radius of the tube, H its depth and T_b and T_t are the bottom and top temperatures.

In this study velocity and temperature boundary conditions appropriate to rigid and conducting boundaries have been used. Initial calculations with either a rigid top or Rayleigh free surface did not show significant difference in the results. The integration in time is carried out using the Adams-Bashforth Scheme. The finite-difference form of the divergence equation for the pressure field is derived directly from the finite-difference forms of the momentum and continuity equations. In so doing a divergence correction to the forcing function is applied (Harlow & Welch, 1965). The divergence equation which is of the elliptical form is solved simultaneously at all the grid points at each time step using two different relaxation methods, namely the accelerated point Liebmann method (Thompson, 1960) and the modified extrapolated implicit line Liebmann method (Deardorff, 1966).

3. Discussion of the results

Figures 1, 2 and 3 show the state of motion at 201.9 seconds (the critical region has just formed) through contours of temperature, horizontal velocity and vertical velocity for a rectangular slot with $\nu = 0.28$ cm$^2\cdot$sec^{-1}, and with an initial symmetric perturbation. From these diagrams it is clear that hot water flows upward near the axis and cold water flows downward near the walls with corresponding radial motions. The temperature varied by about 25°C from the axis to the wall at the

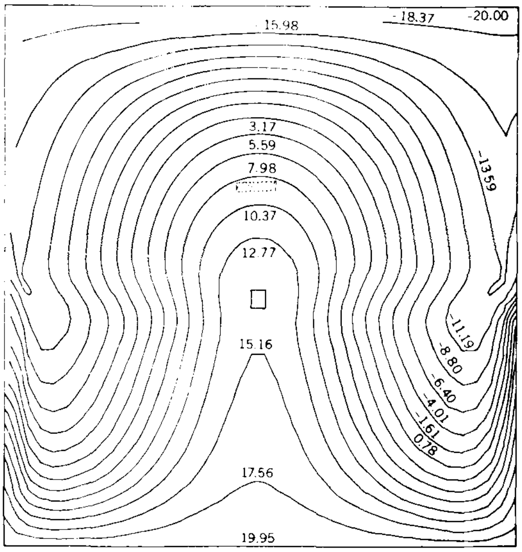

Fig. 1. Rectangular slot. $\nu = 0.28\ \mathrm{cm^2 \cdot sec^{-1}}$. Contours of temperature at 201.9 seconds.

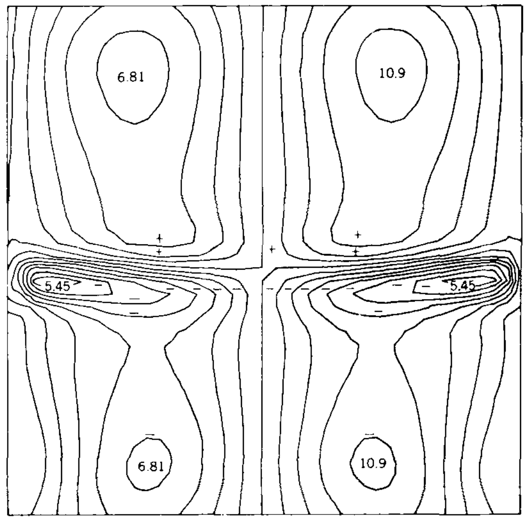

Fig. 2. Rectangular slot. $\nu = 0.28\ \mathrm{cm^2 \cdot sec^{-1}}$. Contours of horizontal velocity at 201.9 seconds. Contour interval is 1.362 cm/sec. The velocity is defined positive towards the wall and negative towards the centre.

level of the critical region. Near the top and bottom the temperature varied by about 4°C across a radius. This circulation pattern which is consistent with the flow pattern shown by the countours of U and W in Figures 2 and 3 shows that the mode excited is the fundamental symmetric mode. The maximum values of the radial and vertical velocity fields developed are 10.9 and 125.8 cm/sec respectively for this case. This flow pattern created a critical region between 1/2 and 1/3 depth on the axis, the area of which is about 1/40th to 1/20th of the area of the vent.

If we consider the state when the velocity field is of the order of one centimeter per second as the initial state, then all the significant temperature increase and development of motion takes place in only about 22 seconds for this case. These rapidly developing and unrealistic motions are thought to be due to the use of inappropriate values for ν and $\varkappa$. However, the time scales tend to become realistic once the coefficients are substantially increased as can be seen from Table 1. When molecular values are used the computation became unstable. When eddy values which are 10^6 times the molecular values are used, the motions developed very fast and then died down. Thus, the largest value of ν used puts an upper limit on the eddy value consistent with the hypothesis of mean convection. The range of 20 to 45 minutes for the time in which all significant development of motion took place for $\nu = 280\ \mathrm{cm^2 \cdot sec^{-1}}$ is obtained by extrapolating from the results of other cases on a logarithamic graph.

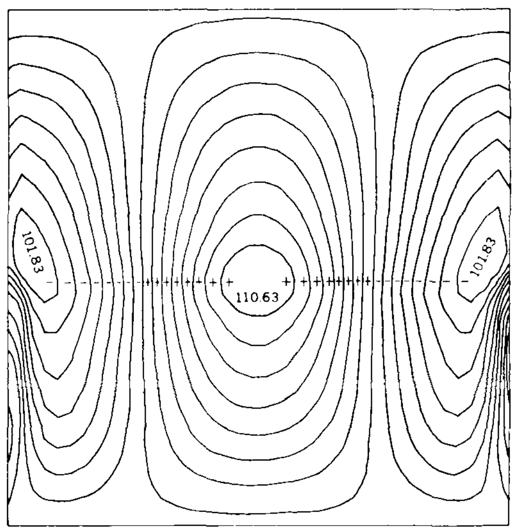

Fig. 3. Rectangular slot. $\nu = 0.28\ \mathrm{cm^2 \cdot sec^{-1}}$. Contours of vertical velocity at 209.9 seconds. Contour interval is 15.17 cm/sec. The velocity is defined positive upwards and negative downwards.

Table 1. *Variation of relevant times with eddy coefficients*

ν cm$^2 \cdot$sec^{-1}	Rayleigh no.	Time of formation of a critical region (sec.)	Time when W_{max} is 1 cm/sec (sec.)	Time in which all the significant development took place (sec.)
0.0028	1.228×10^{12}	Computation is unstable	—	—
0.28	1.228×10^{8}	202.4	179	23.4
5.6	0.307×10^{6}	209.1	180	29.1
56	3070	310.6	183	127.6
280 (extrapolated)	122.8	1380 to 2880	200	1180 to 2680
2240	1.92		—	—

No direct computation with $\nu = 280$ over a long enough time to form a critical region could be made because of the very stringent limit on the time increment imposed by the diffusion criterion. A calculation with $\nu = 280$ up to 100 seconds gave W_{max} of 0.44 cm/sec and an extrapolation of this gives for the time for the critical region formation anywhere from 20 minutes to 2.25 hours.

To confirm this tendency of realistic times with increased coefficients, an examination is made of the variation of the magnitudes of the terms in the vertical momentum equation. Again for $\nu = 280$, each term is extrapolated from the corresponding values for other coefficients. This examination showed that when eddy values which are 10^5 times the molecular values are used, the viscous terms are just about the magnitude to balance the vertical advection terms and consequently the conditions for a neutral or damped state are being approached. When the coefficients are 10^6 times the molecular values, the viscous term completely dominates the other terms and the motion is damped. Thus, one can say that the order of magnitude of 10^5 times the molecular values for ν and $\varkappa$ is nicely bracketed between 56 cm$^2 \cdot$sec^{-1} (4 orders) where the motions are too fast and 2240 cm$^2 \cdot$sec^{-1} (6 orders) where they are definitely damped.

Under the working hypothesis that an upper limit for ν is given by a critical Rayleigh number, the next question is to investigate the appropriate value of this number for the present problem. The value of 10^5 times the molecular value of ν gives 122.8 for the critical Rayleigh number (Ra_c) for the fundamental symmetric mode in a vertical rectangular slot. Yih's (1959) theoretical study with insulated walls for the same geometry puts it at 237.6. Donaldson's (1961) laboratory study places it between 84 and 290.[1] Our value falls in the range given by Donaldson but is only slightly greater than half the value given by Yih. The possible reasons for this discrepancy will be explained later.

More serious than the discrepancy in the value of the Ra_c for the fundamental symmetric mode is the question of the most unstable mode in a vertical rectangular slot (or tube). Hales and Yih showed that it is the fundamental asymmetric mode that should be excited because it has the lowest Ra_c. The authors' computations suggested (though do not prove conclusively) that the mode actually excited is the fundamental symmetric mode. In the author's computations in rectangular slot with asymmetric perturbation of initial amplitude one tenth of the symmetric perturbation, for some time after the initial state the asymmetric mode grew faster than the symmetric mode relatively speaking (its amplitude became 1/7th of the symmetric perturbation) but after the motion developed strongly the symmetric mode started to grow faster (the amplitude of the asymmetric mode decreased to 1/36th of that of the symmetric mode when a critical region has formed). This indicates that the asymmetric mode is more unstable and grows faster in the beginning, but it can be suppressed by the symmetric mode later. This result, however, is not conclusive because no cases were studied where the

[1] This is inferred by the author after a careful examination of Donaldson's work.

initial amplitudes of the asymmetric and symmetric perturbations were equal or reversed in relative magnitudes.

One can reconcile the results of the linear theories of Hales and Yih, the laboratory experimental results of Donaldson and the author's results with the actual observations in geysers if one adapts the following argument. Since linear theories can only predict which mode is the most unstable one at the initial time and since they cannot predict what happens at subsequent times, their result that the fundamental asymmetric mode is the most unstable one is probably correct. The excitation of the fundamental symmetric mode in natural geysers and in laboratory experiments is consistent with the suggestion that the symmetric mode can overtake and suppress the asymmetric mode after the motion develops. Another possibility is that the conducting walls (in the present work) may be significant, at least to the extent of determining the mode, so that the results of the linear theories with insulated walls should not be compared on an identical basis with the results for conducting walls.

Because of the possible differences in the behaviours between conducting and insulating walls, the following question can be asked for the geyser problem. Do the walls exhibit more of an insulating or a conducting nature? To answer this question the following situation may be studied. Let the fluid and the wall be at the same temperature initially. Then at $t = 0$, let the fluid temperature be increased by ΔT. After a given time, one can calculate the time-averaged heat-flux in the fluid, say, a few centimeters away from the wall. The temperature change ΔT_W in the wall which is required so that the time averaged heat-flux matches the fluid-flux can be calculated now. If this temperature change in the wall is of the same order as ΔT (temperature change in the fluid), the conclusion is that the wall behaves more like insulated. On the other hand, if it is an order of magnitude less than ΔT, then it means that the wall is more like conducting. In the present study, the conclusion is that the wall behaves more or less like an insulated one for small eddy coefficients with unrealistically fast development of motion. However, with appropriate eddy coefficients the development of motion is realistic and the wall behaves more like a conducting one.

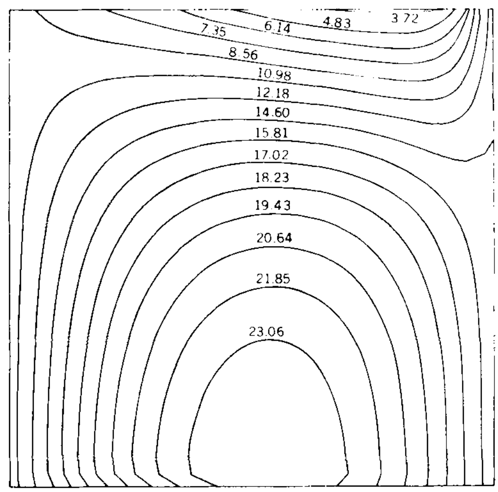

Fig. 4. Circular tube. $\nu = 5.6$ cm$^2 \cdot$sec^{-1}. Contours of boiling temperature minus local temperature at 201.8 seconds.

Figure 4 shows the contours of DBT (difference between boiling temperature and actual temperature) at 201.8 seconds for the circular tube case with $\nu = 5.6$ and $\varkappa = 3.6$ cm$^2 \cdot$sec^{-1}. A critical region whose area is very small is about to form at 209.1 seconds at about 1/3 depth. The area of the critical region is at least an order of magnitude less than the area of the vent. The fact that the critical region is small in its horizontal extent is associated with the result that strong horizontal temperature gradients (25 to 30°C in 150 cms) exist in the vent across a radius at the level of the critical region. Though the value of DBT (boiling temperature minus local temperature) is highest at about 1/3 depth initially, it is here that the critical region finally forms because the correlation between the vertical velocity and temperature is highest here.

Table 2 compares some of the observed and computed parameters in geysers. Items 2, 6 and 9 need some explanation. In the computations, an extrapolated value (with appropriate ν and $\varkappa$) for the time of formation of a critical region is between 20 and 45 minutes. In the observed cases, strictly speaking it is very difficult to know exactly when a critical region has formed. The only physical time that is known with reasonable certainty is the period of eruption. For the Great Geyser of Iceland abortive eruptions indicating probably the formation of a

Table 2

Parameter	Extrapolated value at $\nu = 280$, $cm^2 \cdot sec^{-1}$, $\varkappa = 180\ cm^2 \cdot sec^{-1}$	Observed value
1. Nature of the mode excited as inferred from circulation pattern	Fundamental symmetric mode?	Fundamental symmetric mode. Bunsen (1847), Lang (1880)
2. Time of formation of a critical region (or zone)	20 to 45 minutes	Few minutes to several days. Bunsen (1847). Allen & Day (1935)
3. Maximum velocity field	80 cm per second	50 to 150 cm/sec. Bunsen (1847)
4. Depth of the critical region from the top	About $\frac{1}{3}$ of total depth	$\frac{1}{3}$ to $\frac{1}{2}$ of total depth. Bunsen (1847), Birch *et al.* (1949), Sigurgeirsson (1949)
5. Vertical temperature gradient	40°C in 22.5 meters	26 to 40°C in 22.5 meters. Bunsen (1847), McNitt (1965)
6. Horizontal temperature gradient	At the level of the critical region it is 25°C in 150 cm. Near top and bottom it is about 4°C in 150 cm	—
7. Time averaged heat flux at the critical region	0.017 $cal/cm^2/sec$	0.01 to 0.1 $cal/cm^2/sec$. Elder (1965) for geothermal regions
8. Instantaneous maximum heat flux at the critical region	0.46 $cal/cm^2/sec$	0.4 $cal/cm^2/sec$. Dawson (1964) for geothermal regions
9. Heat flux at the top	6 $cal/cm^2/sec$	About 1 $cal/cm^2/sec$. Benseman (1959)

critical region may occur every hour while the eruption period varies from 11 to 24 hours. Regarding item 6, no observational value is available for the horizontal temperature gradient. Elder (1965), talking about geothermal regions in general, stated that strong horizontal gradients of temperature are found in a distance of one meter. Regarding item 9 the extrapolated heat flux at the top in the computations is about 6 $cal/cm^2/sec$ whereas some observed values (Berseman 1959) have a maximum of about 1 $cal/cm^2/sec$. Thus, the computed value seems to be somewhat high. The effective $\varkappa$ at the top may not be as high as 180 $cm^2 \cdot sec^{-1}$. For the computed value to agree with the observed value, the effective $\varkappa$ at the top should be about 30 $cm^2 \cdot sec^{-1}$.

In the light of the basic hypothesis of this paper, namely the presence of active mean convection throughout the preliminary stage, and in view of the computational results discussed above, a most important question is how it is for the Great Geyser and some other geysers of this class, the time scales are too long. Probably part of the reason lies in the fact that the heat-flux available to these long-period geysers is low. There is also the possibility that after an eruption due to the filling of the vent, there is quite an agitation and hence the relevant exchange coefficients may be large, with the result that the Great Geyser may operate in a Hales' sense, the temperature gradient just oscillating around the critical value.

4. Conclusions and suggestions for observational work

In conclusion the following remarks may be made. On the hypothesis of active mean convection throughout the preliminary stage before boiling starts in geyser vents, the eddy coefficients needed to produce the proper time scales are roughly 10^5 times the molecular values. This particular result is in agreement with Hales' work; though he has not considered the question of time scales. Unlike the conduction theories which produce a critical zone at the bottom, the present work produced a critical region whose area is at least one order of magnitude less than the area of the vent. The depth of the critical region, the magnitude of the velocity field produced, the heat-flux etc. agree quite well with observed values in geysers.

An alternative to the boiling theories is the gas theory of Thorkelsson (1928) which assumes that the so-called permanent gases are responsible for causing geyser eruptions. But Allen and Day (1935) clarified after careful observations that permanent gases are extremely scarce in geysers. Actually the gas theory is quite similar to the boiling theory of Bunsen in many respects. The equilibrium concentration of the gas increases with the increase of pressure and decreases with the increase of temperature. This is very similar to the two opposing influences on the boling temperature in the boiling theory (discussed in section 1). The gas theory has many features closely corresponding to the boiling theories that are just as difficult to fit into a consistent picture of the process. For example, the gas has to be transported to the critical region (or zone) in the proper time. Here, by critical zone is meant the region in the vent where the eruption is initially triggered. If gas and water are forced simply through the geyser vents, one would expect that the mixture should come up to the surface in a few seconds. Then it is difficult to see why the eruption period should be around an hour (even ignoring still longer periods).

The author would like to make the following suggestions for further observational work in geysers.

1. Measurements of the temperature field at various radial positions, for the same vertical level, preferably at several depths and times. This should settle the question of a critical zone spreading from wall to wall.
2. Temperature fluctuation statistics can be obtained and these could give an indirect estimate of velocity fluctuations and eddy coefficients, should direct velocity measurements be too difficult.
3. Some estimates of the wall heat flux are very desirable.
4. To explain the peculiarities of individual geysers, an accurate plumbing of their vents is necessary.

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ТЕРМИЧЕСКАЯ КОНВЕКЦИЯ В ВЕРТИКАЛЬНЫХ ТРУБАХ С ПРИЛОЖЕНИЕМ К ГЕЙЗЕРАМ

Термическая конвекция в вертикальных трубах изучается теоретически как задача с начальными условиями с использованием численных расчетов. Термический и вязкий пограничные слои у стенки учитываются приближенно. Особое внимание уделяется геофизическим приложениям этой задачи, в частности, при рассмотрении конвективных теорий гейзерных извержений с гидродинамической точки зрения и определении форм конвекции в начальной стадии, прежде чем возникает кипение. Характерный масштаб времени и другие свойства объясняются через соответствующие коэффициенты турбулентного обмена.