On the influence of convective clouds on the large scale stratification

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ABSTRACT

Changes in the large scale stratification resulting from non-adiabatic processes within convective clouds are investigated. The changes are considered as a result of net entrainment, a factor which appears highly sensitive to the presence and dynamics of precipitation. It is shown that Cumulus or Cumulonimbus convection can be a selfamplifying process, under certain circumstances, producing an upper, cold-core low pressure center. This process may be a factor in tropical cyclogenesis.

Introduction

The interaction between the large scale field of atmospheric motions and cloud cover is one of the most important topics in modern dynamical meteorology, especially in the perspective of numerical weather prediction for longer periods, for which the presence of such interactions and particularlily the presence of a feedback chain: large scale pattern → cloud cover → large scale pattern, cannot be disregarded. The first link of this feedback is probably the best understood, at least for the influence of the large scale pattern on the development of a single cloud. With respect to Cu and Cb clouds (which are the main subject of the present article) this problem is relatively well, though not satisfactorily, investigated by many authors and there exists an extensive litterature. Results of these investigations are in wide use in predicting cloudiness, thunderstorms and showers, when forecast of the large scale background is known beforehand. But the very important problem of how the large scale pattern (and perhaps other factors like local surface properties) controls the spacing and size distribution of convective clouds over greater areas is rather far from being solved. Even the most recent papers (for instance Asai & Kasahara, 1967), though fairly sophisticated, use extremly simplified assumptions for the physics and geometry of the convective system.

The second link of the chain, the influence of cloud cover on the large scale pattern, is at the present time almost completely unknown. The presence of clouds affects the large scale pattern by: (a) modification of the radiation balance between the surface, atmosphere and outer space, (b) acting as a source or sink for sensible heat released or absorbed to the latent form in the water phase transformations and (c) vertical mixing and, connected with it, vertical transport of energy, momentum and water vapour which appears in large scale models in the form of sources and sinks for these physical properties. But both theoretical and empirical knowledge on details of these processes is still very fragmentary and unsatisfactory.

For the problem of the influence of cloud cover on the large scale pattern, it seems reasonable to distinguish the situations in which the direct influence of clouds is restricted to a relatively shallow layer (for instance, Cumulus humilis convection in the lowest 1–2 km) and can be probably treated as a boundary layer and then introduced to the large scale model in the form of suitably adjusted boundary conditions, from situations in which clouds affect a considerable part of meteorologically interesting atmospheric layers (for instance, deeply penetrating Cumulonimbus convection). The range of applicability of each approach may of course vary in particular problems.

The aim of the present article is limited to the investigations of certain aspects of the role which deeply penetrating convection plays in

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the interscale interactions, namely, in modifying the large scale temperature/humidity stratification within the convective layer. This sort of interaction is certainly very essential, since, due to the quasi-hydrostatic balance of the large scale pattern, temperature and humidity stratification controls the pressure field, which, in general, is also in balance with a prevailing component of the air flow. On the other hand, temperature and humidity stratification controls to a high degree the convection itself.

In order to avoid complications, the investigations concentrate on the effects of convective transport of mass, heat and water vapour. Thus, the model of the atmosphere used here excludes changes in stratification caused by radiative heat transfer, large scale effects (mass convergence, differential advection etc.), microturbulent diffusion, friction and any type of momentum exchange. The Cumulus and Cumulonimbus convection is thus assumed to be the only means of vertical transport.

These restrictions are summarized in the following assumptions:

A. The model consists of a fragment of atmospheric air, which occupies the space over a ground surface of area S.[1] Its horizontal size is large when compared to the size of a single convective cell, so that the model as a whole can represent a large scale meteorological system. For sake of simplicity, the ground surface is assumed to be flat.

B. No kind of physical exchange through the lateral boundary of the system is allowed (or, if present, causes no change in the state of the model), so that the model can be imagined as closed in a vertical cylindrical vessel with ideally isolating lateral walls. Mass exchange due to eventual fallout of precipitation or evaporation from the ground is assumed negligible, so that the total mass of the model (M) is invariant.

C. Net radiative and conductive heating or cooling as well as vertical diffusion of water vapour in free air is absent, though they may be present in a relatively thin subcloud layer near the surface. Frictional production of heat is negligible.

D. The net momentum flux is negligible, at least as it affects stratification.

Possibilities of the direct realization of this model in the true atmosphere depend strongly upon the characteristic times and sizes of the investigated convection. For instance, in the case of small cumulus clouds traced during the time corresponding to the life cycle of a single cell, assumptions –D seem to be frequently valid and are more or less explicitly accepted in most of the present Cumulus or Cumulonimbus models (often with infinite M and S). But on the other hand, in the case of large thunderstorm systems traced during a few days, assumptions A–D seem to hold mainly in extended air masses, moving slowly within horizontal, large scale, circulatory cells. This limits the direct applicability of the model, although the conclusions it yields may be indirectly applied to a much broader range of problems.

Changes in stratification caused by developing convective clouds can be roughly estimated by means of the old, simple "slice" method developed by J. Bjerknes (1938). This well known model is based upon the assumption that the atmosphere remains in hydrostatic equilibrium and that convection does not essentially alter the mean vertical mass distribution. Any upward mass transport through any isobaric surface p must be nearly compensated by equal downward transport. Since upward motion is associated with clouds and downward motion associated with cloudless air, the air parcels of cloudless air initially at pressure p will move down to the level p'. If tops of cylindrical clouds are reaching pressure p_t, there pressures are related by the formula

$$p' = p + \sigma(p - p_t), \tag{1}$$

where σ gives the ratio of surfaces covered by clouds and cloudless air respectively.

The corresponding change of temperature can be estimated by assuming adiabatic descent in the cloudless air. Since, for deeply penetrating convective clouds in the active stage, σ rarely exceeds a few percent (see for instance Riehl, 1954), and the difference $(p - p_t)$, in general, cannot exceed 600–800 mb, one can easily see that the difference $p' - p$ can be, at most, of the order of a few tens of milibars at the cloud base, which, for typical lapse rates, gives an increase of temperature of the order 1–2° centigrade; for smaller p, this increase will be correspondingly less. Such a change could conceivably affect the development of later convective cells, since it increases the initial energy threshold which

[1] For a complete list of the symbols used here, see p. 42.

should be crossed by the moving thermal before free convections starts (see Pettersen, 1956), but its direct effect on the large scale pattern seems to be marginal when compared, for instance, with even weak differential advection (neglected in the present model, but generally present in reality).

Since the active stage of an individual convective cell seldom exceeds 1–2 hours, after which time the cell dissipates, the deep Cumulus or Cumulonimbus convection can be considered as an important factor in modifying the large scale pattern only when certain modifying influences can survive the dissipation of the cell and the effects of subsequently developing cells accumulate.

A complete investigation of the thermodynamics and hydrodynamics of the dissipation of clouds and its influence upon the large scale stratification seems to be a hopeless task due to the extreme complexity of the problem and due to the number of different ways in which it can be realized. The problem can be simplified, however, by making use of the hydrostatic balance as a characteristic feature of the large scale pattern. Namely, it can be noticed that, before the convection develops, the atmosphere is generally in a certain state of hydrostatic equilibrium. Convection disturbs this equilibrium, but, when the development of new convective cells stops, the process of dissipation of the existing cells can be expected to build up, asymptotically, a new hydrostatic balance. This asymptotic equilibrium state can be determined in a relatively easy way and, due to the rapidity of hydrostatic adjustment, can in many cases be considered as a good approximation of the real state of the atmosphere. Both the initial and final state can be assumed to be stable with respect to dry adiabatic motions, if weak superadiabatic lapse rates in the undercloud layer are disregarded.

In the present article, changes in the stratification are described as resulting from the redistribution of mass by the vertical drafts. This approach (used also by Braham, 1952) seems to be more convenient than the usual one which operates with the convective heat fluxes. Due to the hydrostatic assumption both approaches are in fact equivalent.[1]

[1] Quite recently the author became aquainted with the manuscript by A. Fraser (1968) where similar ideas are developed.

The present paper consists of five sections. The first section deals with the general methodical problems. In Sections 2 and 3, certain particular classes of cloud developments are investigated qualitatively. Conclusions of these two sections are then used to support certain hypotheses on the mechanism by which the convective clouds may participate in the early stages of development of tropical hurricanes. This is presented in the fourth section. Section 5 contains a short summary.

List of symbols

Latin letters

- c_p Specific heat per unit mass for dry air under constant pressure;
- c_p' the same for humid air;
- c_v specific heat per unit mass for dry air under constant volume;
- g gravitational acceleration;
- M mass of air in the model atmosphere;
- m mass flux in a vertical draft;
- $R = c_p - c_v$ gas constant for dry air;
- Q net heat release over a certain time;
- S area of the horizontal cross section of the model;
- T temperature;
- T^* virtual temperature ($T^* = T(1 + 0.00061\, q)$ where q = specific humidity).

Greek letters

- δ Dirac symbol;
- $\theta = T(1000\ \text{mb}/p)^{R/c_p}$ potential temperature;
- $\theta^* = T^*(1000\ \text{mb}/p)^{R/c_p}$ virtual potential temperature;
- Φ geopotential;
- $\nu = -dp/d\theta^*$;
- $\Delta\psi = \psi_f - \psi_i$ difference between a certain function ψ in the final (ψ_f) and initial (ψ_i) state of hydrostatic equilibrium.

Indices

- "c" Refers to the convection ceiling; i.e., to the highest level reached by the convective currents;
- "f" refers to the final state of hydrostatic equilibrium;
- "i" refers to the initial state of hydrostatic equilibrium;
- "o" refers to the ground level.

The remaining indices ($_1$, $_2$, α, β, γ, m, n) are explained in the text of the proof of the theorem in sect. 3.

Note. In symbols of intervals, parenthesis [denotes that the respective end is included with the interval, while parenthesis (denotes that the end is excluded.

1. General method

Let us consider a model atmosphere, described by conditions A–D from the Introduction, in hydrostatic equilibrium, stable (or may be partly neutral) with respect to dry adiabatic motions, and with surface pressure p_0. Since the model does not permit lateral mass exchange and since small mass variations due to eventual rainfall or evaporation from the surface are to be disregarded, $p_0 = Mg/S$ is an invariant.

In this article the state of the atmosphere will be described by means of the functions $T^*(p)$ or $\theta^*(p)$ which, for simplicity, will be assumed continuous.

Since, due to the assumption of stability, θ^* is a nonincreasing function of p, the reverse function $p(\theta^*)$ can be used as well; it will have jumps for θ^* corresponding to the neutrally stratified layers. The function

$$\nu(\theta^*) = -\frac{dp}{d\theta^*} \geqslant 0$$

can be also used as a unique description of the atmospheric state of hydrostatic equilibrium since $p(\theta^*)$ is uniquely determined by the formula:

$$p(\theta^*) = p_0 - \int_0^{\theta^*} \nu(\theta^*)\, d\theta^*, \tag{2}$$

where the Dirac δ symbolism should be used for neutrally stratified layers. For simplicity, integration in (2) is carried out from $\theta^* = 0$, $\nu(\theta^*)$ being extended below the ground surface as equal identically to zero.

It is easy to see that $\nu(\theta^*)$ can be treated as a mass distribution with respect to the virtual potential temperature, normalized with respect to the surface area and gravity; i.e., for $d\theta^* \to 0$, the differential expression

$$\frac{S}{g}\, \nu(\theta^*)\, d\theta^*$$

represents the mass of air whose virtual-potential temperature is between θ^* and $\theta^* + d\theta^*$.

During convection, heating and cooling due to the phase transformations of water and mixing (all of which are essentially nonadiabatic processes) alter the initial distribution $\nu_i(\theta^*)$. One should, however, notice that, since θ^* is an invariant of dry adiabatic processes and since net radiative and conductive heating outside the surface layer is here excluded, $\nu(\theta^*)$ is not altered by processes going on in the clear air far from the direct vicinity of the clouds and surface. The final state of hydrostatic equilibrium is thus determined by a new mass distribution

$$\nu_f(\theta^*) = \nu_i(\theta^*) + \Delta\nu(\theta^*) \tag{3}$$

with $\Delta\nu(\theta^*)$ depending principally upon the processes which take place in the clouds or their near vicinity. From the definition of ν, it follows at once that the stability of the hydrostatic equilibrium is increased in the layers where $\Delta\nu < 0$ and decreased where $\Delta\nu > 0$.

The task of determining the new stratification reduces thus to finding the function $\Delta\nu$. This approach, although very general, may, in certain cases, require a very detailed knowledge of all the thermo dynamical properties of convection in order to determine $\Delta\nu$. For such cases, this approach has very little use, since detailed knowledge of the convection may permit a more direct and exact analysis of the large scale changes. But there exists also a wide class of important problems in which the final $\nu_f(\theta^*)$ can be approximately determined by means of relatively simple models of convective clouds in a steady environment. The case of deeply penetrating convection seems to belong to this class.

Essentially, as already mentioned, active updrafts in Cumulus and Cumulonimbus clouds, which develop in an unsaturated, conditionally unstable atmosphere, cover a small fraction of the sky, while the rest of the atmosphere undergoes dry-adiabatic changes which do not participate in changing the function $\nu(\theta^*)$. It is easy to see that, if nonadiabatic processes are limited to a cloud and its nearest environment, the following equality holds:

$$\Delta\nu(\theta^*) = -\frac{dm}{d\theta^*}\frac{g}{S}, \tag{4}$$

where m denotes the vertical mass flux in cloud drafts, integrated over the life-time of the cloud. $dm/d\theta^*$ represents here the timeintegrated net entrainment,[1] i.e., the sum of inflow and outflow of environmental air associated with the convective current. This parameter, or at least its "inflow" part, though generally refered to pressure or altitude, rather than to θ^*, is commonly used in several single cloud models based on experimental data as well as on theoretical considerations (see for instance Stommel, 1947; Byers & Braham, 1949; Austin, 1951; Ludlam & Scorer, 1953; Levine, 1959; Squires & Turner, 1962; Haman, 1967). Although detrainment (outflow) is generally not considered, it can be included by a certain modification of many of the above models, assuming, for instance, that an individual air parcel "detrains" at the level where both its buoyancy and velocity vanish.

Most of these models refer to an environment invariant during the life of the cloud, which is an essential simplification. Since it seems acceptable in the present case of sparse clouds, one can expect that the net entrainment $dm/d\theta^*$, calculated in frames of such a model, should differ little from that calculated without this simplification. If the basic parameters (size, spacing, life-time, initial mass flux, etc.) for simultaneously developing clouds, could be deduced from the large scale situation itself, then $\Delta\nu$, corresponding to the total effect of such a group of clouds is found by adding $dm/d\theta^*$ for the individual clouds in the group. If this holds, the following approximate procedure could be used for finding $\Delta\nu$ over a longer, large-scale time interval in which the assumption of steady environmental stratification may be too strong.

(a) Divide the assumed "large scale" time interval into subintervals corresponding to the life-times of groups of simultaneously developing cells.

(b) Calculate $\Delta\nu$ over the first subinterval from (4) using one of the simplified cloud models, assuming invariant stratification during the cloud development.

(c) Use $\Delta\nu$ in order to determine the new stratification by means of (2) and (4)

(d) Repeat procedure (b) over the next subinterval with the new environmental stratification.

(e) Repeat procedures (c) and (d) until the initial large scale interval is exhausted.

From a purely practical point of view it may be sometimes convenient to replace the entrainment with respect to θ^* by its analogue with respect to altitude or to hydrostatic pressure. The assumption that the environment remains unchanged during a given cycle of evaluation makes this change a simple substitution.

Applicability of this relatively simple method for quantitative studies is nevertheless limited (at least at the present state of knowledge of cumulus dynamics) by the very low accuracy with which the entrainment can be determined and by the already mentioned lack of sufficiently precise knowledge as to how the ensemble of clouds depends upon the large scale pattern. But, as will be shown in the following sections, certain interesting conclusions can be deduced from crude qualitative estimates based upon simplified models and observational evidence.[2]

Before proceeding further, let us notice that the functions $\Delta\nu(\theta^*)$, $\Delta p(\theta^*)$ and $\Delta T^*(p)$ are subject to the following constraints:

$$\Delta p(\infty) = -\int_0^\infty \Delta\nu(\theta^*)\,d\theta^* = 0, \qquad (5)$$

which follows directly from the mass conservation in our model,

$$\Delta p(\theta^*) = 0, \quad \text{for} \quad \theta^* > \theta_c^*, \qquad (6)$$

which follows from (5) and the fact that $\Delta\nu(\theta^*) = 0$ above the convection ceiling (for $\theta^* > \theta_c^*$), and

$$gS\int_0^\infty \Delta(c_p' T)\,dp \approx gSc_p\int_0^\infty \Delta T^*\,dp = Q, \qquad (7)$$

which states that the change of the internal and gravitational energy (which, according to any handbook of dynamical meteorology, integral (7) represents) is due to the net release of heat Q.

In our model Q consists of latent heat and possible heat flux from the surface. The approximation, indicated in (7) by the sign $\approx$, derives from disregarding the difference between R/c_v for dry and humid air, which introduces only marginal error here.

[1] Strictly speaking, this is the rate of entrainment with respect to θ^*, but we shall refer to it simply as to entrainment.

[2] Certain simplified models are now under quantitative investigation by the author.

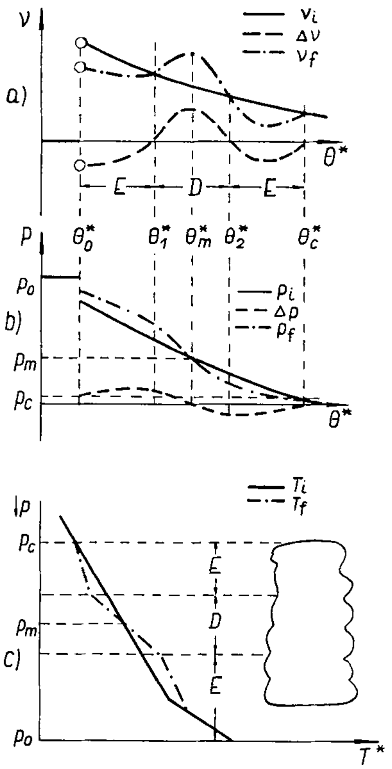

Fig. 1. Schematic illustration of the Case I with the lowest layer neutrally (dry adiabatically) stratified. (a) curves $\nu_i(\theta^*)$, $\nu_f(\theta^*)$, $\Delta\nu(\theta^*)$, (b) curves $p_i(\theta^*)$, $p_f(\theta^*)$, $\Delta p(\theta^*)$, (c) curves $T_i^*(p)$ and $T_f^*(p)$. *D*, Detrainment zone; *E*, entrainment zone. Points with indices other than "*o*" and "*c*" (bottom and top of the convection layer) refer to the proof of the theorem in Sect. 3. Rings at θ_o^* on the Fig. 1*a* symbolize Dirac "delta".

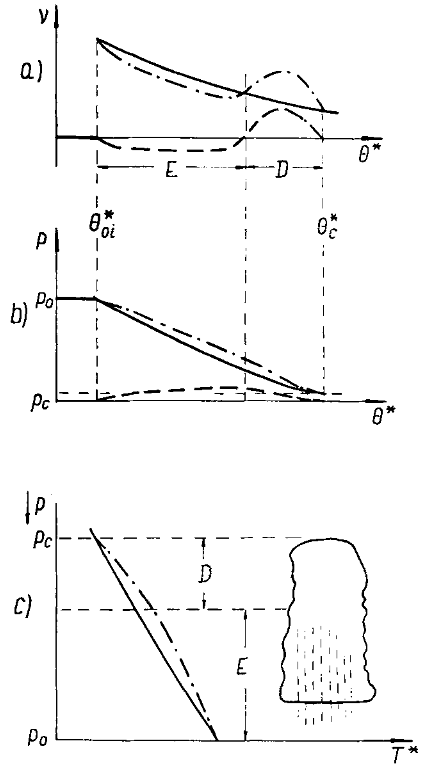

Fig. 2. Schematic illustration of the Case IIa with one detrainment zone at the top of the convection layer. See Fig. 1 for explanations.

Let us also notice that for $\theta^* < \theta_{oi}^*$

$$\Delta\nu(\theta^*) \geqslant 0; \quad \Delta p(\theta^*) \leqslant 0; \tag{8}$$

which follows directly from the fact that both ν_i and ν_f are non-negative and that $\nu_i(\theta^*) = 0$ for $\theta^* < \theta_{oi}^*$.

2. Net entrainment in certain typical situations

In order to link the present considerations to developments in a real atmosphere, let us consider distributions of layers with positive and negative values of $dm/d\theta^*$ (to which we shall refer as entrainment and detrainment zones respectively) in the following, rather typical, cases of convective phenomena:

I. A Cumulus cloud which does not form precipitation, so that the condensed water can be treated as thought it were moving with air.

II. A Cumulus or Cumulonimbus cloud in which the formation of precipitation occurs so that a certain amount of condensed water is removed from the air mass in which the condensation occured. This precipitation is assumed to evaporate completely in more or less vigorous downdrafts which terminate above (Case II*a*) or at the ground surface (Case II*b*).

III. A Cumulus or Cumulonimbus cloud with precipitation falling down to the ground surface. In all cases environmental air is assumed unsaturated with water vapour, and the temperature near the surface is assumed not to rise essentially during the life cycle of a convective cell.

Figs. 1 to 4 present anticipated distributions of $\Delta\nu$ and Δp, as well as corresponding changes

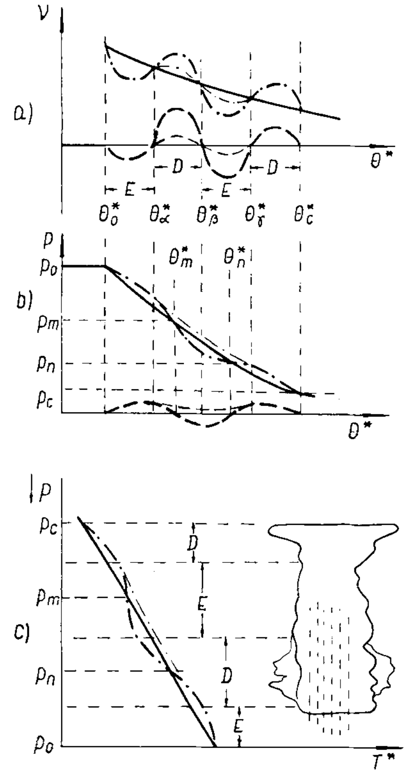

Fig. 3. Schematic illustration of the Case IIa with two entrainment and two detrainment zones. See Fig. 1 for explanations. Variant α marked with bold lines, variant β with thin lines.

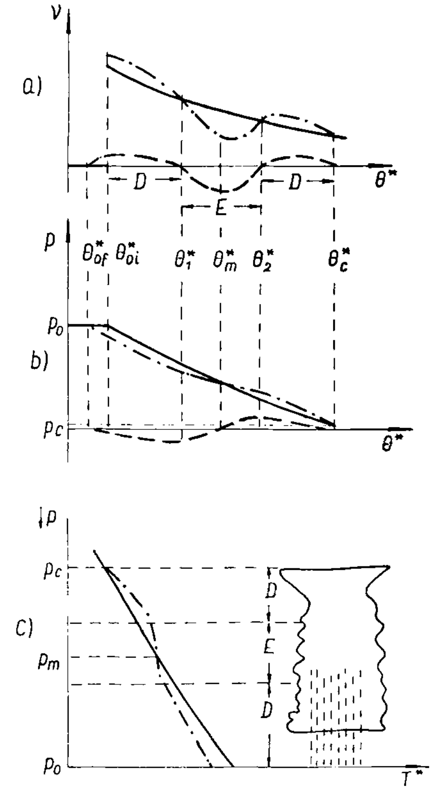

Fig. 4. Schematic illustration of the Cases IIb and III. See Fig. 1 for explanations. Note positve $\Delta\nu$ values for $\theta^* < \theta_{oi}$.

in the p *vs* θ^* and p *vs* T^* relations under the assumption that formulas (2) and (4) hold. Exact proof of the correspondence between parts (a) and parts (b), and (c) of Figs. 1, 3 and 4 well be given in the next section with certain additional assumptions. Shapes of the $\Delta\nu$ curves are here deduced under the plausible assumption that the inflow into the vertical current is a characteristic feature of its active stage and present at all altitudes, while outflow is rather characteristic of the decay stage and takes place at levels where the moving air tends towards mechanical and thermodynamical equilibrium with its environment.

For a single cloud, such levels are generally concentrated within a relatively thin layer; for example, in the vicinity of the convection ceiling, inversions or other layers with increased hydrostatic stability, or the ground surface (as a mechanical obstacle). This can be understood, since, with nearly invariant environment and considerably long lasting, quasi-steady states of vertical motions, a large part of the air involved in the cloud development goes through nearly the same cycle of transformations with final states within a relatively narrow region. In case of ensembles of clouds which may have a wide dispersion of individual properties, the resulting outflow region may be diffused. In any case, it can be expected not to fill the whole convection layer, as the inflow region does.

Superposition of the above sketched distributions of inflow and outflow regions gives the picture of net entrainment with alternating distinct entrainment and detrainment zones, as shown in Figs. 1 to 4. It seems reasonable to treat it as a typical, though not necessarily a universal, pattern.

It should be noted that such a distribution of entrainment and detrainment zones, was found by Braham (1952) in several cases of thunderstorms.

In Case I, the air is supposed to move upwards to the convection ceiling, mixing with the environment, and then return down under negative buoyancy induced by evaporation during mixing with the dry environmental air. It can be shown with use of formulas given by Austin (1951)[1] that, under a typical, conditionally unstable distribution of temperature and unsaturated water vapour, the descent should stop in the central part of the cloudy layer so that the detrainment zones with positive $\Delta\nu$ values should be found there. With no particular irregularities in the shapes and dynamics of the clouds, one can reasonably expect that only one coherent detrainment zone will be present. Fig. 1c) indicates that this corresponds to warming (in the sense of increasing the virtual temperature) in the lower part of the convection layer and cooling in its upper part.

Existence of the region of cooling follows directly from the geometry of the curve $\Delta\nu$ and formula (6), while the region of warming must be present if essential heat sinks other than evaporation are absent, since otherwise the integral (7) would be negative (see the next section).

In Case II, the air in the updraft, in the developing stage of the cloud life, moves upwards, mixing with the environment as in Case I. At a certain level (called here the precipitation level), precipitation forms and part of the water which was condensed in the upper portion of the cloud is in the form of precipitation displaced to other portions of the cloud. Finally, precipitation induces a downdraft and evaporates within it.

In this case, the distribution of entrainment and detrainment zones may be complicated. In the simplest case, if the precipitation removes a sufficiently large amount of liquid water from the air which rises above the precipitation level, the top of the cloud can evaporate during the upward motion, possessing still positive buoyancy. The detrainment zone will be then localized at the top of the convection layer, if inertial overshooting of the cloud tops can be disregarded.

Formation of other detrainment zones can be connected with precipitation driven downdrafts so they must appear below the precipitation level. The dynamics of such downdrafts is rather complicated and poorly investigated; however, some more detailed information can be found, for instance, in works of Byers & Braham (1949), Braham (1952), Das (1964) and Srivastava (1967).[2] The intensity of downward mass transport in such downdrafts depends in a complicated way upon the intensity of precipitation, external stratification, entrainment to the downdraft, etc.

In the simplest case, the outflow from the downdrafts appears to be too weak to overcome the inflow into the updrafts, in the early stage of cloud development, and no detrainment zones form below the precipitation level. This is shown in Fig. 2. Here, the effect of the clouds results in heating of the entire convection layer. Since the only source of this heating is the released latent heat,[3] this situation is possible only when a certain amount of condensed water does not reevaporate (see the next section).

The second simple case takes place when only one detrainment zone forms below the precipitation level, not reaching the ground surface (Case IIa) or reaching it (Case IIb). This is shown in Figs. 3 and 4. In IIa the lowest portion of the convection layer is warmed, the central part is cooled and the top part is warmed (variant α, Fig. 3). The warming of the entire convection layer is also possible (variant β, Fig. 3) if a certain amount of condensed water does not reevaporate and in this way provides net sources of sensible heat.

Case IIb (Fig. 4) presents the reversal of the situation in Case I; i.e. cooling in the lower portion and heating in the upper portion of the convection layer. Note that $\Delta\nu(\theta^*)$ can be now positive for $\theta^* < \theta^*_{oi}$, which corresponds to the production (due to the evaporation of precipitation) of potential temperatures lower than those which were present at the beginning of the cycle. Cooling near the surface increases the direct flux of heat from the ground so that the former temperature distribution near the ground surface may be, occasionally, quickly restored.

The simplest version of Case III does not differ essentially from Case IIb so that Fig. 4 can also serve as an illustration. In this case,

[1] See also Fraser (1968).

[2] An article on the rain driven downdrafts is now under preparation by the author.

[3] Over the life time of a single cell, direct heating from the surface can generally affect only a thin layer near the ground. This may be important for the further development of the convection itself, but has little effect upon the integral (7).

evaporation of rainfall may also induce cooling near the ground while the latent heat released by the condensation of the precipitated water remains in the atmosphere, increasing the value of the integral (7).

If certain factors, as radiation or large scale motions, disturb relation (4), curves of $\Delta\nu$ in Figs. 1 to 4 can be nevertheless interpreted as a qualitative representation of $-dm/d\theta^*$ with respect to the initial distribution of θ^*.

Resulting entrainment and detrainment zones produced by several subsequently developing groups of clouds (for instance over one day) can be expected to retain the general character deduced here for single clouds, providing that there were no major changes in the cloud type.

It should be once more pointed out that the above discussion of Cases II and III is reduced to only a few situations, rather arbirarily chosen, among many which can be realized by different distributions of the condensed water into the precipitating and nonprecipitating part.[1] This illustrates the key importance of precipitation in the interaction between clouds and the large scale pattern. Precipitation is important here not only as a source of latent heat release, but, as an agent transporting latent heat between different air parcels, it affects also the details of the geometry of the new stratification.

3. A theorem for the completely evaporating clouds

In order to give a more rigorous proof of rather intuitive statements in Section 2, let us now consider a class of clouds which give no precipitation at the ground and evaporate entirely after completing their life cycle, belonging therefore to Cases I and II of the previous section.

This class can be represented, for instance, by Cumulus or Cumulonimbus clouds which disappear in the late afternoon, leaving a clear sky for the night. We shall now consider a problem which corresponds to evaluation of day to day changes in stratification caused by the development of this sort of clouds. The essential conclusions derived in this section can probably be extended to certain cases where the assumption of no precipitation and complete evaporation does not hold, for which cases there is no general proof.

[1] Discussion of Case I is more general.

We shall prove the following theorem:

THEOREM. *If, in the general frames of the model defined in the Introduction, clouds develop in such a way that:*

(a) *no precipitation reaches the ground surface,*

(b) *all condensed water reevaporates completely,*

(c_1) *the detrainment zone is reduced to one coherent layer in the central part of the convection layer (Case I, Section 2),*

or

(c_2) *there are two separate coherent detrainment zones, one on the top and one in the central part of the convection layer (Case II*a*, Section 2) or at the bottom (Case II*b*, Section 2),*

(d) *sensible heat exchange with the surface can be disregarded,*

then:

(a) *above and partly within the lower detrainment zone (or the unique detrainment zone in the case of assumption* c_1*), there exists a layer with negative values of* $\Delta T^*(p)$ *(for the case of assumption* c_1 *this layer reaches the top of the convection layer (pressure* p_c*)), and*

(b) *the absolute geopotential of the top of the layer with negative* $\Delta T^*(p)$ *(pressure denoted* p_m *on Figs. 1, 3 and 4) decreases during the convective process; i.e.,* $\Delta\Phi(p_m) < 0$.

Proof. Assumptions (a) and (b) imply that there is no net release of latent heat during convection (if one neglects small differences in latent heat release for condensation and evaporation which may take place at different temperatures). Since assumption (d) excludes any exchange of sensible heat with the surface, while other sensible heat fluxes are excluded by Condition D in the Introduction, and there is no kinetic energy before or after convection, the gravitational-internal energy of the model remains unaltered by the convection, i.e.

$$c_p \int_0^{p_0} \Delta T^*(p)\, dp = 0 = c_p \int_{p_c}^{p_0} \Delta T^*(p)\, dp. \qquad (9)$$

Let us now consider the functions:

$$\Delta\nu(\theta^*),$$

$$\Delta p(\theta^*) = -\int_0^{\theta^*} \Delta\nu(\theta^*)\, d\theta^*,$$

$$\Delta\theta^*(p),$$

and $$\Delta T^*(p) = \left(\frac{p}{1000}\right)^{R/c_p} \Delta\theta^*(p).$$

Assumption (c_1) implies that there exist points θ_1^*, θ_2^* such that $0<\theta_1^*<\theta_2^*<\theta_c^*$[1] and

$$\Delta\nu(\theta^*)\leqslant 0 \quad \text{if} \quad 0\leqslant\theta^*<\theta_1^*,$$

$$\Delta\nu(\theta^*)\geqslant 0 \quad \text{if} \quad \theta_1^*<\theta^*<\theta_2^*,$$

$$\Delta\nu(\theta^*)\leqslant 0 \quad \text{if} \quad \theta_2^*<\theta^*\leqslant\theta_c^*.$$

So we can conclude that $\Delta p(\theta^*)$ is non-decreasing both in $[0, \theta_1^*)$ and (θ_2^*, θ_c^*); since $\Delta p(0)=0$, and because of (6) this means that $\Delta p(\theta^*)\leqslant 0$ for $\theta^*<\theta_1^*$ and $\Delta p(\theta^*)\geqslant 0$ for $\theta_2^*<\theta^*<\theta_c^*$. Thus, if not identically equal to zero, $\Delta p(\theta^*)$ must change sign (perhaps in the form of a jump) within $[\theta_1^*, \theta_2^*]$, but, since $\Delta\nu(\theta^*)$ is non-negative within (θ_1^*, θ_2^*), only one such change is possible. Hence there exists a value θ_m such that

$$\Delta p(\theta^*)\geqslant 0 \quad \text{for} \quad \theta^*<\theta_m^*,$$

$$\Delta p(\theta^*)\leqslant 0 \quad \text{for} \quad \theta^*>\theta_m^*.$$

Since

$$\theta_i^*(p)=\theta_f^*(p+\Delta p(\theta_i^*(p))), \tag{10}$$

$$\theta_f^*(p)=\theta_i^*(p-\Delta p(\theta_f^*(p))), \tag{11}$$

applicability of each form being limited by the fact that neither $\theta_i^*(p)$ nor $\theta_f^*(p)$ are determined for $p>p_0$, the function $\Delta\theta^*(p)$ can be presented in two alternative forms

$$\Delta\theta^*(p)=\theta_f^*(p)-\theta_f^*(p+\Delta p(\theta_i^*(p))), \tag{12}$$

$$\Delta\theta^*(p)=\theta_i^*(p-\Delta p(\theta_f^*(p)))-\theta_i^*(p), \tag{13}$$

at least one of them being applicable for each value of $p\leqslant p_0$. But since both θ_i^* and θ_f^* are continuous and non-increasing functions of p, it follows from (12) and (13) that: if $\Delta\theta^*(p)\neq 0$, then

$$\operatorname{sign}\Delta\theta^*(p)=\operatorname{sign}\Delta T^*(p)=\operatorname{sign}\Delta p(\theta_f^*(p))$$
$$=\operatorname{sign}\Delta p(\theta_i^*(p)). \tag{14}$$

[1] Cases $\theta_1^*=\theta_2^*$ oz $\theta_2^*=\theta_c^*$, which may occure if $\Delta\nu$ is there of Dirac δ form, need only inessential modifications of the proof and are not considered here.

So there exists a point p_m such that $\theta_m^* = \theta_i^*(p_m)=\theta_f^*(p_m)$ and

$$\Delta T^*(p)\geqslant 0, \quad \Delta\theta^*(p)\geqslant 0, \quad p_m\leqslant p\leqslant p_o, \tag{15}$$

$$\Delta T^*(p)\leqslant 0, \quad \Delta\theta^*(p)\leqslant 0, \quad p_c\leqslant p\leqslant p_m, \tag{16}$$

where condition (9) imposes a sharp inequality in certain points of both intervals, if the trivial identiy $\Delta T^*(p)\equiv 0$ is not taken into account. Since p_m obviously belongs to the detrainment zone, thesis (a) for the case of assumption (c_1) is proved. Thesis (b) will be proved later.

If assumption (c_2) is valid and if the lower detrainment zone reaches the surface (Case III, Section 1), then the qualitative shape of the $\Delta\nu(\theta^*)$ distribution differs only in sign from that considered above. So the above proof can be here also applied, with the directions of inequalities (15) and (16) reversed. Thesis (b) follows directly in that case.

If, however, the lower detrainment zone lies above the surface, points θ_α^*, θ_β^*, θ_γ^* must exist such that $0<\theta_\infty^*<\vartheta_\beta^*<\theta_\gamma^*<\theta_c^*$ in the four following segments (case that some of them are reduced to a point is disregared as before):

$$[0, \theta_\alpha^*), \quad (\theta_\alpha^*, \theta_\beta^*), \quad (\theta_\beta^*, \theta_\gamma^*), \quad (\theta_\gamma^*, \theta_c^*],$$

there is, respectively,

$$\Delta\nu(\theta^*)\leqslant 0, \quad \Delta\nu(\theta^*)\geqslant 0, \quad \Delta\nu(\vartheta^*)\leqslant 0, \quad \Delta\nu(\theta^*)\geqslant 0.$$

Hence, $\Delta p(\theta^*)$ is non-decreasing for $\theta^*<\theta_\infty^*$ and non-increasing for $\theta_\gamma^*<\theta^*\leqslant\theta_c^*$; thus, due to (6) $\Delta p(\theta^*)$ must be nonnegative there. $\Delta p(\theta^*)$ can become negative in the interval $[\theta_\infty^*, \theta_\beta^*]$ (i.e. in the interval containing the lower detrainment zone), becoming once more positive to the right of that interval; it is easy to see that only one such oscillation is possible.

Therefore, the interval $[0, \theta_c]$ can be divided into three parts by points θ_m and θ_n, so that

$$\Delta p(\theta^*)\geqslant 0 \quad \text{for} \quad 0<\theta^*<\theta_m^*,$$

$$\Delta p(\theta^*)\leqslant 0 \quad \text{for} \quad \theta_m^*<\theta^*<\theta_n^*,$$

$$\Delta p(\theta^*)\geqslant 0 \quad \text{for} \quad \theta_n^*<\theta^*<\theta_c^*,$$

and hence due to (14) the interval p_c, p_0 can also be divided into three corresponding parts by means of points p_m and p_n, such that $\theta_m^*=\theta_i^*(p_m)=\theta_f^*(p_m)$ and $\theta_n^*=\theta_i^*(p_n)=\theta_f^*(p_n)$, so that

$$\Delta\theta^*(p) \geq 0, \qquad \Delta T^*(p) \geq 0 \quad \text{for} \quad p_m \leq p \leq p_o,$$

$$\Delta\theta^*(p) \leq 0, \qquad \Delta T^*(p) \leq 0 \quad \text{for} \quad p_n \leq p \leq p_m,$$

$$\Delta\theta^*(p) \geq 0, \qquad \Delta T^*(p) \geq 0 \quad \text{for} \quad p_c \leq p \leq p_n,$$

the inequality being sharp at least in certain points if the trivial case $\Delta T(p) \equiv 0$ is disregarded. This follows from (9), which otherwise would be not fulfiled, and proves thesis (a) in the case of assumption (d). Proof of thesis (b) is a follows:

Let us rewrite (9) in the form

$$\int_{p_c}^{p_n} \Delta T^*(p)\,dp + \int_{p_n}^{p_m} \Delta T^*(p)\,dp + \int_{p_m}^{p_o} \Delta T^*(p)\,dp = 0, \quad (17)$$

considering $p_n = p_c$ in the case of assumption (c_1). Since

$$\int_{p_c}^{p_n} \Delta T^*(p)\,dp \geq 0, \quad (18)$$

we find

$$-\int_{p_n}^{p_m} \Delta T^*(p)\,dp - \int_{p_m}^{p_o} \Delta T^*(p)\,dp \geq 0, \quad (19)$$

and since $p_n < p_m < p_o$ we have

$$-\int_{p_n}^{p_m} \Delta T^*(p)\frac{dp}{p} > \int_{p_m}^{p_o} \Delta T^*(p)\frac{dp}{p}, \quad (20)$$

which yields

$$\Delta\Phi(p_n) = R\int_{p_n}^{p_o} \Delta T^*\frac{dp}{p} < 0. \quad (21)$$

4. Three hypotheses on Cumulus convection

In order to avoid possible misunderstandings, it should be pointed out that the ideas presented in this section are hypotheses, for which proof *sensu stricto* is not to be given in this article. The aim of presenting these hypotheses here is to indicate the possibilities of applying the conclusions which follow from the former considerations and to suggest certain directions for further development of the theory of interscale interactions. The following reasoning has only to substantiate the plausibility of the proposed approach, while more complete proof of its validity (which would require rather wide theoretical investigations in the poorly penetrated field of mesometeorology) is beyond the scope of the present publication.

The first hypothesis can be formulated as follows:

1. Cumulus convection without precipitation and with full reevaporation of condensed water may tend to self-intensification in the sense of producing clouds with gradually increasing vertical extent and increasing energy release. If more intensive within a certain area than in its surroundings, such convection may produce a high-level, cold-core barometric low.

Let us consider a large scale meteorological system in which the existing field of motions prevents (for instance by means of gradient or geostrophical balance) intensive mass exchange through the lateral boundaries of the system. Such a system may likely appear as a fairly good realization of the model proposed in the Introduction and, over a time interval corresponding to the life cycle of a Cumulus cloud, may well fulfill the assumptions of the theorem from Section 3. If convection in the system is of the type corresponding to assumption (c_1) of this theorem, the clouds developing in the first cycle produce cooling of the upper part of the convection layer, heating of the lower part, and a decrease of the geopotential at the top of the convection layer. This means that the mean temperature lapse rate in the convection layer increases. It can be also shown that, if the clouds developing in the next convective cycle would have the same condensation level and the same potential pseudo-wet-bulb temperature, the so called instability energy[1] corresponding to them would be greater than in the

[1] This energy is defined as the integral

$$g\int_{p_d}^{p_o} (\hat{T}(p) - T^*(p))\frac{dp}{p},$$

where $\hat{T}(p)$ represents virtual temperature of an air parcel moved dry-adiabatically from the surface up to the condensation level and then wet-adiabatically to the level p_d, above which $\hat{T} - T^*$ becomes definitely negative. With certain restrictions, this magnitude is often used as a predictor for convective phenomena, being positively correlated with the thickness of the convection layer (see also Petterssen, 1956). An increase of this magnitude, in the present case, follows directly from a comparison of the above integral with formula (21).

first cycle. But heating in the lower part of the convection layer means that, if the second cycle of development of Cumulus clouds has to start, *the potential pseudo-wet-bulb temperature must* be increased (for otherwise the starting updraft would not get a positive buoyancy at a reasonable altitude). This additional increase of the potential pseudo-wet-bulb temperature, which must be provided by the ground surface sources of heat and water vapour, increases the instability energy. All this suggests that, at least from the point of view of synoptical practice, an *intensification of the convective processes*, in the sense indicated, can be expected. If traced through a few subsequent days, this intensification can be expected to form a trend superposed upon the diurnal variations of the convection.

The formation of the high-level, cold-core barometric low follows directly from thesis (b) of the theorem in Section 3.[1]

The increased lapse rate and instability energy are of course not sufficient for the intensification of the convective activity. The surface sources of heat and water vapour must be able to provide a sufficient rise in the potential pseudo-wet-bulb temperature and, furthermore, the effect of change in stratification should not be opposed by the increased damping of cloud development by mixing.

The first of the above indicated additional conditions is often likely to be fulfilled. It is easy to see that, if the subcloud layer has, as it often occurs, a nearly dry-adiabatic lapse rate, the effect of convection may not increase the temperature near the surface; under these circumstances the adiabatic layer becomes thinner. (This case is shown in Fig. 1.) This, together with the increased stability above the adiabatic layer, will damp out the upward flux of heat and water vapour, facilitating an increase of the potential pseudo-wet-bulb temperature near the surface.

The damping effect of mixing with environmental air is stronger, the dryer the air and the narrower the currents. Since dissipation of clouds increases the humidity of the convection layer, the subsequently developing clouds can be expected to be correspondingly less affected by mixing if their horizontal size does not decrease, which decrease is not very likely, since the barrier of increased stability just above the neutral layer may often filter out the updrafts, permitting only the greatest to cross it.

The effects of radiative and conductive heat transfer, as well as of large scale motions which were assumed absent, can in certain cases encourage such a development as proposed in Hypothesis 1, and discourage it in the other cases, since these factors are to a high degree independent of the convection. The problem of determining which of the two above possibilities is realized in given situations, requires further investigation. Some investigations are being undertaken by the author, but the results are not yet formulated.

The above picture of selfintensifying convection contradicts the common belief that free convection is a process which always tends to stabilize the mean hydrostatic equilibrium. Although this common belief is true for simple thermal convection, without phase changes involved, convection in a conditionally unstable atmosphere with complete reevaporation of the condensed water resembles rather forced mixing, which decreases the initial hydrostatic stability of the mixed layer. Water vapour behaves here, to some extent, like the cooling agent in absorptive refrigerators.

The eventual selfintensification of the convection is of course limited, being in fact driven by a noneven distribution of humidity. Transport of water vapour by convection, as well as an increase of relative humidity due to the cooling processes, can make the evaporation of clouds impossible after a certain time. Also, an increasing intensity of convection may lead to the development of precipitation. In such cases, net heating of the atmosphere starts and the cold core may be gradually replaced by a warm core.

To summarise, the following cycle of events may take place:

Cumulus convection in a certain area builds up an upper, cold-core barometric low and conditions for more intense convection as well. This process continues until the humidity of the upper air becomes too great and/or precipitation starts. Then the warm core begins to replace the cold core, presumably at first in the upper

[1] The above argumentation ceases to hold if precipitation appears in the clouds and assumption (c_2) of the theorem becomes valid. This is due to the warming at the top of the convective layer, which may, though not necessarily, overcome the effects of cooling in its central part. However, the presence of the cold core low in the central part is still actual.

part (compare with the sequense in Case I – Case II – Case III in Section 2).

This cycle resembles the early stages of tropical hurricane development as described by Yanai (1961, 1963), with characteristic invariance of surface pressure. Thus the following second hypothesis is proposed:

2. It is possible that the process described in Hypothesis 1, joined with a favourable large scale situation, can be of considerable importance in the dynamic of a forming tropical hurricane.

Now let us notice that convective clouds produce their detrainment zones in certain parts of the convection layer, izzespective, of the assumptions of the present model. Particularly, they can be involved in a large scale circulatory system which provides suitable dynamic and thermodynamic mechanisms for the removal of accumulated mass from the detrainment zones and for the compensation of non-isentropic activity of clouds, maintaining in this way the large scale steady state. This seems to be the case in the trades. The air, pumped by the trade Cumuli (which correspond to Case I, Section 2) into the central part of the cloudy layer, is removed by a large scale field of horizontal mass divergence. The diverging air mass moves towards the doldrums belt where precipitating Cumulonimbus clouds, with detrainment zones in the upper troposphere, prevail. Being lifted upwards, the air then moves polewards, slowly restoring the initial distribution of temperature and vapour, mainly by radiation and microturbulent diffusion, in a loop process similar to that proposed by Carlson & Ludlam (1965).[1]

This scheme of steady state works as long as the compensating large scale processes balance the activity of the clouds, particularly, if the removal of mass from the detrainment zones is secured. But it seems possible that, if certain disturbances within the large scale field affect the mechanism of mass removal, convection within the affected area may build up a selfintensifying, cold-core pressure system in a way suggested by Hypothesis 1, and then develop into a hurricane as suggested by Hypothesis 2.

[1] The above idea had been formulated during a personal discussion with Dr. A. Fraser, Imperial College, London.

This reasoning yields the following third hypothesis:

3. It is possible that one of the mechanisms in which an easterly wave develops into a hurricane is based upon its disturbing, in a certain area, the horizontal outflow of air from the detrainment zone of the trade Cumuli.

5. Conclusions

Results presented in this paper can be summarized as follows:

(a) In the case of deeply penetrating Cumulus and Cumulonimbus convection, the interaction between the convective phenomena and large scale stratification can be conveniently described in terms of the entrainment-detrainment notion.

(b) Entrainment and detrainment processes are essentially affected by precipitation, primarily through liquid water and ice (and hence latent heat) exchange between different parcels of air.

(c) Convection with no precipitation and with total reevaporation of the condensed water may be, in certain circumstances, a self-amplifying phenomenon and may lead to formation of a cold-core barometric low in the region of the most intensive convection.

(d) It seems possible that such a process may be an important factor in the early stages of tropical cyclogenesis.

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О ВЛИЯНИИ КОНВЕКТИВНЫХ ОБЛАКОВ НА КРУПНОМАСШТАБНУЮ СТРАТИФИКАЦИЮ

Исследуются изменения в крупномасштабной стратификации, вызванные неадиабатическими процессами в конвективных облаках. Изменения рассматриваются как результат чистого вторжения, фактор, который оказывается сильно чувствительным к существованию и динамике осадков. Показывается, что конвекция в Cumulus или Cumulonimbus может быть самоусиливающимся процессом, при определенных условиях приводящим к появлению наверху центра низкого давления с холодным ядром. Этот процесс может иметь значение для тропического циклогенеза.