The influence of the large-scale heat sources on the dynamics of the ultra-long waves

By BO R. DÖÖS, *Swedish Natural Science Research Council* and *International Meteorological Institute, Stockholm, Sweden*[1]

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ABSTRACT

The main purpose of this investigation is to examine to what extent large-scale heat sources are responsible for the formation and behaviour of ultra-long waves in the atmosphere. This has been performed with the aid of a linear, time-dependent, quasi-geostrophic model in which non-adiabatic heating has been assumed to be proportional to the temperature difference between the atmosphere and the underlying surface.

Results obtained with this model exhibit certain characteristic features which are in agreement with observational studies of these long waves in middle latitudes during winter.

Thus the solution consists of a stationary part and a travelling part. For the lowest longitudinal wave-numbers the stationary part is very close to the observed mean conditions in the lower part of the atmosphere.

The non-stationary part consists of two travelling waves which have decaying amplitudes for the three lowest wave numbers; in the corresponding adiabatic case the solution is amplifying.

The phase-velocity for the wave-component which decays most slowly is close to the Rossby phase velocity modified by a weak divergence effect.

If the initial condition is chosen so that the system is not in thermal equilibrium, the resulting motions of the ultra-long waves are slowly damped fluctuations about a mean state.

1. Introduction

After electronic computers with comparatively large internal memories were introduced in the middle of the fifties it became possible to make barotropic numerical forecasts for regions of almost hemispheric size. By expanding the forecast area it was thus expected that the forecasts should be less subjected to errors arising from the physically unrealistic lateral boundary conditions. This was found to be the case, but at the same time severe errors were introduced due to the fact that the non-divergent barotropic model is insufficient to describe the motions of planetary waves when they are not kept fixed by the boundaries. The longest waves showed a rapid westward propagation. This erroneous behaviour of the ultra-long waves, which in reality are quasi-stationary, was, surprisingly, not expected, although they moved in agreement with the Rossby phase speed formula.

The main reasons for the existence of the long-term average planetary circulation have been explained by demonstrating the effect of the major mountain barriers by, e.g. Charney & Eliassen (1949) and Bolin (1950). Later Smagorinsky (1953) pointed out that the large-scale distribution of heat sources is of equal importance. However, these investigations were based on the assumption that the motion is stationary The actual formation of the very long waves due to these external factors and their subsequent behaviour could therefore not be described and is still not sufficiently understood.

In view of the difficulties in including these effects in the forecast models in a realistic way, particularly in the barotropic model, it was therefore rather natural to reduce these errors by forcing the longest waves to remain stationary (Wolff, 1958) or by taking into account the divergence for the large-scale motion, which has the effect of reducing the phase velocity for the planetary waves significantly (Cressman, 1958).

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Because the combined effects of heating and orography could be explained to be the major reasons for the creation of the longest waves and that they are almost stationary, it is evident that these methods to control the phase speeds of these large-scale disturbances must be considered as rather artificial.

During the last few years there have been several investigations of the observed fluctuations of the quasi-stationary planetary flow pattern, e.g. by Kubota & Iida (1954), Graham (1955), Barret (1961), Deland (1964, 1965), Haney (1961), and Eliasen & Machenhauer (1965), which clearly illustrate that fluctuations in amplitude and phase of the waves with small longitudinal wave-number are the result of a travelling wave being superimposed on a stationary wave with a larger amplitude. If the amplitude of the travelling wave is larger than the stationary wave, then the wave is no longer quasi-stationary, which also frequently occurs. It is also demonstrated that the phase speed of the travelling wave is in close agreement with the phase speed obtained with the Rossby formula, particularly if it is modified with the divergence effect, however, to a much lesser extent than Cressman (l.c.) suggested.

In this investigation, which is devoted to increasing our knowledge regarding these fluctuations, we shall limit ourselves to consider only the effect of exchange of sensible heat between oceans and atmosphere. Since this type of external force is one of the most important effects for the development of these quasi-stationary waves it is expected that it also plays an important role in producing the fluctuations of these waves.

For a more complete treatment of this problem it is evident that other types of non-adiabatic heating and the effect of orography have to be included. Another cause of the fluctuations of the large-scale motion which ought to be taken into account is the transfer of kinetic energy from the smaller scale phenomena (see Saltzman, 1959, and Saltzman & Fleisher, 1960).

2. Scale considerations

Attempts have been made with the aid of scale analysis to explain why the ultra-long waves display a quasi-stationary character. Burger (1958) demonstrated that when the characteristic horizontal scale (half-wave length) S is of the same order of magnitude as the earth's radius a,

$$\frac{S}{a} \sim 1.$$

All terms in the vorticity equation are considerably smaller than the β-term and the divergence term. The vorticity equation then becomes

$$\mathbf{V} \cdot \nabla f + f \operatorname{div} \mathbf{V} = 0.$$

This result was then taken to be the explanation for the lack of agreement between the observed quasi-stationary planetary motion and the theory of the Rossby waves. This is, however, not entirely correct. As was pointed out in the introduction, the observed quasi-stationary ultra-long waves can be split up into two parts, one stationary and one travelling. By following Eliasen & Machenhauer (1965) we write one wave component of the wind field in the following form,

$$\psi = \bar{\psi}^D + (\psi - \bar{\psi}^D),$$

where the bar denotes the mean value over D days. The first part cannot contain fluctuations with periods less than D days. With a proper value of D it is possible to express the observed windfield in such a way that the effect of external forces, which are effective on the planetary scales, viz. large-scale heating and mountain effects, is responsible for the stationary part, $\bar{\psi}^D$.

If now a scale analysis for the determination of the relative importance of the various terms in the basic hydrodynamic and thermodynamic equations is carried out for planetary scale motions of the atmosphere, with the assumption that the effect of external forces can be disregarded, then it is evident that this analysis is not valid for the stationary part of the motion, $\bar{\psi}^D$, which is caused by these forces. The characteristic phase speed c of the streamline configuration must then necessarily be connected with the moving part of the field, $\psi - \bar{\psi}^D$, which for the very largest scale of motion can be considerably greater than the characteristic wind velocity, V.

Since the phase-speed c to some extent is dependent on the external forces, this is not entirely correct; however, it gives a more satisfactory result than to assume that c is less than or equal to the characteristic wind velocity V. This implies that the local rate of change of the relative vorticity cannot be disregarded. This has been demonstrated by Deland (1965), who also pointed out that the inclusion of the latitudinal variation of the planetary scale motion leads to the zonal advection term in the vorticity equation becoming important.

In our treatment of the behaviour of the termally produced planeatary scale motion we can thus confidently make use of the vorticity equation in the form

$$\frac{\partial \zeta}{\partial t} + \mathbf{V} \cdot \nabla(\zeta + f) + f \operatorname{div} \mathbf{V} = 0.$$

3. Basic physical assumptions

Thus, in this study we shall be concerned with the role non-adiabatic heating plays in connexion with the variations of large-scale motions of the atmosphere. In order to obtain a better understanding of this dependence we shall with the aid of a quasi-geostrophic, linearized, time-dependent model investigate how the atmosphere responds to the exchange of sensible heat with the underlying surfaces. This approach to the problem, where time dependence is taken into account, is based to some extent on a previous study by the author (1962).

Heating is assumed to be proportional to the temperature difference between the air and the surface, regardless whether this be land or ocean. The vertical variation of heating is assumed to be a decreasing power of pressure.

The heating rate per unit mass can then be written in the following form

$$Q = K(T_g - T_s)\left(\frac{p}{p_s}\right)^r,$$

where K and r are constants, p_s is the surface pressure, T_g and T_s are the temperatures of the underlying surface and the air respectively.

Concerning the temperature, T_g, we shall assume it is independent of time. This assumption of course excludes the possibility of studying long-term variations in the atmospheric circulation which are due to seasonal variations in the temperature of the oceans. Furthermore, we have with this assumption automatically neglected the feed-back mechanism which results from the variations in the atmospheric circulation causing changes in the temperature of the sea surface through, e.g. upwelling and temperature advection in the sea. Since the variations produced by this mechanism are rather insignificant for short time periods, we can safely disregard this effect as long as we are interested in the large-scale circulations of the atmosphere which have a characteristic time period of only a few weeks.

4. The basic equations

For this treatment of the problem we shall make use of the vorticity equation, the continuity equation, the thermodynamic equation, the hydrostatic equation and the quasi-geostrophic approximation.

The dependent variables are the two components of horizontal wind, u and v; the vertical p-velocity, ω; the height of a pressure surface, z; and the air temperature, T_a.

Since we are going to consider small disturbances which are superimposed on a zonal current which is a function of pressure only, we express the dependent variables in the following form

$$\begin{cases} u = U(p) + u'(x, y, p, t), \\ v = v'(x, y, p, t), \\ \omega = \omega'(x, y, p, t,) \\ z = \bar{z}(y, p) + z'(x, y, p, t) \\ T_a = \bar{T}_a(y, p) + T_a'(x, y, p, t) \end{cases}$$

and $$T_g = \bar{T}_g(y) + T_g'(x, y),$$

where the primed quantities refer to the perturbations.

We shall make use of a local Cartesian coordinate system with pressure as the vertical coordinate together with the mid-latitude "β-plane" approximation. The Coriolis parameter $2\Omega \sin \varphi$ is thus represented by the linear function $f + \beta y$, where f and β are constants.

After linearization the equations become

$$
\begin{cases}
\left(\dfrac{\partial}{\partial t}+U\dfrac{\partial}{\partial x}\right)\left(\dfrac{\partial v'}{\partial x}-\dfrac{\partial u'}{\partial y}\right)+\beta v'+f\left(\dfrac{\partial u'}{\partial x}+\dfrac{\partial v'}{\partial y}\right)-\dfrac{dU}{dp}\dfrac{d\omega'}{\partial y}=0, & (1)\\[2ex]
\dfrac{\partial u'}{\partial x}+\dfrac{\partial v'}{\partial y}+\dfrac{\partial \omega'}{\partial p}=0, & (2)\\[2ex]
\left(\dfrac{\partial}{\partial t}+U\dfrac{\partial}{\partial x}\right)\dfrac{\partial z'}{\partial p}+v'\dfrac{\partial}{\partial y}\dfrac{\partial \bar{z}}{\partial p}+\sigma\omega'=-\dfrac{R}{c_p g}\dfrac{Q}{p}, & (3)\\[2ex]
\dfrac{\partial z'}{\partial p}=-\dfrac{RT'_a}{gp}, & (4)
\end{cases}
$$

where
$$\sigma=\frac{\partial^2\bar{z}}{\partial p^2}+\frac{1}{\varkappa p}\frac{\partial \bar{z}}{\partial p} \tag{5}$$

is a measure of the static stability, which we shall consider to be constant. We shall also assume that in the undisturbed state the surface air temperature is in thermal equilibrium with the underlying surface averaged along the x-axis (latitude).

$$\bar{T}_a(y,p_s)=\bar{T}_g(y).$$

The expression for the non-adiabatic heating can then be written

$$Q=K[T'_g(x,y)-T'_a(x,y,p_s,t)]\left(\frac{p}{p_s}\right)^r. \tag{7}$$

The pressure p_s will be approximated by a constant value p_0 which is assumed to be representative for the top of the friction layer. By making use of the hydrostatic equation we get the following form for the turbulent transfer of sensible heat:

$$Q=K\left[T'_g+\frac{gp_0}{R}\left(\frac{\partial z'}{\partial p}\right)_0\right]\left(\frac{p}{p_0}\right)^r. \tag{8}$$

Now we introduce the geostrophic wind approximation and eliminate the divergence between the vorticity equation and the continuity equation. The vorticity equation and the thermodynamic equation then become

$$
\begin{cases}
\left(\dfrac{\partial}{\partial t}+U\dfrac{\partial}{\partial x}\right)\left(\nabla^2 z'-\dfrac{\beta}{f}\dfrac{\partial z'}{\partial x}\right)+\beta\dfrac{\partial z'}{\partial x}-\dfrac{f^2}{g}\dfrac{\partial\omega'}{\partial p}-\dfrac{f}{g}\dfrac{dU}{dp}\dfrac{\partial\omega'}{\partial y}=0, & (9)\\[2ex]
\left(\dfrac{\partial}{\partial t}+U\dfrac{\partial}{\partial x}\right)\dfrac{\partial z'}{\partial p}-\dfrac{dU}{dp}\dfrac{\partial z'}{\partial x}+\sigma\omega'=-\dfrac{RK}{c_p gp}\left[T'_g+\dfrac{gp_0}{R}\left(\dfrac{\partial z'}{\partial p}\right)_0\right]\left(\dfrac{p}{p_0}\right)^r. & (10)
\end{cases}
$$

We assume that the contribution from the twisting term and the β-term in the expression for the relative vorticity can be disregarded.

By making use of the non-dimensional independent variable

$$P=p/p_0$$

we obtain

$$
\begin{cases}
\left(\dfrac{\partial}{\partial t}+U\dfrac{\partial}{\partial x}\right)\nabla^2 z'+\beta\dfrac{\partial z'}{\partial x}=\dfrac{f^2}{gp_0}\dfrac{\partial\omega'}{\partial P}, & (11)\\[2ex]
\left(\dfrac{\partial}{\partial t}+U\dfrac{\partial}{\partial x}\right)\dfrac{\partial z'}{\partial P}-\dfrac{dU}{dP}\dfrac{\partial z'}{\partial x}+\sigma p_0\omega'=-\dfrac{RK}{c_p g}\left[T'_g+\dfrac{g}{R}\left(\dfrac{\partial z'}{\partial P}\right)_0\right]P^{r-1}. & (12)
\end{cases}
$$

Elimination of ω between these two equations yields

$$\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\left(\frac{\partial^2 z'}{\partial P^2}+\frac{g\sigma p_0^2}{f^2}\nabla^2 z'\right)+\left(\frac{\beta}{f^2}g\sigma p_0^2-\frac{d^2U}{dP^2}\right)\frac{\partial z'}{\partial x}+\frac{RK}{c_p g}\times\left[T'_g+\frac{g}{R}\left(\frac{\partial z'}{\partial P}\right)_0\right]\frac{d}{dP}P^{r-1}=0. \tag{13}$$

Because the procedure used in solving this equation is dependent on the vertical variations of the horizontal mean wind and the non-adiabatic heating, it is necessary to make certain decisions regarding these functions before proceeding to formulate initial and boundary conditions.

As has previously been demonstrated by the author (1962), the choice of a wind profile and the value of r, which determines the rate of decrease in the heating, are not critical. Fairly good agreement with observed variations of these quantities is obtained by using $r = 1$ and the wind profile

$$U = U^*(P - \varepsilon)^2, \tag{14}$$

where
$$U^* = \frac{\beta g \sigma p_0^2}{2f^2} \tag{15}$$

and ε is an arbitrary non-dimensional constant. With these assumptions equation (13) becomes

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\left(\frac{\partial^2 z'}{\partial P^2} + \frac{g\sigma p_0^2}{f^2}\nabla^2 z'\right) = 0. \tag{16}$$

We notice that with this choice of r the term which represents the non-adiabatic heating vanishes from the equation and occurs in the boundary conditions only.

Now we prescribe the deviation T_g' from the zonally averaged temperature, $\bar{T}_g$, of the ground to be harmonic in x and y

$$T_g'(x, y) = D^* \cos(kx - \Delta)\cos ly$$
$$= (D_1 \sin kx + D_2 \cos kx)\cos ly, \tag{17}$$

where k and l are the wave numbers in the longitudinal and latitudinal directions respectively. By expressing T_g' in the above manner we consequently assume the perturbations of the pressure surfaces to be of the following form:

$$z'(x, y, p, t) = z^*(P, t)\cos[\bar{k}x - \psi(P, t)]\cos ly$$
$$= [Z_1(P, t)\sin kx$$
$$+ Z_2(P, t)\cos kx]\cos ly. \tag{18}$$

By defining the new dependent variable

$$Z = Z_1 + iZ_2 \tag{19}$$

we finally arrive at the following form for our equation

$$\left(\frac{\partial}{\partial t} + ikU\right)\left(\frac{\partial^2 Z}{\partial P^2} - \gamma^2 Z\right) = 0, \tag{20}$$

where
$$\gamma^2 = \frac{g\sigma p_0^2}{f^2}(k^2 + l^2). \tag{21}$$

5. Initial and boundary conditions

In order to study the pattern of development of a thermally produced disturbance we shall assume the pressure surfaces are undisturbed initially.

Regarding the boundary conditions, we require the vertical p-velocity to be zero at the top of the atmosphere. At the bottom of the atmosphere, which is defined as the top of the friction layer, we make use of the expression for the frictionally produced vertical velocity derived by Charney & Eliassen (1949).

Thus the initial and boundary conditions become

$$\begin{cases} z' = 0 & \text{at } t = 0, & (22) \\ \omega' = 0 & \text{at } P = 0, & (23) \\ w' = -\dfrac{g}{f}\left(\dfrac{2K^*}{f}\right)^{\frac{1}{2}}\sin 2\alpha & & \\ \qquad \times (k^2 + l^2)z' & \text{at } P = 1, & (24) \end{cases}$$

where K^* is the eddy viscosity coefficient and α is the acute angle between the surface wind and the isobars.

In view of the geostrophic approximation we have

$$\omega' = g\bar{\varrho}\frac{\partial z'}{\partial t} - g\bar{\varrho}w'. \tag{25}$$

We can now express the boundary conditions in terms of the dependent variable Z by making use of the thermodynamic equation (12) with $r = 1$, together with (18), (19) and (25). This gives

$$\begin{cases} Z = 0 & \text{at } t = 0, & (26) \\ \left(\dfrac{\partial}{\partial t} + ikU\right)\dfrac{\partial Z}{\partial P} - ik\dfrac{dU}{dP}Z & & \\ \qquad + \dfrac{RK}{c_p g}D + \dfrac{K}{c_p}\left(\dfrac{\partial Z}{\partial P}\right)_0 = 0 & \text{at } P = 0, & (27) \\ \left(\dfrac{\partial}{\partial t} + ikU\right)\dfrac{\partial Z}{\partial P} - ik\dfrac{dU}{dP} + \dfrac{RK}{c_p g}D + \dfrac{K}{c_p}\left(\dfrac{\partial Z}{\partial P}\right)_0 & & \\ \qquad + \dfrac{g\sigma p_0^2}{RT_0}\dfrac{\partial Z}{\partial t} + \dfrac{g^2 p_0^2 \sigma}{2fRT_0}\left(\dfrac{2K^*}{f}\right)^{\frac{1}{2}}\sin 2\alpha & & \\ \qquad \times (k^2 + l^2)Z = 0 & \text{at } P = 1, & (28) \end{cases}$$

where $D = D_1 + iD_2$.

6. Solution

After introducing a non-dimensional variable, τ, for time, defined as

$$\tau = kU^*t$$

the solution of equation (20) can be written in the form

$$Z(P,\tau) = F(\tau)e^{\gamma P} + G(\tau)e^{-\gamma P}, \tag{29}$$

where $F(\tau)$ and $G(\tau)$ represent arbitrary functions of time. These functions are determined, with the aid of the conditions stated in the previous paragraph, in the following way.

By substituting the solution (29) into the boundary conditions (27) and (28) we obtain

$$\left\{\begin{aligned}
&\left[\gamma\frac{d}{d\tau} + \mu\gamma e^{\gamma} + i(\gamma\varepsilon^2 + 2\varepsilon)\right]F \\
&\qquad - \left[\gamma\frac{d}{d\tau} + \mu\gamma e^{-\gamma} + i(\gamma\varepsilon^2 - 2\varepsilon)\right]G + aD = 0, \qquad (30)\\
&\left[(\gamma+\lambda)\frac{d}{d\tau} + (\mu\gamma+\nu) + i\gamma(1-\varepsilon)^2 - 2i(1-\varepsilon)\right]e^{\gamma}F \\
&\qquad - \left[(\gamma-\lambda)\frac{d}{d\tau} + (\mu\gamma-\nu) + i\gamma(1-\varepsilon)^2 \right.\\
&\qquad \left. + 2i(1-\varepsilon)\right]e^{-\gamma}G + aD = 0, \qquad (31)
\end{aligned}\right.$$

where

$$\left\{\begin{aligned}
\lambda &= \frac{g\sigma p_0^2}{RT_0},\\
\mu &= \frac{K}{c_p kU^*},\\
\nu &= \frac{g^2\sigma p_0^2}{2U^*fRT_0}\left(\frac{2K^*}{f}\right)^{\frac{1}{2}}\frac{k^2+l^2}{k}\sin 2\alpha, \qquad (32)\\
a &= \frac{RK}{c_p gkU^*}.
\end{aligned}\right.$$

By solving this system of equations we get

$$\left\{\begin{aligned}
F(\tau) &= A_1 e^{\Omega_{\rm I}\tau} + A_2 e^{\Omega_{\rm II}\tau} + F_{\rm st} \qquad (33)\\
G(\tau) &= A_3 e^{\Omega_{\rm I}\tau} + A_4 e^{\Omega_{\rm II}\tau} + G_{\rm st} \qquad (34)
\end{aligned}\right.$$

where $F_{\rm st}$ and $G_{\rm st}$ represent the solution for the stationary case. The arbitrary constants are determined with the aid of the initial condition (26) which, in view of (29), we now write

$$\left.\begin{aligned} F&=0\\ G&=0\end{aligned}\right\} \text{ at } \tau = 0. \tag{35}$$

For the complex constants

$$A_n = a_n + ib_n \quad \text{for} \quad n = 1,2,3,4 \tag{36}$$

we then obtain the following expressions

$$\left\{\begin{aligned}
A_1 &= \frac{\Omega_{\rm II}}{\Omega_{\rm I}-\Omega_{\rm II}} + \frac{aD}{M}[(\gamma-\lambda)e^{-\gamma}-\gamma],\\
A_2 &= -\frac{\Omega_{\rm I}}{\Omega_{\rm I}-\Omega_{\rm II}} - \frac{aD}{M}[(\gamma-\lambda)e^{-\gamma}-\gamma],\\
A_3 &= \frac{\Omega_{\rm II}}{\Omega_{\rm I}-\Omega_{\rm II}} + \frac{aD}{M}[(\gamma+\lambda)e^{\gamma}+\gamma], \qquad (37)\\
A_4 &= -\frac{\Omega_{\rm I}}{\Omega_{\rm I}-\Omega_{\rm II}} - \frac{aD}{M}[(\gamma+\lambda)e^{\gamma}+\gamma],
\end{aligned}\right.$$

where $\quad M = \gamma[(\gamma+\lambda)e^{\gamma} - (\gamma-\lambda)e^{-\gamma}].$

We write the complex frequencies in the form

$$\left\{\begin{aligned}
\Omega_{\rm I} &= \Omega_1 + i\omega_1,\\
\Omega_{\rm II} &= \Omega_2 + i\omega_2
\end{aligned}\right. \tag{38}$$

and introduce the following function of

$$\left\{\begin{aligned}
C_1 &= a_1 e^{\gamma P} + a_3 e^{-\gamma P},\\
C_2 &= b_1 e^{\gamma P} + b_3 e^{-\gamma P},\\
C_3 &= a_2 e^{\gamma P} + a_4 e^{-\gamma P},\\
C_4 &= b_2 e^{\gamma P} + b_4 e^{-\gamma P}.
\end{aligned}\right. \tag{39}$$

According to (29), the solution (18) for the perturbations of the height of the pressure surfaces becomes

$$\begin{aligned}
z' = \{&[F_r(\tau)e^{\gamma P} + G_r(\tau)e^{-\gamma P}]\sin kx\\
&+ [F_i(\tau)e^{\gamma P} + G_i(\tau)e^{-\gamma P}]\cos kx\}\cos ly, \qquad (40)
\end{aligned}$$

where r denotes the real part and i the imaginary part of the solutions F and G. By making use of (33)–(39) we then obtain after some manipulation

$$z' = [\sqrt{C_1^2+C_2^2}\{e^{\Omega_1\tau}\cos[kx+\omega_1\tau-\delta_1(P)] - \cos[kx-\delta_1(P)]\} + \sqrt{C_3^2+C_4^2}\{e^{\Omega_2\tau}\cos[kx+\omega_2\tau-\delta_2(P) - \cos[kx-\delta_2(P)]\}]\cos ly, \quad (41)$$

where the phase angles δ_1 and δ_2 are given by

$$\begin{cases} \delta_1 = \operatorname{arctg}\dfrac{C_1(P)}{C_2(P)}, \\ \delta_2 = \operatorname{arctg}\dfrac{C_3(P)}{C_4(P)}. \end{cases} \quad (42)$$

An alternative form of the solution (41) is given by

$$z' = z^*(P,\tau)\cos[kx-\psi(P,\tau)]\cos ly, \quad (43)$$

where

$$\begin{cases} z^*(P,\tau) = \sqrt{Z_1^2+Z_2^2}, & (44) \\ \psi^*(P,\tau) = \operatorname{arctg}\dfrac{Z_1}{Z_2}, & (45) \end{cases}$$

$$\begin{cases} Z_1 = -\sqrt{C_1^2+C_2^2}\{e^{\Omega_1\tau}\sin[\omega_1\tau-\delta_1(P)] + \sin\delta_1(P)\} - \sqrt{C_3^2+C_4^2}\{e^{\Omega_2\tau}\sin[\omega_2\tau-\delta_2(P)] + \sin\delta_2(P)\}, \\ Z_2 = \sqrt{C_1^2+C_2^2}\{e^{\Omega_1\tau}\cos[\omega_1\tau-\delta_1(P)] - \cos\delta_1(P)\} + \sqrt{C_3^2+C_4^2}\{e^{\Omega_2\tau}\cos[\omega_2\tau-\delta_2(P)] - \cos\delta_2(P)\}. \end{cases} \quad (46)$$

7. Properties of the solution

From the two versions of the solution we have obtained, it is possible to make some general comments regarding the behaviour of the motion caused by the external force Q.

The solution thus consists of a time-dependent part which is superimposed on a stationary perturbation

$$z'(x,y,P,\tau) = z'_{\text{st}}(x,y,P) + z'_{\text{tr}}(x,y,P,\tau).$$

The time-dependent part is represented by two harmonic waves, whose amplitudes are functions of time and pressure

$$z'_{\text{tr}} = \sum_{\nu=1}^{2} B_\nu(P)\,e^{\Omega_\nu\tau}\cos[kx+\omega_\nu\tau-\delta_\nu(P)]\cos ly, \quad (47)$$

where

$$\begin{cases} B_1(P) = \sqrt{C_1^2+C_2^2}, \\ B_2(P) = \sqrt{C_3^2+C_4^2}. \end{cases} \quad (48)$$

The phase speeds for these individual waves are independent of P and τ.

$$c_\nu = -U^*\omega_\nu. \quad (49)$$

The phase speed for the total motion, however, becomes a function of both pressure and time. It can be calculated from

$$c(P,\tau) = \frac{1}{k}\frac{\partial\psi}{\partial t} = U^*\frac{\partial\psi}{\partial\tau} = z^{*-2}\left(Z_2\frac{\partial Z_1}{\partial\tau} - Z_1\frac{\partial Z_2}{\partial\tau}\right). \quad (50)$$

The stationary part of the solution can also be written in the following form

$$z'_{\text{st}} = -B^*(P)\cos[kx-\delta(P)]\cos ly, \quad (51)$$

where

$$\begin{cases} B^*(P) = [(C_1+C_3)^2+(C_2+C_4)^2]^{\frac{1}{2}} & (52) \\ \delta(P) = \operatorname{arctg}\dfrac{C_1+C_3}{C_2+C_4}. & (53) \end{cases}$$

The development from the initial state when the time-dependent part of the solution compensates the stationary part ($z'_{\text{tr}} = -z'_{\text{st}}$), is unstable when Ω_1 and/or Ω_2 are larger than zero.

In the stable case (Ω_1 and Ω_2 are both less than zero) the time-dependent part of the solution is oscillatory with a steadily decreasing amplitude. The disturbance then approaches the steady-state solution asymptotically.

If $\Omega_1 \leqslant \Omega_2$ and $\Omega_2 = 0$, the amplitude of the resulting motion, $z^*(P,\tau)$ is oscillatory around the steady-state solution.

The question whether the disturbance as given by (43) is quasi-stationary or travelling, is determined by the relative magnitude of the amplitudes of the two travelling waves (48) and the amplitude of the stationary wave (52). With the aid of the ratios R_1, and R_2

Table 1. *The amplitude in metres and the phase-angle tn degrees of longitude*

n	D_n^*	Δ_n/n
0	3.80	—
1	2.64	238
2	4.31	166
3	3.49	78
4	1.67	56
5	0.30	46

$$\left\{\begin{aligned} R_1^2 &= \frac{C_1^2+C_2^2}{(C_1+C_3)^2+(C_2+C_4)^2}e^{2\Omega_1\tau}, \\ R_2^2 &= \frac{C_3^2+C_4^2}{(C_1+C_3)^2+(C_2+C_4)^2}e^{2\Omega_2\tau} \end{aligned}\right. \tag{54}$$

we can state that

the disturbance is quasi-stationary if

$$\left.\begin{matrix} R_1 \\ R_2 \end{matrix}\right\} < 1,$$

the disturbance is travelling if

$$R_1 \quad \text{or} \quad R_2 > 1.$$

These ratios are either increasing or decreasing with time, depending on the signs of Ω_1 and Ω_2. In the stable case, e.g., it is thus possible that a disturbance is travelling during a certain period of time and at a later stage becomes stationary or quasi-stationary.

We notice also that these ratios are functions of pressure. A disturbance can therefore be stationary or quasi-stationary in certain layers and travelling in other layers.

8. Numerical results

In the evaluation of the solution we shall make use of such values of the various parameters and constants as are representative for the winter season in mid-latitudes (45° N).

By a zonal harmonic analysis of observed surface temperatures of the oceans we obtained the following approximate expression for the deviation from the zonally averaged temperature

$$T_g'(x,y) = [D_0^* + \sum_{n=1}^{N} D_n^* \cos(nk_1 x - \Delta_n)] \cos ly, \tag{55}$$

where n denotes the number of waves around the latitude circle 45° N. Then

$$k = nk_1 = n \cdot 2.22 \cdot 10^{-9}\,\text{cm}^{-1}.$$

The wavelength which corresponds to this wave number is

$$L_x = \frac{2\pi}{k} = \frac{28\,200}{n}\ \text{km}.$$

The meridional wave number is chosen to be

$$l = 1.26 \cdot 10^{-8}\,\text{cm}^{-1}, \quad L_y = 5000\,\text{km}.$$

Table 1 shows the values of the amplitude and the phase angle for the first five harmonics.

Furthermore, we shall in the computations make use of the following numerical values:

$K = 10^2$ erg gm^{-1} °K^{-1} sec^{-1}

$\sigma = 0.24$ cm mb^{-2}

$\varrho_0 = 900$ mb

$T_0 = 273$°K

$K^* = 10^5$ cm^2 sec^{-1}

$\alpha = 15°$

$U^* = 1450$ cm sec^{-1}

$\varepsilon = 1.55$

$f = 1.03 \cdot 10^{-4}$ sec^{-1}

$\beta = 1.62 \cdot 10^{-13}$ cm^{-1} sec^{-1}

Then

$\lambda = 0.25$

$\mu = 3.10/n$

$\nu = 0.039\,(n^2+32)/n$

$a = 400/n$ cm°K^{-1}

The stationary case:

We shall first consider some of the results which are obtained for the stationary case. These results are considerably simpler to compare with observed conditions than the time-dependent solution. In this way we can also get some evidence that the values chosen for the various parameters are realistic.

The observed values which we are going to make use of in this comparison are obtained

Table 2. *Comparison of the observed and computed values for the amplitude, B^* (in metres), and the phase angle, δ (in degrees of longitude), in the stationary case*

			$n=1$		$n=2$		$n=3$	
			B^*	δ	B^*	δ	B^*	δ
Comp.	$P=0.5$		19	10	18	67	10	18
Obs.	500 mb.	40° N	84	−34	34	42	64	−22
		50° N	84	6	79	43	80	−14
Comp.	$P=1.0$		33	54	48	85	33	28
Obs.	1000 mb.	40° N	39	46	48	89	32	−12
		50° N	76	66	83	85	33	2

from Eliasen (1958), who has performed a zonal harmonic analysis of the height field of certain pressure surfaces for different latitudes. We thus consider the deviations from the zonal mean value of the pressure surfaces, 500 mb and 1000 mb, in January. For the three lowest wave numbers we get the values of amplitudes (B^*) and phase angles (δ) for the computed (51) and the observed height fields which are given in Table 2.

Since the observed height field exhibits a pronounced variation with respect to latitude, we present the values for this field for the two latitudes 40° N and 50° N.

The computed heights refer to the two levels $P=0.5$ and $P=1$.

Quite generally we can state there is a fairly good agreement between observed and computed height fields for the lower pressure surface ($p=1000$ mb, $P=1$) and less good for the upper pressure surface ($p=500$ mb, $P=0.5$). Before proceeding to a somewhat more detailed discussion of this comparison we may also state that this is by and large what one could expect. As a matter of fact, it would have been astonishing if agreement had been much better since we have only taken into account one of the effects which are important in this connexion. Thus it has e.g. been pointed out by Eliasen (1958) that the large-scale distribution of heat sources plays an important role in the formation of the planetary waves in the lowest part of the troposphere, whereas the effect of large mountain barriers becomes more important in the middle and upper troposphere. The plausibility of this statement has been demonstrated by Saltzman (1963, 1965). This is, however, not necessarily the entire explanation of the fact that the amplitude of the computed disturbance at $P=0.5$ is much less than the observed. As has been demonstrated by the author (1962), the choice of the parameter, r, determines the vertical variation of the heating and has a strong influence in this respect. As the value of r is increased, i.e. the heating is more pronounced in the lowest part of the atmosphere, the amplitude becomes

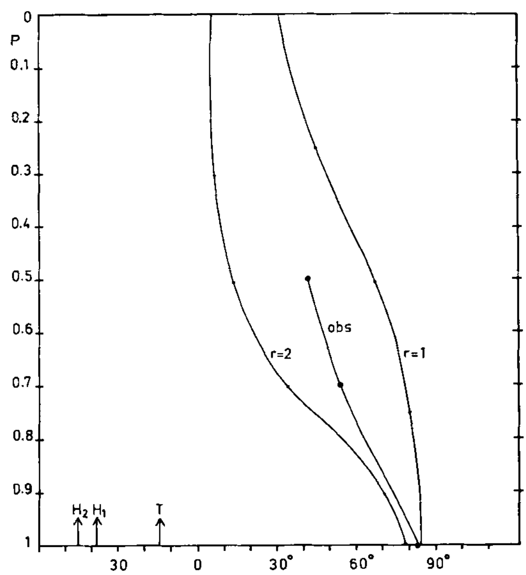


Fig. 1. The vertical variation of the position of a ridge for $n=2$ for two values of the parameter r. The observed variation (*obs*) represents the normal values in January at 45°N. T represents the position of the maximum deviation of the ground temperature from the zonal mean value. H_1 and H_2 give the position of the maximum rate of heating for $r=1$ and 2 respectively.

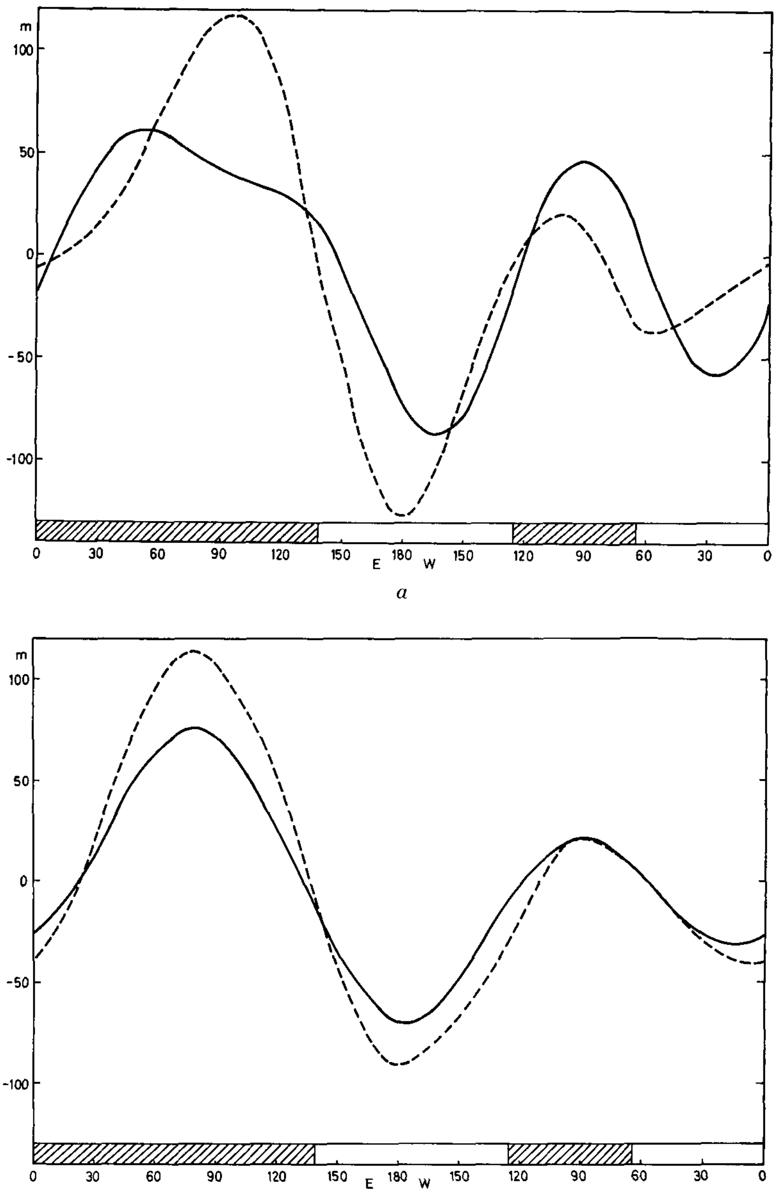


Fig. 2. (*a*) The dashed line gives the observed mean profile of the 1000 mb surface at 45°N in January as represented by the sum of the first three longitudinal wave numbers ($n = 1$, 2 and 3) according to Eliasen (1958). The solid line represents the corresponding profile ($P = 1$) for the stationary solution (56). (*b*) Same as Fig. 2, except that only the first two longitudinal wave numbers ($n = 1$ and 2) are taken into account.

weaker in the lower layers and stronger in the upper layers. Furthermore, the variation of phase with respect to height becomes stronger, but the positions of troughs and ridges do not change significantly at the surface. At the level $P = 0.5$, however, they get a more westerly position. This is illustrated for $r = 2$ in Fig. 1, in which we compare observed and computed variations of phase angle and amplitude along the vertical for two values of this parameter

($r = 1$ and 2). By studying this figure we see that a somewhat better agreement can be achieved by choosing another value of r, say 1.5.

The result of this comparison for the lower layer ($p = 1000$ mb, $P = 1$) has been illustrated in Fig. 2a, where all three terms ($n = 1$, 2 and 3) have been taken into account, and in Fig. 2b, where only the first two terms are included.

$$z' = -\sum_{n=1}^{N} B_n^* \cos[nk, x - \delta_n] \cos ly \quad \text{for} \quad N = \begin{cases} 3 & \text{Fig. } 2a, \\ 2 & \text{Fig. } 2b. \end{cases} \tag{56}$$

These figures illustrate quite clearly that the agreement between observed and computed conditions is rather good for the first two wave numbers and less good for the third wave number. Without trying to draw too far going conclusions we may state that this is in agreement with the interpretation put forward by van Mieghem (1960) of the seasonal displacements of the observed positions of troughs and ridges. The contribution from Fourier wave number $n = 3$ shows, in contrast with wave numbers $n = 1$ and $n = 2$, very small changes with season. It is therefore likely that wave number 3 is mainly of orographic origin.

The time-dependent case:

The calculations of the height (z') of a pressure surface which is given by (41) or (43), has been performed for four pressure surfaces, $P = 0$, $\frac{1}{4}$, $\frac{1}{2}$ and 1. The results of some of these calculations are shown in Figs. 3a ($n = 1$), 3b ($n = 2$), and 3c ($n = 3$). For these three longitudinal wave numbers we have illustrated the variation with respect to time of the amplitude (44) and the phase angle (45) of a ridge for two pressure surfaces, $P = \frac{1}{2}$ and 1. We have for the sake of comparison also shown in the figures the values of these quantities as obtained from normal charts in January.

The amplitudes B_1 and B_2 (48) of the two components of the non-stationary part of the solution and the amplitude B^* (52) of the stationary part are given in Table 3. This table also contain the values of Ω and c which express the exponential growth rate of the travelling waves and the phase velocity respectively. Furthermore, for $n = 2$, we have included in this table corresponding quantities for a larger value of the meridional wave-length (L_y = 20 000 km).

In order to isolate the effect of the non-adiabatic heating we have also evaluated Ω and c in the adiabatic case, both with and without friction. The results of these calculations are given in Table 4. We have also presented the phase velocities $c_R(q = 0)$ and $c_{RD}(q = 10^{-13}\ \text{m}^{-2})$ which are obtained with the linearized barotropic model

$$c_{RD} = \frac{U(k^2 + l^2) - \beta}{k^2 + l^2 + q}, \tag{57}$$

where the parameter q represents large-scale divergence (Cressman, 1958). The value used for q is approximately the same as Eliasen & Machenhauer (1965) found gave good agreement in their comparison of the Rossby-Haurwitz velocity with the velocity of the observed moving part of the fluctuations.

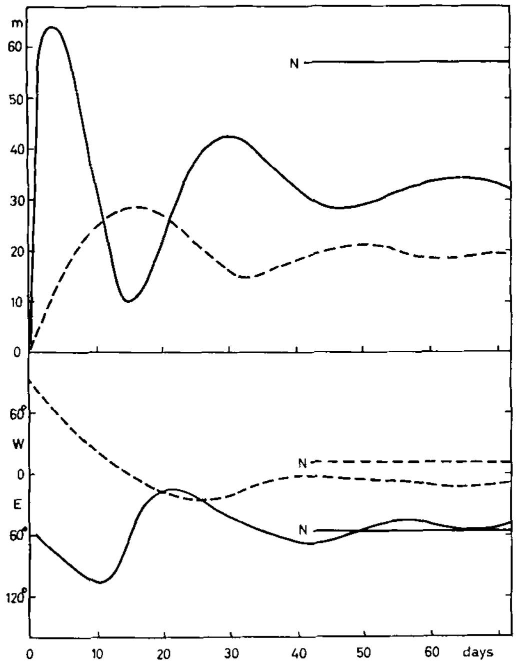
It should be pointed out that this value of q is much less than the value Cressman suggested ($q = 0.75\ 10^{-12}\ \text{m}^{-2}$), which is commonly used in numerical forecasting with the barotropic model. The reason for this considerable difference in the value of q is evidently that the entire field of motion is taken into account in numerical weather prediction, i.e. also the stationary part. If a model does not contain mechanisms which are responsible for the long stationary waves, as is e.g. the case with the barotropic model, it is convenient to use this artificial method of reducing the large negative phase speeds of these long waves.

9. Discussion

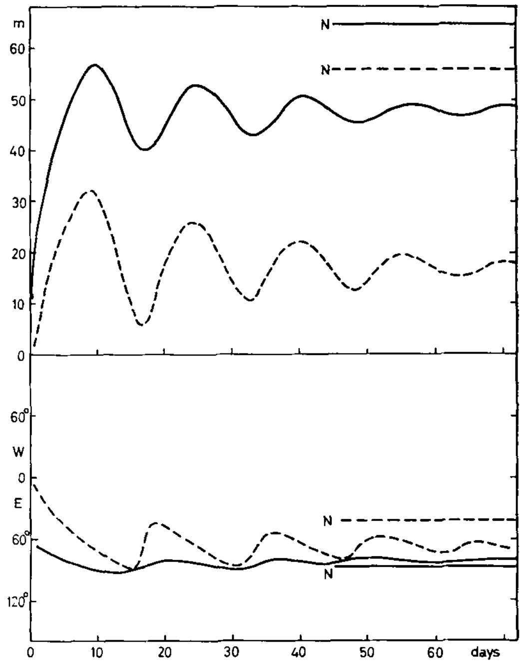
In view of the numerous simplifying assumptions we have introduced it is evident we cannot make a detailed comparison of the results with the observed motion of the atmosphere. It may, however, be possible to make certain general comparisons which can help to improve our understanding regarding the behaviour of the ultra-long waves.

In discussing the main features of the solution we may first state that it simulates the observed fact that the planetary waves consist of a stationary and a travelling component.

Regarding the stationary part of the solution we have previously demonstrated that there is good agreement with the observed conditions for the lowest longitudinal wave numbers. We



a

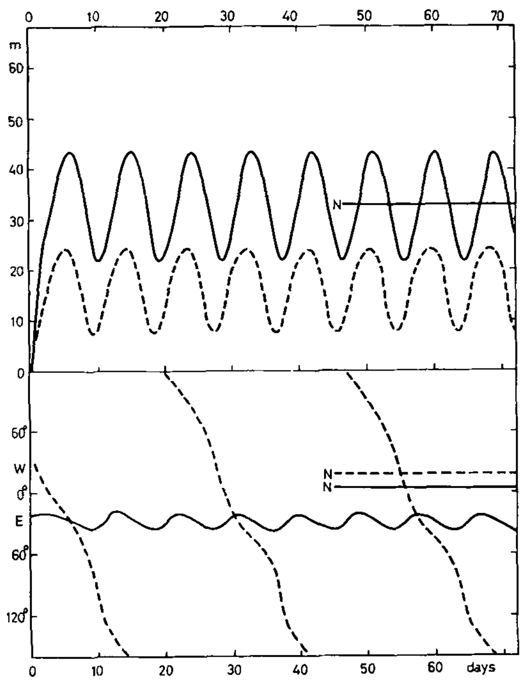


b

Fig. 3 (*a*) The computed variation with time of the amplitude (44) and the phase angle (45) of a ridge for the longitudinal wave number, $n = 1$. The solid lines refer to the level $P = 1$ and the dashed line to $P = 0.5$. The lines marked N represent the normal values in January at 45°N (the normal amplitude at 500 mb, 84 m, is not shown).

(*b*) Same as Fig. 3*a*, except that $n = 2$.

(*c*) Same as Fig. 3*a*, except that $n = 3$ (the normal amplitude at 500 mb, 72 m, is not shown).



c

should remember the computed values refer to one latitudinal scale ($L_y = 5000$ km) while the observed mean values refer to the actual values. This strongly indicates that in winter the chosen value for L_y is the dominant one.

Concerning the observed non-stationary part of the planetary disturbances it has been clearly illustrated that they move in conformity with the Rossby phase speed formula if it is modified with a weak divergence effect (57). This is also in agreement with the present baroclinic theory since this gives similar values for the phase velocity (cf. Table 4). We notice that the effects of friction and non-diabatic heating have only a minor influence on the phase-velocity, as also has been demonstrated in special studies of the effects of friction (Holopainen, 1961) and the exchange of sensible heat (Haltiner, 1967) on the dynamics of baroclinic waves.

Table 3. *The values of Ω_ν, c_ν, and B_ν ($\nu = 1, 2$) for the travelling waves and for the stationary wave*

n	L_y (km)	Ω (day^{-1})	c (m sec^{-1})		$P=1$	$P=\frac{3}{4}$	$P=\frac{1}{2}$	$P=\frac{1}{4}$
1	5 000	$\Omega_1 = -0.053$	$c_1 = 9.8$	B_1	51	21	22	53
		$\Omega_2 = -0.926$	$c_2 = 9.8$	B_2	83	36	3	45
				B^*	33	21	19	24
2	5 000	$\Omega_1 = -0.034$	$c_1 = 10.4$	B_1	13	16	22	32
		$\Omega_2 = -0.953$	$c_2 = 10.3$	B_2	39	19	8	18
				B^*	48	29	18	17
3	5 000	$\Omega_1 = 0$	$c_1 = 12.3$	B_1	11	12	15	23
		$\Omega_2 = -1.004$	$c_2 = 9.5$	B_2	29	14	8	18
				B^*	33	19	10	7
2	20 000	$\Omega_1 = -0.804$	$c_1 = 9.6$	B_1	56	17	23	63
		$\Omega_2 = -0.084$	$c_2 = -26.7$	B_2	90	29	31	93
				B^*	45	19	9	34

Table 4. *Comparison of the growth rates (Ω) and phase velocities (c) in the non-adiabatic and adiabatic cases (with and without friction) and the phase velocity in the barotropic case (with and without a free surface)*

		Heating Friction		No heating Friction		No heating No friction		Barotropic	
								$q=0$	$q=10^{-13}$ m^{-2}
n	L_y (km)	Ω (day^{-1})	c (m sec^{-1})	Ω (day^{-1})	c (m sec^{-1})	Ω (day^{-1})	c (m sec^{-1})	c_R (m sec^{-1})	c_{RD} (m sec^{-1})
1	5 000	−0.053	9.8	0.064	9.8	±0.080	9.8	10	9.4
		−0.926	9.8	−0.097					
2	5 000	−0.034	10.4	0.122	10.5	±0.153	10.35	10.9	10.3
		−0.953	10.3	−0.189	10.2				
3	5 000	0	12.3	0.163	11.3	±0.208	10.9	12	11.4
		−1.004	9.5	−0.263	10.7				
4	5 000	0.020	13.8	0.155	12.3	±0.215	11.7	13.2	12.6
		−1.608	9.7	−0.287	11.2				
2	20 000	−0.804	9.6	−0.034	9.6	0	9.6	−34.8	−26.1
		−0.084	−26.7	−0.058	−26.7		−26.7		

In this way we may say that we have been able to make simultaneous comparisons of the stationary waves with the phase velocity of the travelling waves, which is certainly more realistic than comparing only the travelling part of the waves with a barotropic theory. However, with these comparisons we have not yet discussed the important fact that these planetary waves according to observations only show small fluctuations about the mean state.

Eliasen & Machenhauer (1965) thus found in their spherical-harmonic analysis of the fluctuations of these long waves that for almost all of the latitudinal scales they considered, the amplitude in the mean flow was only slightly smaller than the mean value of the amplitude when the longitudinal wave number was less than three. Evidently this implies that the amplitude of the travelling wave is less than the amplitude of the stationary wave. It also indicates that these travelling waves are baroclinically stable, which is not in agreement with the adiabatic, linear theories of baroclinic instability. However, as has been demonstrated here, the inclusion of the transfer of sensible heat changes the condition for the baroclinic instability considerably (cf. Haltiner, 1967). We thus obtain in similarity with the observations that the amplitudes of the travelling waves for $n = 1$ and 2 decays slowly with time (cf. Tables 3 and 4). This implies that the ratios R_1 and R_2 (54) decrease with time. Consequently these waves become stationary (cf. Fig. 3*a*, *b*). Regarding wave number 3, we notice that $\Omega_1 = 0$ and that B_1/B^* is less than unity in the lower layers only. This wave is therefore quasi-stationary in the lower layers and travelling in the upper layers (cf. Fig. 3*c*).

In spite of the uncertainty of the results obtained here due to the approximations made or the choice of the values of the various parameters it is not unlikely that the type of process described here is important for the maintenance of the ultra-long waves in the atmosphere.

It may also be mentioned in this context that in the treatment of these long waves it is to a certain extent possible to justify the neglection of the non-linear effects. Based on the computations of observed energy transfer and conversions performed by Murakami & Tomatsu (1965), Dutton & Johnson (1967) have thus pointed out that the conversion of available potential energy at each wave number to the kinetic energy of that wave number exceeds exchange of kinetic energy with all other waves by at least four times, and exceeds the exchange of kinetic energy with the zonal current by about 40 times. Kinetic energy at each wave number is primarily lost by transfer to the atmosphere above 500 mb, with the transfer at wave numbers two and three being strongly dominant.

The choice of the initial condition with the height field undisturbed is undoubtedly rather artificial. However, this choice is less important in view of the fact that we are not aiming at describing the actual motion, but rather are interested to study how the atmosphere responds when it is not in thermal equilibrium with the underlying surface. Of course, in the real atmosphere this balance is repeatedly disturbed.

With any other initial condition we should have obtained a similar type of response, namely slowly damped fluctuations of the amplitude and the phase angle about the stationary state.

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ВЛИЯНИЕ КРУПНОМАСШТАБНЫХ ИСТОЧНИКОВ ТЕПЛА НА ДИНАМИКУ УЛЬТРА-ДЛИННЫХ ВОЛН

Главная цель этого исследования — изучить, до какой степени крупномасштабные источники тепла ответственны за образование и поведение ультра-длинных волн в атмосфере. Это проделано с помощью линейной, зависящей от времени, квазигеострофической модели, в которой неадиабатическое нагревание предполагается пропорциональным разнице температур атмосферы и подстилающей поверхности. Результаты, полученные с помощью этой модели, показывают некоторые характерные особенности, которые согласуются с результатами наблюдений этих длинных волн в средних широтах зимой. Таким образом, решение состоит из стационарной и перемещающейся частей. Для наименьших долготных волновых чисел стационарная часть очень близка к наблюдаемым средним условиям в нижней части атмосферы. Нестационарная часть состоит из двух перемещающихся волн, амплитуды которых убывают для трех первых волновых чисел; в соответствующем адиабатическом случае решение растет по амплитуде. Фазовая скорость для той из волновых компонент, которая затухает наиболее медленно, близка к россбиевской фазовой скорости, измененной с помощью слабого эффекта дивергенции.

Если начальное состояние выбрано так, что система не находится в термическом равновесии, то результирующее движение ультра-длинных волн представляет собою медленно затухающие флуктуации около среднего состояния.