A note on lower ionosphere equivalent height diurnal variation

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ABSTRACT

Five months of very-low-frequency phase measurements along the propagation paths Balboa (Panama)–Sao Paulo and Criggion (GB)–São Paulo, were studied in order to find apparent and equivalent diurnal height variation for the upper boundary of the earth-ionosphere waveguide. The results suggest that an explanation of the apparent height variation dependence on the wavelength and propagation path may be found if the second order mode of propagation is considered during nighttime and the diurnal change of the ionospheric conductivity gradient is taken into account. The results fit well with an "average ionosphere" model presenting nighttime conductivity gradient of 0.5 km^{-1} and daytime gradient of 0.3 km^{-1}. As a consequence, the equivalent diurnal height variation found is situated near 10 km, the assumption of a single waveguide configuration for both propagation paths being valid.

1. General

The diurnal change of ionospheric heights can be studied from phase velocity measurements at very low frequencies (VLF), assuming a sharply bounded and isotropic ionosphere, concentric with the earth's surface, constituting by this way a spherical waveguide. Assuming the first order mode of propagation as dominant in the VLF range and using a simplified form for the theory (Wait, 1959), a number of authors have studied the diurnal height changes for the upper boundary of the waveguide (Chilton *et al.*, 1963; Volland, 1964; Bates & Albee, 1964; Kaufmann *et al.*, 1967). It has been noticed that such height variations present a marked dependence on the considered propagation path and on the season of the year. In order to get an explanation for this effect it seems necessary to consider the significance of the second order mode during nighttime, and to take into account the phase lags that happen upon the waves interaction with the upper boundary of the earth-ionosphere waveguide. The variation of the lower ionosphere conductivity gradient from nighttime to daytime introduce a difference in the total phase shift measured, informing about a ficticious height variation. The present analysis is based on diurnal changes of the first order mode, being corrected for the influence of the second order mode during nighttime.

1.1. *The behaviour of the first order mode of propagation*

Considering the behaviour of the first order mode alone, the diurnal phase shift can be related to the height variation of the lower ionosphere, from nighttime to daytime, by Wait (1959):

$$\Delta\phi_1 \cong -(d/\lambda)(1/2a + \lambda^2/16h^3)\,\Delta h', \qquad (1)$$

where $\Delta\phi_1$ is the phase difference between nighttime and daytime average levels, for mode 1; $\Delta h'$ is the corresponding variation of the apparent heights of the upper boundary of the earth-ionosphere waveguide; h is the intermediate value for such boundary between nighttime and daytime values (being adopted, for the purpose of this paper, $h = 80$ km); d is the propagation great circle path length; λ is the transmitted wavelength; and a is the mean earth's radius.

Considering now vertical polarized VLF waves interacting with an ideal earth-ionosphere waveguide, whose walls are constituted by the ground (assumed as a nearly perfect conductor) and by the lower ionosphere, a certain reflection coefficient can be defined for the upper boundary as shown by Wait & Walters (1963):

$$R = -\exp\,[(\alpha_1 + j\alpha_2)C], \qquad (2)$$

where α_1 and α_2 are real, and C corresponds to the cosine of the angle of incidence of the waves. For highly oblique incident waves (i.e., $C \leqslant 0.15$), the phase lags upon reflection will be

$$\arg R = -\pi + \alpha_2 C. \tag{3}$$

It is possible to associate α_2 to an equivalent height for the spherical waveguide that is situated below the reference one by an amount of

$$\delta h = \frac{\alpha_2 \lambda}{4\pi} \quad (\text{with } C \leqslant 0.15) \tag{4}$$

after Wait & Walters (1963) and Frisius (1966).

The phase lag term α_2 can be calculated after Wait & Walters (1963) and Wait & Spies (1964), for different conductivity gradients adopted for the lower ionosphere, and as a function of the wavelength and C, being neglected the effect of the magnetic field (that is approximately valid for the transmissions considered here). The height deviation apparent effect, as expressed by (4), occur at each interaction of the waves with the upper boundary. For long propagation paths a parameter m must be taken into account, informing about how many ionospheric interactions may be considered (i.e., the approximate number of hops), in such a form that the total apparent height deviation will be $m \cdot \delta h$. If d is the length of the propagation path, and $2d'$ is the distance containing two successive ionospheric interactions, thus $m \approx d/2d'$. The value of $2d'$ can be obtained within a good approximation when one measures the distance moved by the terminator between two successive signal fadings in amplitude (or correspondent effects in phase) during the sunrise or sunset transitions, as it has been done, for example, by Lynn (1967) and Ries (1967). But, as these effects are not always clearly observable, a relation was derived by Watt & Croghan (1967), based on Wait (1962) theory for the earth detached mode approximations, that gives approximately the same value as above. It is

$$2d' \approx 4.6 \times 10^7 f^{-1/3} \tag{5}$$

for frequencies f of the order of 20 kHz and above.

Thus, the altitude diurnal variations as calculated by (1) would be more adequately represented by

$$\begin{aligned}\Delta h' &= h'_N - h'_D = (h_N - h_D) + m(\delta h_D - \delta h_N)\\ &= \Delta h + m(\delta h_D - \delta h_N),\end{aligned} \tag{6}$$

where the indices N and D correspond respectively to nighttime and daytime conditions, and

h'_N, h'_D: are the apparent heights (being respectively $h_N - m\delta h_N$ and $h_D - m\delta h_D$),
h_N, h_D: are the equivalent heights,
δh_N, δh_D: are the height differences, after (4).

Under these conditions Δh will correspond to the effective diurnal equivalent height variation, and the term

$$m(\delta h_D - \delta h_N) = m\frac{\lambda}{4\pi}[(\alpha_2)_D - (\alpha_2)_N] = m\frac{\lambda}{4\pi}D(\alpha_2) \tag{7}$$

corresponds to an apparent height additional variation caused by the upper boundary reflecting wall imperfections.

When a second transmission is simultaneously considered, it may be assumed that the same equivalent height variation Δh is valid for both paths, i.e., a single ideal waveguide configuration is applicable for the analyis of the phenomena. For two transmissions it will be

$$\Delta h'_1 = \Delta h + m\frac{\lambda_1}{4\pi}D_1(\alpha_2), \tag{8}$$

$$\Delta h'_2 = \Delta h + p\frac{\lambda_2}{4\pi}D_2(\alpha_2), \tag{9}$$

where m corresponds to the transmitter situated at a distance d_1 from the receiver, operating at a wavelength λ_1, and p corresponds to the second transmitter (d_2, λ_2). The observed difference in apparent height variation, mentioned in the beginning of this section, should then satisfy the following equation

$$\Delta H = \Delta h'_1 - \Delta h'_2 = \frac{1}{4\pi}[m\lambda_1 D_1(\alpha_2) - p\lambda_2 D_2(\alpha_2)]. \tag{10}$$

1.2. *The contribution of the second order mode during nighttime*

The above formulation may be used if the diurnal phase variation for the first order mode alone is known. The total phase change is determined from observation, and some considerations are now required in order to estimate the significance of the second order mode on the measured data. From attenuation rates and excitation factors calculated by Wait & Spies (1964) it can be foreseen that the second order mode of propagation may assume important strength, in comparison to the first order mode, during nighttime even at long propagation paths (with the upper boundary, during nighttime, at 90 km, and ionospheric conductivity gradient of 0.5 km^{-1}). During daytime the first order mode is dominant, and the second order mode is practically extinguished after sunrise transition, when the path distance is bigger than 4000 km (Crombie, 1965). The total observed diurnal phase variation will have the form

$$\Delta\phi = \Delta\phi_1 - \delta\phi \qquad (11)$$

assuming that the component due to the second mode is neglectable during daytime. $\delta\phi$ is the phase difference between the resultant electric field actually received during nighttime E, and the mode 1 component E_1 (see Fig. 1). With the knowledge of the relative intensities of the two modes components, E_1/E_2, and the phase angle between them, it is a simple matter to show that

$$\cot \delta\phi = (E_1/E_2) \csc \Delta\phi_{2,1} + \cot \Delta\phi_{2,1}. \qquad (12)$$

The phase difference between the two modes can be derived from their phase velocities in the considered waveguide (Bates & Albee, 1964), being

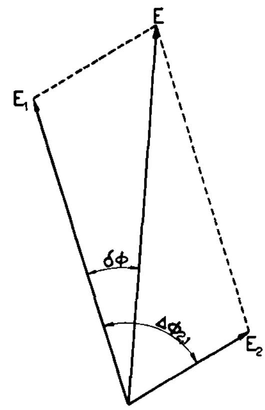

Fig. 1. A simplified diagram illustrating the relative position of the vertical electric fields for modes 1, 2 and resulting received field during nighttime. During daytime the value of E_2 becomes neglectable, and thus $E_1 \approx E$.

$$\Delta\phi_{2,1} \cong -(d\lambda/4h^2) \qquad (13)$$

in cycles. It can be shown that approximately the same values will be found for $\Delta\phi_{2,1}$ if one considers directly the phase velocities for modes 1 (v_1) and 2 (v_2), taken from Wait & Spies (1964) curves, and deduce the final phase difference along a given path (i.e., $\Delta\phi_{2,1} = d(1/v_1 - 1/v_2)$). On the other hand, the vertical electric field produced by the nth mode, in dB relative to

Table 1. *Experimental results used in the present discussion*

The diurnal phase changes in the 3rd and 4th columns are for the first order mode alone, already corrected for the contribution in phase due to the presence of the second order mode of propagation during nighttime. Standard deviations for each period of observations were determined for nighttime and daytime intervals of the transmissions

		$\Delta\phi_1 = \Delta\phi + \delta\phi$ diurnal phase shift (cycles)		Standard deviation (cycles)						
				Nighttime		Daytime		GBZ	NBA	
No.	Period 1966	GBZ	NBA	GBZ	NBA	GBZ	NBA	$\Delta h_1'$ (km)	$\Delta h_2'$ (km)	ΔH (km)
1	23 Mar.–7 Apr.	2.37	0.99	0.10	0.09	0.07	0.06	34.0	24.7	9.3
2	8 Apr.–23 Apr.	2.37	0.98	0.09	0.05	0.04	0.04	34.0	24.4	9.6
3	23 Apr.–8 May	2.32	1.01	0.09	0.05	0.05	0.03	3.33	25.2	8.1
4	9 May–16 May	2.31	0.92	0.08	0.12	0.08	0.06	33.2	23.0	10.2
5	18 May–25 May	2.25	0.99	0.05	0.07	0.02	0.02	32.4	24.7	7.7
6	25 May–9 June	2.33	1.03	0.06	0.04	0.03	0.01	33.4	25.6	7.8
7	10 June–25 June	2.14	1.03	0.06	0.04	0.03	0.01	30.9	25.6	5.3
8	25 June–10 July	2.40	1.06	0.05	0.07	0.03	0.02	34.4	26.3	8.1
9	10 July–25 July	2.39	1.03	0.07	0.05	0.05	0.02	34.2	25.6	8.6
10	25 July–9 Aug.	2.40	1.04	0.06	0.06	0.03	0.01	34.4	25.8	8.6
11	9 Aug.–14 Aug.	2.45	1.05	0.05	0.05	0.03	0.01	35.1	26.1	9.0
Averages		2.34	1.02					33.6	25.2	8.4

1 volt/m, can be derived from the waveguide mode theory (Wait, 1962), and presented in a practical form, as used by Watt & Croghan (1964) and Bickel (1967), and extensively discussed by Watt & Croghan (1967):

$$E_n = 10 \log P + K_n' - 10 \log f - 20 \log h + 20 \log \Lambda_n - 10 \log (a \sin d/a) - \alpha_n d 10^{-6}, \quad (14)$$

where $n = 1$, 2 is the mode number, P is the power in watts radiated into space by the transmitter, K_n' is a constant, being equal to 104.3 for the first two modes, f the frequency in Hz, h in meters is the height of the upper boundary during nighttime, Λ_n is the excitation factor, and α_n the attenuation rate in dB/1000 km. In the present case it was considered the whole path under nighttime, ground located receiver and transmitter, and a propagation path without any important discontinuity.

2. Analysis of the experimental data

The experimental data available for this study consisted of five months recordings obtained at Umuarama Radio Observatory, Campos do Jordão, São Paulo, Brazil, during 1966. The two VLF tracking stations, controlled by a rubidium standard oscillator, were described elsewhere (Kaufmann *et al.*, 1967), and they are part of the international research program INT–VLF articulated by the Institute for Exploratory Research (IER), N.J., USA. During the mentioned period of observation, it has been constantly received signals from GBZ (Criggion, GB, $\lambda_1 = 15.3$ km, $d_1 = 9.5$ Mm, $m \cong 5$-hop path) and NBA (Balboa, Panama, $\lambda_2 = 12.5$ km, $d_2 = 5.1$ Mm, $p \cong 3$-hop path) transmissions.

The contribution in phase $\delta\phi$, due to the presence of the second order mode of propagation during nighttime was calculated, with attenuation rates and excitation factors data published by Wait & Spies (1964), and data on transmitters' radiated power made available by IER (Reder & Viccione, 1967). It has been found that, during nighttime, it should be for GBZ signals received at São Paulo: $E_1/E_2 \cong 1.47$, $\Delta\phi_{2,1} \cong 0.71$ cycles, and $\delta\phi \cong 0.86$ cycles. For NBA it was estimated $E_1/E_2 \cong 0.14$, $\Delta\phi_{2,1} \cong 0.31$ cycles, and $\delta\phi \cong 0.30$ cycles.

The results are presented in Table 1. The nighttime and daytime phase values were averaged at time intervals well separated from the sunrise and sunset transitions, which peculiar phenomena have no direct use for the present paper. Such transition effects, however, presented clear periodic fadings in phase only on NBA transmission, and this was used just for checking the hop distance, expressed by equation (5). The standard deviations found during nighttime and daytime intervals are also listed in Table 1, and the average phase curves for

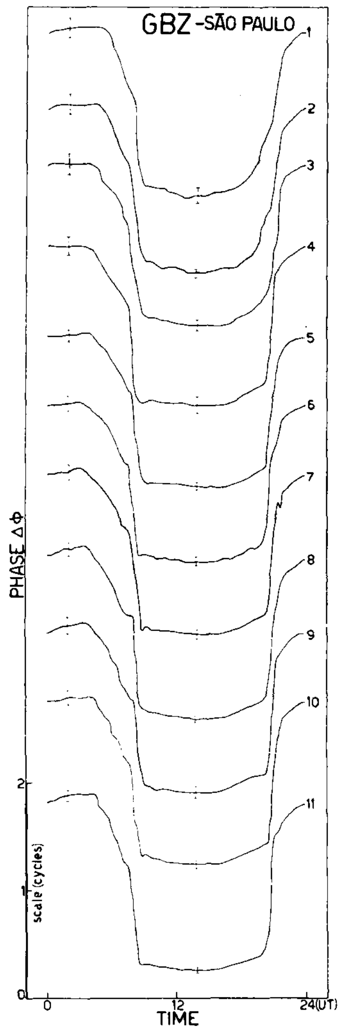

Fig. 2. Phase curves averaged for each of the periods selected (see Table 1), for GBZ–São Paulo transmissions. These data were corrected for a phase slipping effect occurring during the sunrise transition. Bars corresponding to standard deviation ranges are also indicated in the curves.

the eleven periods studied, are presented in Figs. 2 and 3. The average ΔH found for the complete series of observations is near 8.4 km, and it can be noticed that over shorter periods important departures from this value can occur, and this may be even more pronounced when day-to-day analysis is performed. On the other hand, it is very significant to observe that the solution for ΔH in (10) gives apparent height difference near 8 km (with $m(\lambda_1/4\pi)D_1(\alpha_2) = 23$ km, and $p(\lambda_2/4\pi)D_2(\alpha_2) = 15$ km), for a lower ionosphere model proposed by Wait & Spies (1964) for which the conductivity gradients are 0.5 km^{-1} during nighttime and 0.3 km^{-1} during daytime—presenting close agreement with the experimental average results, when the correction in phase due to the presence of the second order mode during nighttime is considered.

From the numerical values at (8) and (9), a physical meaning can be suggested for the obtained results. The effective equivalent diurnal height variation for the upper boundary of the earth-ionosphere waveguide would be near 10 km, being approximately equal for both paths, and the apparent additional height variation (i.e., of the order of 15 km for NBA and 23 km for GBZ) are due to the variation of arg R from nighttime to daytime conditions, as a consequence from the diurnal variation of the ionospheric conductivity gradient.

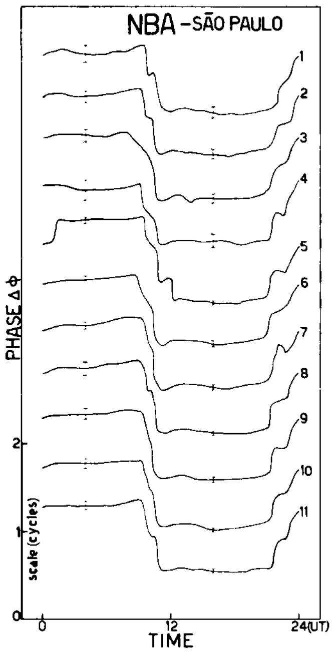

Fig. 3. Phase curves averaged for each of the periods selected (see Table 1), for the NBA–São Paulo transmissions. Vertical bars indicate the standard deviation ranges during nighttime and during daytime, where the signal is more stable. Some of the curves show clear phase fadings during the onset transitions.

3. Concluding remarks

From these results it can be concluded that the "average ionosphere" characteristics proposed by Wait & Spies (1964) is well suited for data averaged during long periods. A single waveguide configuration for both propagation paths may be assumed, with an upper boundary characterized by a conductivity gradient of 0.5 km^{-1} during nighttime and 0.3 km^{-1} during daytime. Good agreement with theory can be then obtained, and the results are quite independent from the choice of the reference height h in (1). The consideration of the second order mode of propagation during nighttime, even on long paths, constitute an important factor for the adequate study of the problem.

It was also shown that this treatment may be applied successfully only for VLF phase data averaged on long time. The discrepancies found for daily measures, or averages from short periods, emphasize the need of exact knowledge on the peculiar waveguide configuration that can possibly be different for each propagation path, since at each season of the year part of the paths receives different amounts of solar ionizing radiation, and, anytime, excess in solar activity can introduce additional "deformation" of the lower ionosphere shape, affecting in particular the conductivity gradient values.

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ЗАМЕЧАНИЕ О СУТОЧНЫХ ВАРИАЦИЯХ ЭКВИВАЛЕНТНОЙ ВЫСОТЫ НИЖНЕЙ ИОНОСФЕРЫ

С целью определения суточных вариаций наблюдаемой и эквивалентной верхней границы волновода земля-ионосфера были рассмотрены пятимесячные измерения фазы очень низкочастотных волн на трассах Бальбоа (Панама) – Сан Паулу (Бразилия) и Криджион (Великобритания) – Сан Паулу. Результаты показывают, что объяснение зависимости вариаций наблюдаемой высоты от длины волны и от трассы может быть получено, если для ночи рассматривать распространяющуюся волну второго порядка и принять во внимание суточные изменения градиента проводимости в поносфере. Результаты хорошо подходят к «осредненной» модели ионосферы, в которой градиент проводимости ночью равен 0,5 км$^{-1}$, а днем — 0,3 км$^{-1}$. В результате найдено, что суточные вариации эквивалентной высоты составляют приблизительно 10 км при условии, что справедливо предположение о существовании лишь одного волновода на обеих трассах.