On the transient part of the atmospheric planetary waves[1]

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ABSTRACT

The structure and motion of the atmospheric planetary waves were studied using a simple time filter and the method of characteristic patterns. Three universal pressure modes are found, independent of the zonal and meridional wavenumbers and of the speed of the wave: modes 1 and 3 have a large vertical mean value, and move with a speed close to the Rossby-Haurwitz speed of the dominant meridional characteristic pattern. The strongly divergent vertical mode 2 has a mean velocity close to the solid rotation of the atmosphere in the longest waves, and close to the Rossby-Haurwitz velocity in the short waves.

Four meridional characteristic patterns are universal for the fastest moving waves, but individual meridional structures are found for modes forced by the seasonal cycle and by the interactions of the fast moving modes.

Typical periods for the exchange of energy between modes are much shorter than the typical periods of fluctuations in the observed phase velocities.

1. Introduction

The transient part of the planetary waves has been studied observationally in surface harmonics at one or two levels by Eliasen & Machenauer (1965) and Deland & Lin (1967). Theoretical studies and spectral forecast models have been conducted by Blinova (1953, 1956, 1957, 1961, 1964, 1965, 1966), Mashkovich (1957, 1960, 1961, 1963, 1964*b*), Eliasen & Machenauer (1965) and Deland & Lin (1967). Mashkovich (1961, 1964*b*) predicted the existence of slow moving modes forced by the interactions of the fast moving modes, which were observed but not recognized by Eliasen & Machenauer (1965). All attempts to use a barotropic divergence model of the form proposed by Cressman (1958), but with an empirical coefficient, have been unsuccessful, which is now shown to be due to the fact that the strongly divergent vertical mode 2 is statistically uncorrelated with the weakly divergent dominant mode 1.

When the vertical structure is taken into account, the dependence of phase angle and wave velocity on meridional wavenumber observed by Eliasen & Machenauer disappears, due to the normal dominance of one meridional mode. The vacillation is also much reduced, and the remainder is attributable to the excitation of sub-dominant meridional modes, to imperfections of the time filter, and to contingent errors (aliases) of the objective analysis system.

2. Characteristic patterns

The formalism of characteristic patterns is described by Lorenz (1956), Obukhov (1960), Mateer (1965) and others. Briefly, a statistical ensemble is formed of related observations over a period of time; the time mean removed; the covariance matrix formed; and the eigenvalues and eigenvectors of the covariance matrix are respectively proportional to the fractional variance explained and to the amplitude of each characteristic pattern.

For example, the statistical ensemble may be formed of the geopotential at eight pressure

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levels at each observation for a month, to give a matrix

$$\{y_{ij}\}, \quad 1 \leqslant i \leqslant 8, \quad 1 \leqslant j \leqslant 62 \quad \text{(approx.)}. \quad (1)$$

The time mean is removed by making

$$\sum_{j=1}^{62} y_{ij} = 0 \quad (1 \leqslant i \leqslant 8). \quad (2)$$

The square covariance matrix

$$U = YY', \quad (3)$$

where Y' is the transpose of Y.

The eigenvalues and eigenvectors of U are proportional to the desired fractional variances and characteristic patterns.

3. Time filters

Two time filters with a fixed and a running average were used in order to separate the long period time average or standing part, the slow moving forced modes, and the fast moving modes of the planetary waves. As the basic data covered January, February and March 1963, a fixed three months average was used to approximate the standing part. With the standing part removed, a 5.5 day or 10 day running average was used to separate the forced modes, and the residue is the fast moving or free modes. The length of the running average must be suitable for the phase velocities of the free modes, which may be found from Eliasen & Machenauer (1965), from this paper, or from the results of preliminary computations.

Refined time filters will be needed to separate observationally those forced modes which Mashkovich (1964*b*), predicted would be generated by the interactions of the free modes from the seasonally forced modes which may be regarded as the seasonal cycle of the standing part. This study is concerned mainly with the free, fast moving modes.

4. Aliases

In order to estimate the contingent errors of the data, which are believed responsible for a substantial part of the irregularities which remain in the results of this study, a model experiment was performed on an idealized objective analysis system.

Given a knowledge of the vector W of amplitudes of every atmospheric wave, one could generate the vector of station soundings S by some non-square matrix A:

$$S = AW. \quad (4)$$

Proceeding through the linear algebraic procedures of the analysis system, and neglecting the constraint of the analysis by the preceding forecast and various minor difficulties, one can generate a vector Z which is an estimate of W, with some non-square matrix B:

$$Z = BS. \quad (5)$$

The error matrix

$$E = AB - I, \quad (6)$$

where I is the identity matrix.

A simple approximation to E was obtained by treating only one pressure level, and by using a field of RMS error of interpolation similar to that which Mashkovich (1964*a*), derived from the theory of optimum interpolation, after Gandin (1963). The fixed error field multiplied by a single surface harmonic Y^M_{N+M} defines an error field for Y^M_{N+M} which may be Fourier analyzed into a complete set of surface harmonics.

The explicit form of a surface harmonic is

$$Y^M_N = \exp(iM\lambda)\, P^M_{N+M}(\cos\theta) \quad (7)$$

where λ is the longitude, θ the colatitude, M the zonal wavenumber, $N+M$ the degree, N the meridional wavenumber, and P^M_{N+M} an associated Legendre polynomial. All the values used in this study were computed by the algorithms of Belousov (1956), because of his extensive study of the errors involved, and were checked against his listings.

The usual meaning of the term alias was further extended by defining a direct alias as the real part and an indirect alias as the imaginary part of the alias between any two surface harmonics (M, N) and (MP, NP); an auto-alias was defined as the alias between any surface harmonic and itself.

The hemispheric mean is the wave vector $(0, 0)$; the alias from $(0, 0)$ to $(0, 0)$ is 32%, to $(0, 1)$ 32%, to $(0, 2)$ 4%, to $(0, 4)$ -7%, and to $(0, 6)$ 25%. These figures indicate that the meridional profile of the zonal mean geopotential or zonal mean wind is among the least reliable information now available.

Separation of zonal or meridional wavenumbers tends to reduce the alias, which falls to the order of 1% for the direct alias between wave vectors near (0,6) and (6,0). The indirect aliases tend to be one or two orders of magnitude smaller than the corresponding direct aliases. The direct auto-aliases vary between about 15 and 50%.

The numbers obtained in this simplified experiment should be used only as estimates of error bounds, but not for the correction of data. No results in this paper are affected by random, systematic or contingent quantitative errors in the order of 30%; but detailed examination would be needed in any attempts to give statistics on the vacillation of the dominant modes, the phase velocities of sub-dominant meridional modes, the possible existence of zonal characteristic patterns, or many forms of correlation coefficient.

5. Motion of pressure modes

The data consisted of zonal harmonics 0 to 15 of the geopotential at 2.5° increments of latitude from 20° N to 87.5° N at eight pressure levels from January to June 1963. Most of the work used the January to March period; in particular, the standing part was evaluated only as a three month mean and subtracted to provide tapes of transient zonal harmonics. Derived data sets carried transient surface harmonics at eight pressures, transient surface harmonics resolved in vertical characteristic patterns, running means and deviations of vertical modes of transient surface harmonics, and vertical modes of transient zonal harmonics at 28 latitudes. It will generally be clear from what data set any particular conclusion is derived.

Cases were discarded with more than 2 adjacent pressure levels missing, or with more than 3 levels missing in all. Missing levels were generated by linear interpolation or extrapolation. Complete missing or discarded observations were ignored in evaluating running means and deviations; no significant change would have resulted from the use of interpolation.

The statistical ensembles used in the evaluation of characteristic patterns were either 62 observations or all available observations in a given calendar month.

It was found that three vertical pressure modes explain 98% or more of the variance of the geopotential of any statistical ensemble, which ensemble may be composed of transient zonal harmonics, transient surface harmonics, or the running means or deviations of either transient zonal or transient surface harmonics. These three modes are indistinguishable from those reported by Obukhov (1960), and Holmström (1963), from ensembles of station radiosonde soundings, and have a case-to-case variation similar to that shown by Holmström. Consequently, the modes given in Holmström's figure 6 were arbitrarily chosen as standard, orthonormalized over the eight pressure levels of this study. The fact that mode 2 is of comparable magnitudes but opposite signs in the lower and upper atmosphere indicates that it is relatively more divergent than the other modes. The percentage variances explained by each mode are usually about 70–90, 10–20, and 1–10%, except in February 1963 when mode 2 is more important, in agreement with the findings of Wiin-Nielsen & Drake (1966), on divergent and non-divergent energy conversions.

If the vertical mean is removed from the statistical ensemble, only two significant modes result, with percentage variances explained about 70–80% by the vertical mean, 10–20% by the new mode 1, and 1–10% by the new mode 2. The observed modes are similar to modes obtained by orthornormalizing the standard modes to a constant, but the observed modes show a small variation of the pressure level of the extremum with meridional wavenumber which reflects the variation of tropopause height with latitude.

When the running time mean and deviation of surface harmonics are expanded in vertical characteristic patterns, the zonal phase angles are seen to be a function of zonal wavenumber and vertical mode number but not normally of meridional wavenumber: the meridional wavenumber appears briefly when a sub-dominant meridional characteristic pattern (see below) is excited.

Vertical modes 1 and 3 of any wave normally have their phase angles close together, and move with a mean speed near the Rossby-Haurwitz value computed with a meridional wavenumber of 2, 3 or 4, whichever most closely corresponds to the expansion of the dominant standard meridional mode in the appropriate associated Legendre polynomials. The meridional wavenumber of largest amplitude may also be used in estimating the applicable Rossby-Haurwitz

Table 1. *Observed mean zonal phase velocities of free and forced modes separately, Jan.–March 1963, deg. phase and deg. longitude per day*

Zonal W. N.	1	2	3	4	5	6
Vert. mode (free modes)						
1. Phase	−32.5	−33.0	−9.0	18.0	43.9	45.8
Long.	−32.5	−16.5	−3.0	4.5	8.8	7.6
2. Phase	16.0	38.3[a]	36.0	45.0	54.8	50.6
Long.	16.0	19.1[a]	12.0	11.3	11.0	8.4
Vert. mode (forced modes)						
1. Phase	−7.0	−6.8	−1.5	−2.0	−2.5	5.0
Long.	−7.0	−3.4	−.5	−.5	−.5	.8
2. Phase	−7.0	2.3	5.5	1.5	1.5	4.5
Long.	−7.0	1.1	1.8	.4	.3	.8

[a] In periods when small amplitudes do not require interpolations, this value is 15.0 degrees longitude per day.

speed of vertical modes 1 and 3 of any zonal wavenumber.

Vertical mode 2 of zonal wavenumbers 1 and 2 has a mean speed close to the angular velocity of solid rotation of the atmosphere; with increasing zonal wavenumber the speed approaches the Rossby-Haurwitz velocity computed in the same way as for vertical modes 1 and 3.

Mean rotational speeds obtained by examination of plots are shown in Table 1. Rossby-Haurwitz speeds are shown in Table 2. When the amplitude of any wave fell below one decametre, the phase angle was carefully scrutinized because much noise appears at such times.

The mean phase speeds of the running mean or forced modes are very approximate and not necessarily significantly different from zero. They exhibit much vacillation, and this study made no attempt to separate the seasonally forced modes from the modes forced by the interactions of the free modes.

No meaningful statistics can be computed on the unsteadiness of the phase speed or the vacillation of either the free or forced modes unless allowances are made for all the following effects:

1. The solid rotation of the atmosphere.
2. Aliases.
3. Excitation of sub-dominant meridional modes.
4. Imperfections of the simple time filters of this study.

Expansions in sines and cosines of pressure were uninformative.

6. Meridional characteristic patterns

The free modes exhibit four universal meridional characteristic patterns, independent of zonal wavenumber and vertical mode, except that meridional mode 1 is absent from monthly or comparable ensembles of zonal wavenumber

Table 2. *Rossby-Haurwitz speeds in degrees longitude per day, with the index of solid rotation* $\bar{\alpha} = 15.58$ *degrees long. per day*

$$\omega_{R-H} = \bar{\alpha} - \frac{2(360+\bar{\alpha})}{N(N+1)},$$

where the degree N is the sum of the zonal and meridional wave numbers

Degree N ...	1	2	3	4	5	6	7	8	9	10	11	12
ω_{R-H}	−359.5	−109.5	−42.3	−21.9	−9.4	−2.3	2.2	5.2	7.2	8.8	9.9	10.8

Table 3. *Standard meridional modes: A coefficient, zonal wave number 7, vertical mode 1, March 1963*

Lat. ...	20.0	22.5	25.0	27.5	30.0	32.5	35.0	37.5	40.0	42.5	45.0	47.5	50.0	52.5
	55.0	57.5	60.0	62.5	65.0	67.5	70.0	72.5	75.0	77.5	80.0	82.5	85.0	87.5
K: 1	.1154	.1698	.2314	.2947	.3493	.3826	.3886	.3700	.3293	.2710	.2056	.1446	.0986	.0663
	.0401	.0172	.0013	−.0052	−.0044	−.0009	.0006	.0001	−.0005	−.0005	−.0001	.0001	−.0000	−.0000
2	−.1162	−.1550	−.1961	−.2285	−.2327	−.1983	−.1134	.0200	.1731	.3035	.3829	.4043	.3740	.3109
	.2387	.1659	.0980	.0446	.0106	−.0053	−.0093	−.0067	−.0037	−.0010	−.0002	−.0000	.0001	.0000
3	.2611	.3026	.3073	.2525	.1312	−.0208	−.1674	−.2645	−.2836	−.2188	−.0847	.0778	.2166	.2950
	.3095	.2817	.2315	.1801	.1341	.0920	.0520	.0232	.0077	.0022	.0001	.0000	−.0000	.0000
4	−.2567	−.2920	−.2631	−.1498	.0352	.2157	.3045	.2344	.0279	−.1847	−.2839	−.2234	−.0600	.1203
	.2567	.3159	.3036	.2520	.1894	.1256	.0680	.0263	.0046	−.0015	−.0014	−.0002	.0002	.0000
5	.3483	.2431	.0891	−.1100	−.2652	−.2600	−.0910	.1374	.2947	.2672	.0637	−.1778	−.3087	−.2609
	−.0677	.1648	.3046	.3065	.2147	.1074	.0321	−.0010	−.0063	−.0035	−.0006	−.0005	−.0002	.0000
6	.2704	.2120	.0499	−.1742	−.2946	−.1276	.1732	.2817	.1061	−.1497	−.2581	−.1636	.0514	.2253
	.2521	.1154	−.1086	−.2832	−.3481	−.3118	−.2235	−.1266	−.0520	−.0151	−.0030	−.0006	−.0002	−.0001

Table 4. *Expansion of standard meridional modes in associated Legendre polynomials*

	$M = 1$						$M = 2$						$M = 3$					
N ...	0	2	4	6	8	10	0	2	4	6	8	10	0	2	4	6	8	10
K: 1	.173	.092	−.160	.028	.065	−.044	.159	.154	−.128	−.025	.067	−.025	.143	.192	−.082	−.060	.055	−.009
2	.009	.128	.049	−.171	−.003	.110	−.011	.116	.131	−.129	−.079	.088	−.024	.089	.182	−.059	−.117	.048
3	.074	.040	.049	.063	−.163	−.021	.066	.035	.046	.134	−.111	−.100	.061	.029	.021	.175	−.031	−.139
4	−.017	.075	.065	−.020	.018	−.116	−.029	.052	.083	.009	.069	−.085	−.035	.030	.084	.015	.108	−.014
5	.031	.013	.035	.040	−.033	−.003	.028	.006	.025	.061	−.003	.035	.027	.002	.010	.064	.010	.073
6	.005	−.044	−.046	−.032	−.042	−.051	.013	−.027	−.041	−.035	−.079	−.002	.016	−.015	−.031	−.019	−.087	−.041

	$M = 4$						$M = 5$						$M = 6$					
N ...	0	2	4	6	8	10	0	2	4	6	8	10	0	2	4	6	8	10
K: 1	.128	.215	−.034	−.077	.038	.000	.113	.227	.011	−.080	.021	.005	.100	.230	.051	−.073	.007	.007
2	−.033	.056	.205	.014	−.121	.008	−.037	.025	.206	.080	−.101	−.022	−.039	−.002	.192	.131	−.067	−.039
3	.058	.027	−.011	.184	.053	−.138	.056	.029	−.042	.166	.123	−.109	.054	.035	−.067	.132	.173	−.065
4	−.038	.012	.079	.006	.120	.068	−.039	−.002	.074	−.009	.107	.139	−.039	−.013	.071	−.022	.074	.186
5	.026	.001	−.005	.057	.006	.091	.026	.003	−.016	.050	−.007	.080	.026	.006	−.023	.045	−.020	.046
6	.018	−.008	−.025	−.000	−.074	−.056	.018	−.003	−.023	.015	−.054	−.053	.019	.001	−.024	.024	−.035	−.042

Table 5. *Tilt of surface harmonics of the geopotential, less 3 month mean, January, February, March 1963*

Differences of east-west phase angle in radians of phase, 850–500 mb, 850–200 mb values

Meridional wavenumber...		0		2		4		6		8	
Zonal wavenumber...		850–500	850–200	850–500	850–200	850–500	850–200	850–500	850–200	850–500	850–200
1	Jan.	.20	.25	.03	−.16	.37	.69	.15	.25	.20	.33
	Feb.	.29	−.25	.11	.01	−.05	−.03	−.01	−.13	.39	.47
	Mar.	.39	.75	.05	.10	.27	.21	.17	.31	.25	.21
2	Jan.	.53	.29	.03	.06	.25	.23	.51	.57	.37	.51
	Feb.	.45	.60	.41	.48	.32	−.00	.35	−.08	.32	.25
	Mar.	.59	.84	.58	.93	.05	.03	.25	.39	.55	.65
3	Jan.	.46	.57	.20	.32	.28	.54	.43	.66	.32	.40
	Feb.	.41	.41	.69	1.05	.33	.33	.52	.60	.14	.53
	Mar.	.60	.56	.25	.25	.08	−.13	.35	.25	.29	.26
4	Jan.	.51	.64	.17	.32	.20	.27	.25	.39	.11	.08
	Feb.	.20	.28	.34	.38	.21	.29	.16	.21	.16	.11
	Mar.	.19	.40	.27	.52	.24	.31	−.09	.16	.35	.35
5	Jan.	.18	.23	.22	.64	.29	.44	.26	.38	.22	.18
	Feb.	.55	.86	.37	.54	.41	.63	.35	.74	.31	.45
	Mar.	.43	.56	.31	.38	.50	.47	.40	.39	.24	.08
6	Jan.	.36	.36	.27	.26	.49	.48	.17	.26	.44	−.22
	Feb.	.08	.02	.12	.01	.21	.38	.30	.35	.43	.42
	Mar.	.47	.76	.60	.60	.41	.55	.45	.44	.18	.33

0. The four modes have, respectively, 1 to 4 extrema of significant amplitude between 20° N and 70° N, are near zero from 70° N to the pole, and explain typically 60–70, 10–30, 5–10 and 1–5% of the variance, respectively. The case-to-case variation suggests that mode 4 is about half noise; there are also modes 5 and 6 which appear to be not entirely noise, with 5 and 6 extrema respectively.

The forced modes exhibit individual meridional characteristic patterns: better time filters and a separation of the seasonally forced modes are desirable for their detailed examination. Modes 5 and 6 of the free waves appear, at least partly, to be modes of the forced waves which remain because of imperfections of the time filters.

The modes of the A coefficient (cosine, east from the Greenwich Meridian), zonal wavenumber 7, vertical mode 1, March 1963, were arbitrarily adopted as standard because they appeared near the average. They are listed in Table 3 and their expansion as linear combinations of associated Legendre polynomials is given in Table 4. The residuals are too large to allow a transformation of data from surface harmonics to meridional modes, except for modes 1 and 2.

Tropical and inter-hemispheric effects which cannot be analyzed in this study may contribute to some of the modes. This may be judged from the ratio of the amplitude at 20° N to the extrema; the ratio is near 1 for modes 4, 5 and 6, about 0.8 for mode 3, and about 0.25 for modes 1 and 2.

7. Characteristic patterns of longitude

The existence of characteristic patterns of longitude would imply that the values of the A and B coefficients of zonal harmonics were correlated. Because the raw data of this study were already expressed in zonal harmonics, a procedure was adopted which would have given the coefficients of the expansion of zonal characteristic patterns in zonal harmonics. The 31 A and B coefficients of zonal wavenumbers 0 to 15 at any one latitude and any one pressure level for any one month were used to form a covariance matrix in the usual manner.

Table 6. *Tilt of surface harmonics of the geopotential: 5.5 day mean and deviation, less 3 month mean, Jan.–March 1963*

Differences of east-west phase angle in radians of phase modulo 2π; 850 mb–500 mb, 850 mb–200 mb values

Meridional wavenumber...		0		2		4		6		8	
Zonal wavenumber...		850–500	850–200	850–500	850–200	850–500	850–200	850–500	850–200	850–500	850–200
1	Free	1.43	1.37	1.41	1.22	1.32	1.19	1.05	1.06	1.03	1.12
	Forced	0.96	1.31	1.25	1.25	0.76	0.92	1.64	1.44	0.78	0.91
2	Free	1.43	1.22	1.12	1.05	1.10	1.29	0.89	1.00	1.16	1.04
	Forced	0.57	0.77	0.84	0.70	1.13	1.07	0.43	0.62	0.76	0.77
3	Free	0.96	1.07	0.65	0.93	1.34	1.25	0.94	1.01	0.94	1.07
	Forced	0.25	0.77	0.28	0.17	1.10	0.99	0.68	0.76	1.41	1.48
4	Free	0.97	0.97	1.04	1.00	0.95	0.96	0.77	0.91	0.96	0.97
	Forced	0.68	0.78	0.54	0.66	0.86	0.79	1.58	1.44	1.19	1.27
5	Free	0.76	0.92	0.95	0.93	1.07	1.16	1.04	0.98	0.98	0.92
	Forced	1.23	0.93	0.68	0.90	0.94	0.98	1.27	1.39	1.23	1.49
6	Free	0.73	0.85	0.84	1.03	1.07	1.09	0.74	0.74	0.94	0.94
	Forced	1.21	1.18	0.97	1.01	1.23	1.17	1.13	0.96	1.21	1.50

The resultant characteristic patterns were so unstable from one latitude circle or month to another as to be entirely attributable to the forced modes and aliases.

Analyses of data resolved in vertical modes, meridional characteristic patterns, and time filtered showed that no stable characteristic patterns exist for the free modes. It is also characteristic of such cases that the eigenvalues decrease only slowly, and that the largest eigenvalue corresponds to only a small fraction of the variance, in the order of 20 to 50%.

For the forced modes, many months of data would be required for a significant result.

8. Tilt of surface harmonics expansions

The tilt of the surface harmonics of the geopotential was examined only by the mean differences of the 850 mb, 500 mb, and 200 mb phase angles (1000, 150 and 100 mb data being unavailable for January 1963), equally weighted over the availble observations without regard to amplitude. All differences were brought into the interval $(-\pi, \pi)$ before averaging.

Table 5 shows the monthly values in radians for January, February and March.

The considerable month to month changes have no evident systematic character.

Table 6 shows the three monthly values for the free and forced modes separately.

The tilts of the free and forced modes are mostly similar, to the west from 850 to 500 mb, and approximately vertical from 500 to 200 mb. The recurvature above 500 mb usually increases with increasing meridional wavenumber and decreasing zonal wavenumber: the variation with meridional wavenumber may merely reflect the variation of tropopause height with latitude. The 500 to 200 mb differences are consistent with the prediction of Mashkovich (1961, 1964*b*), of a tilt to the east in the upper troposphere and lower stratosphere.

No calculations were made in meridional characteristic patterns.

9. Comparison with other publications

The main observational studies of the transient part of the planetary waves are those of Eliasen & Machenauer (1965), and Deland & Lin (1967). Both these studies were mainly of the stream function at 500 mb, where the value of vertical mode 2 is near zero, although Eliasen & Machenauer also studied the 1000 mb field. No previous study has used three or more levels.

Eliasen & Machenauer also applied a simple time filter with an averaging period which they made a function of meridional wavenumber because of their finding that phase speeds depend on meridional wavenumber. Their observed phase velocities are generally similar to those found for vertical modes 1 and 3 in the present study; their finding that the phase angles at 500 mb and 1000 mb are usually little different was also observed in this study, but both this observation and occasional departures from it are due to the ratio of the percentage variances explained by vertical modes 1 and 2: cases were noted when the 200 mb and 850 mb phase velocities were large and of opposite signs.

The dependence of phase velocity on meridional wavenumber is due mainly to a time filtering technique which retained the forced modes: if Eliasen & Machenauer had worked at levels far from 500 mb, then vertical mode 2 would have been an additional complication, but no qualitative change would have been necessary in their conclusions. The use of cosine or other vertical modes also leads to a dependence of phase velocity on meridional wavenumber over short time periods.

Their other conclusions are consistent with everything observed in the present study, although their opinion of the nature of slow moving waves conflicts with the results of Mashkovich (1964*b*).

The main result of Deland & Lin (1967), on which the present study casts new light, and which is not comparable to any result of Eliasen & Machenauer (1965), is that surface harmonics are slightly correlated. The existence of characteristic patterns leads one to expect such correlations, though substantial calculations would be required for quantitative comparisons.

It must also be pointed out that many of the correlation coefficients reported by Deland & Lin are comparable in magnitude to the space aliases found in the model experiment of this report: because the simple model experiment says nothing about the sign of contingent errors, a further investigation is needed before anything more can be said.

Although no regression of one day's phase velocity on another has been made in this study, it appears likely from the available graphs that the separation of modes by time filtering and characteristic patterns would improve on the maximum correlation coefficient of 0.69 reported in Deland & Lin's Table 4.

The difficulty which Deland & Lin found in choosing a satisfactory empirical coefficient for the divergence is due to the facts that they did not use a time filter, and that the main part of the divergence is in vertical mode 2, which is by definition statistically uncorrelated with the dominant vertical mode 1. Operational analysis and forecast models also fail to separate the standing, forced and free modes.

Scale analyses of the planetary waves have been published by Burger (1958), Murakami (1963), and Deland (1965*b*).

Burger & Murakami show that in slow moving planetary waves the vorticity equation and the divergence equation both become diagnostic. Vertical mode 2 of the observed free modes may meet their assumptions; vertical mode 2 of zonal wavenumber 1 and 2 have a mean phase speed very close to the angular velocity of solid rotation of the atmosphere, although periods of considerable vacillation are observed.

The basic difference between the scale analysis of Deland (1965*b*), and those of Burger & Murakami is that Deland assumes wave speeds in the order of the Rossby-Haurwitz velocity. The horizontal advection of vorticity is not then negligible, and Deland concludes that there may exist modes for which the vorticity equation has prognostic value and which move with speeds in the order of the Rossby-Haurwitz values. This study has shown the existence of vertical modes 1 and 3 of the free waves, which meet Deland's assumptions. Zonal wavenumbers 5 and 6 of vertical mode 2 of the free waves also approach Deland's assumptions and conclusions.

Zonal wavenumbers 3 and 4 of vertical mode 2 of the free waves have properties intermediate between those assumed by Deland & Murakami.

It remains to be clarified whether any scale analysis is possible for the forced modes.

Unknown to the author, many of the results of this study had been predicted by Mashkovich (1957, 1960, 1961, 1963, 1964*b*), in a wide range of studies using grid point and spectral models, using instantaneous tendencies of surface harmonics in a time-invariant basic flow, and using low order models based on those of Lorenz (1962, 1963*a*, *b*, 1965).

Points 4 and 7 to 14 of the conclusions of Mashkovich (1961), are supported and point 1 appears to be contradicted by this study. In particular, the motion and development of disturbances depends on their vertical structure and the vertical structure normally changes rapidly with time.

The more rapid amplification of disturbances in the upper troposphere predicted by Mashkovich agrees with the strong vertical shears of the horizontal wind shown in the observed characteristic patterns.

The predicted *inclinations* of the trough and ridge lines to the west in the lower troposphere and to the east in the upper troposphere are observed for all waves expressed in surface harmonics.

Wave motions of unbounded amplitude are not observed in the atmosphere, nor in low order models (Mashkovich, 1963, 1964*b*), but only in models with a time invariant basic flow (Mashkovich, 1961), i.e. with an infinite source of energy for the disturbances. No explanation is offered for the apparent breakdown of Mashkovich's (1961), prediction 1 that the region of baroclinic instability is a function of the degree of a surface harmonic; the higher meridional wavenumbers were found to be of comparatively small amplitude in agreement with the observations of Eliasen & Machenauer (1965).

10. Conclusion

1. Any planetary wave or station sounding of the geopotential may be represented by three universal vertical characteristic patterns, or by a vertical mean and two characteristic patterns. Typical percentage variances explained by each mode are 70–90, 10–20 and 1–10%, with the total greater than 98%. Vertical modes 1 and 3 have non-zero vertical means, and their motions are very similar in all cases studied. Mode 2 is of opposite signs but comparable magnitudes in the upper and lower troposphere, with a vertical mean near zero, and is therefore thought to explain most of the horizontal divergence and vertical velocity of the wind. These three vertical modes are indistinguishable from those found by Obukhov (1960), and Holmström (1963).

The latitudinal structure of the free modes of all zonal wavenumbers may be represented by four universal characteristic patterns, with typical percentage variances explained 60–70, 10–30, 5–10 and 1–5%, with the total about 95%. Modes 1, 2, 3 and 4 have respectively 1, 2, 3 and 4 extrema of significant amplitude between 20° N and the North Pole, and are close to zero from 70° N to the pole. The fourth meridional mode is about equally signal and noise: it and higher modes have large values at 20° N, and are thus not correctly represented in the present study or the available data, and may be substantially influenced by inter-hemispheric exchanges.

2. The characteristic patterns of pressure, and the C.P.'s of latitude of fast moving waves are sufficiently universal that the conclusions of this report are not significantly influenced by their case-to-case variations, and sufficiently universal that diagnostic or prognostic spectral dynamic models may be formulated in terms of them.

3. Slower moving waves exhibit individual meridional characteristic patterns; the meridional C.P.'s of zonal wavenumbers greater than 6 in winter and greater than 3 in June approximate the universal functions, because the slow moving parts are small relative to the fast moving parts.

4. The divergent vertical mode 2 was relatively important in February 1963; no results are available for January 1963. This is consistent with the results of Wiin-Nielsen & Drake (1966).

5. The vertical C.P.'s obtained when the vertical mean is removed are approximately those found by orthonormalizing the usual vertical modes to a constant. Changes in the pressure levels of the extrema with meridional wavenumber correspond to the variation of tropopause height with latitude.

6. No zonal characteristic patterns were identified by this study.

7. The zonal phase angle and speed of any given wave are usually approximately independent of meridional wavenumber for both fast and slow moving waves. This observation, and occasional departures from it, are explained by the usual dominance of one meridional characteristic pattern.

8. The transient planetary waves (less the 3 month mean) exhibit rotational periods about 21 to 28 days.

9. Vertical modes 1 and 3 of any given zonal wavenumber are normally close together (modulo π) in zonal phase angle, for both fast and slow moving waves.

10. Vertical mode 2 of any given zonal wavenumber moves predominantly eastward unless a time filter is applied: modes 1 and 3 require more detailed descriptions.

11. Vertical modes 1 and 3 of zonal wave-

numbers 1 through 5 move predominantly west with much vacillation, unless a time filter is applied: vertical modes 1 and 3 of zonal wavenumber 6 move predominantly rapidly eastward.

12. The vertical mean wave has the same characteristics as vertical modes 1 and 3, but the meridional wavenumbers are less closely coupled. Expansions in vertical cosine modes are uninformative, and expansions have not been made in characteristic patterns with the vertical mean removed.

13. The 5.5 day mean of the waves show no mean motion significantly different from zero relative to the earth.

14. The deviation from a 5.5 day mean has characteristics that all vertical modes of zonal wavenumbers greater than 3 move east. For zonal wavenumbers 1, 2 and 3, vertical modes 1 and 3 move west with much vacillation, and vertical mode 2 moves east with much vacillation.

These results are independent of meridional wavenumber except rarely when a sub-dominant meridional mode is excited.

15. The zonal phase velocities of the transient, 5.5 day average, and fast-moving modes are all independent of the amplitude of the corresponding wave; large energy exchanges occur on a time scale short compared to changes in the zonal phase velocity.

16. The vertical modes are closely coupled by the vertical shear of the quasi-horizontal wind; the typical times for 100 % energy changes by this mechanism are comparable to the periods of the free modes.

17. Vertical modes 1 and 3 have phase speeds close to non-divergent Rossby-Haurwitz values assuming a meridional wavenumber of 2, 3, or 4 which is an approximation to the dominant meridional mode.

18. The divergent modes have zonal phase speeds absolutely smaller than the relevant Rossby-Haurwitz values, and independent of meridional wavenumber.

19. The predominant zonal phase relation of the several meridional wavenumbers of any given wave indicates a NE–SW tilt of the ridge or trough lines: this result is explained by the fact that the dominant meridional mode has a maximum in middle latitudes.

20. The vertical structure of surface harmonic expansions was investigated only by differences of the 850, 500 and 200 mb zonal phase angles. The tilt of all waves is westward from 850 to 500 mb, and vertical or slightly eastward from 500 to 200 mb, independently of time averaging period.

21. Little attention was given to the time spectrum of energy in this study, but the broad indications are of bands around 5–10 days (free modes) and 28 days (forced modes). These groups of modes have different meridional structures; the clearest results are obtained if the expansions are performed in the order zonal, pressure or time, and lastly meridional.

22. No theory known to the author adequately describes the observations; the treatment of Murakami (1963, p. 136) is consistent with the observations for the 5.5 day mean of vertical modes 1 and 3 or of the vertical mean of zonal wavenumbers less than 6. The scale analysis of Deland (1965*b*), applies to the weakly divergent free modes of all wavenumbers. Conclusions 4 and 7 through 14 of Mashkovich (1961), are confirmed. His conclusion 1 appears to be contradicted, and no light is cast on his other points.

23. The observations of previous authors of the apparent variation of the zonal phase speed of zonal harmonics with amplitude (Eliasen, 1958), and the vacillation and dependence on meridional wavenumber of the zonal phase speed of surface harmonics (Eliasen & Machenauer, 1965; Deland & Lin, 1967), appear to be due to the omission of the vertical structure and the non-separation of free and forced modes in previous studies.

24. The main errors of analysis are contingent, i.e. a function of the state of the atmosphere. They cannot, therefore, be eliminated by any purely statistical technique. In the author's opinion, a carefully designed study of the actual analysis system would enable greater value to be extracted from the available data. Even if critical points did not exist, the amplitudes of waves often fall within the quasi-random noise level.

11. Suggestions for future work

Direct extensions of the present work are by the study of data in the tropics, Southern Hemisphere, and above 100 mb. Satellite observations of the vertical and horizontal structure of the atmosphere may conveniently employ characteristic patterns as an efficient noise filter.

Existing observational studies of the transports and conversions of available potential energy, kinetic energy, momentum, angular momentum and matter (air, water, ozone, dust) may be formulated in characteristic patterns. In particular, the causes and roles of the characteristic patterns are not understood.

The budgets of all the above parameters may be evaluated for the standing, forced and free modes of the atmosphere: a few illustrative references are Oort (1964), Wiin-Nielsen & Drake (1966), Gavrilin (1965), Holopainen (1966).

The forced modes need further description but little can be added to the description of the free modes without a detailed knowledge of the contingent errors of the analysis system.

Characteristic patterns may be applied to scale analyses (Murakami, 1963; Deland, 1965*b*), to prognostic models (Mashkovich, 1964*b*; Baer, 1964), and non-linear diagnostic calculations. They are more economical than grid points, surface harmonics or solid harmonics for diagnostic calculations, and are more economical for prognostic models provided the boundary conditions can be formulated in characteristic patterns, e.g. friction, topography and diabatic effects.

The role and mechanism of geostrophic ballance are questions which can probably be treated more economically and meaningfully in characteristic patterns than in any other formulation on a spherical earth without friction, topography or diabatic effects.

REFERENCES

Baer, F., 1964, Integration with the spectral vorticity equation. *Journal of the Atmospheric Sciences, 21*, No. 3, 260–276.

Belousov, S. L., 1956, Tablitsi normirovanikh prisoedinennikh polinomov Lezhandra. Akademiya Nauk SSSR. Reprinted as: *Normalized Associated Legendre Polynomials*, Mathematical Table Series, vol. 18, Pergamon, 1962.

Blinova, E. N., 1953, K voprosu ob opredelinii davleniya na urovne morya. *Dokladi Akademii Nauk SSSR, 92*, No. 3, 557–560. (On the question of determining the sea level pressure.)

Blinova, E. N., 1956, Metod resheniya nelineinoi zadachi ob atmosfernikh dvizheniyakh planetarnogo masshtaba. *Dokladi Akademii Nauk SSSR, 110*, No. 6, 975–977. (A method of solving the non-linear problem of atmospheric motions of planetary scale.)

Blinova, E. N., 1957, Hydrodynamic methods of short and long range forecasting. *Tellus, 9*, No. 4.

Blinova, 1961, Gidrodinamicheskoe issledovanie nestatsionarnikh atmosfernikh protsessov planetarnogo masshtaba. *Dokladi Akademii Nauk SSSR, 140*, No. 2, 354–357. (A hydrodynamic study of non-stationary atmospheric processes of planetary scale.)

Blinova, E. N., 1964, *Long Range Hydrodynamic Weather Forecasting*. Israel Program for Scientific Translations, 1965, 123 pp.

Blinova, E. N., 1965, Dinamika atmosfernikh dvizhenii planetarnogo masshtaba i gidrodinamicheskoi dolgosrochnii prognoz pogodi. *Trudi Mirovogo Meteorologicheskogo Tsentra*, Vipusk *5*, 84 pp. (The dynamics of atmospheric planetary scale motions and hydrodynamic long period weather forecasting.)

Blinova, E. N., 1966, Ob obespechenii danimi nablyudenii operativnoi i issledovatelskoi raboti po gidrodinamicheskomu dolgoschrochnomu prognozu pogodi. *Trudi Mirovgo Meteorologicheskogo Tsentra*, Vipusk *12*, 3–20. (On the provision of observational data for operational and research work on hydrodynamic long period weather forecasting.)

Burger, A. P., 1958, Scale considerations of planetary motions of the atmosphere. *Tellus, 11*, 195–205.

Cressman, G. P., 1958, Barotropic divergence and very long Atmospheric Waves. *Monthly Weather Review, 86*, 293–297.

Deland, R. J., 1965, On the scale analysis of traveling planetary waves. *Tellus, 17*, No. 4, 527–528.

Deland, R. J. & Yeong-jer Lin, 1967, On the movement and prediction of travelling planetary-scale waves. *Monthly Weather Review, 95*, No. 1, 21–31.

Eliasen, E., 1958, A study of the long atmospheric waves on the basis of zonal harmonic analysis. *Tellus, 10*, 206–215.

Eliasen, E. & Machenauer, B., 1965, A study of the fluctuations of the atmospheric planetary patterns represented by spherical harmonics. *Tellus, 17*, No. 2, 220–238.

Gandin, L. S., 1963, Obektivnii analiz meteorologicheskikh polei. Gidrometeoizdat. Translated as: *Objective Analysis of Meteorological fields*. Israel Program for Scientific Translations, 1965: Oldbourne Press, London, 1965.

Gavrilin, B. L., 1965, On the description of the vertical structure of synoptic processes. *Izvestiya Akademii Nauk SSSR, Seriya Atm. i Okean. Fiziki, No. 1*, 8–17.

Holmström, I., 1963, On a method for parametric representation of the state of the atmosphere. *Tellus, 15*, No 2, 127–149.

Holopainen, E. O., 1966, *A Diagnostic Study of the Maintenance of Stationary Disturbances in the Atmosphere*. University of Michigan, Office of Research Administration, Report 06372-3-T.
Lorenz, E. N., 1956, *Empirical Orthogonal Functions and Statistical Weather Prediction*. Sci. Rep. No. 1, Statistical Forecasting Project, Dept. of Met., Mass. Inst. Tech.
Lorenz, E. N., 1962, The statistical prediction of solutions of dynamic equations. *Proceedings of the International Symposium on Numerical Weather Prediction in Tokyo*, November 7–13, 1960.
Lorenz, E. N., 1963*a*, The predictability of hydrodynamic flow. *Trans. New York Academy of Sciences*, Ser. II, vol. 25, No. 4, 409–432.
Lorenz, E. N., 1963*b*, The mechanics of vacillation. *Journal of the Atmospheric Sciences*, *20*, No. 5, 448–464.
Lorenz, E. N., 1965, A study of the predictability of a 28-variable atmospheric model. *Tellus*, *17*, No. 3, 321–333.
Mashkovich, S. A., 1957, K uchetu nelineinikh chlenov uravnenii pri reshenii zadachi o prognoze davleniya i temperaturi maloi zablagovremennosti. *Trudi Tsentralnogo Instituta Prognozov*, Vipusk *60*. (On the calculation of non-linear terms in the equations in short period pressure and temperature forecasts.)
Mashkovich, S. A. & Kheifets, Ya. M., 1960, K teorii dolgosrochnogo prognoza s uchetom vertikalnoi stratifikatsii atmosferi i turbulentnogo peremeshivaniya. *Trudi Tsentralnogo Instituta Prognozov*, Vipusk *93*, 3–15. (On the theory of long range forecasting, taking account of the vertical stratification of the atmosphere and turbulent exchange.)
Mashkovich, S. A., 1961, K teorii voln davleniya v baroklinoi atmosfere. *Trudi Tsentralnogo Instituta Prognozov*, Vipusk *111*, 13–28. (On the theory of pressure waves in a baroclinic atmosphere.)
Mashkovich, S. A., 1963, O svyazi kolebanii zonalnoi tsirkulyatsii atmosferi s razvitiem dlinnikh voln. *Sbornik "Obyektivnii analiz i prognoz Meteorologicheskikh elementov"*. Izdatelstvo Akademii Nauk SSSR. (On the connection of fluctuations of the zonal atmospheric circulation with the development of long waves. Collection "Objective analysis and prognosis of meteorological elements".)
Mashkovich, S. A., 1964*a*, Obektivnii analiz aerologicheskikh nablyudenii i trebovanie k razmeshcheniya seti stantsii. *Izvestiya Akademii Nauk SSSR, Seriya Geofizicheskaya*, No. 2, 285–292. (Objective analysis of aerological observations and the need for the placement of a station network.)
Mashkovich, S. A., 1964*b*, O vzaimodeistvii zonalnoi tzirkulyatsii i nalozhennikh na nee nezonalnikh vozmushchenii. *Izvestiya Akademii Nauk SSSR*, No. 6, 930–946. (On the interaction of the zonal circulation and superposed non-zonal disturbances.)
Mateer, C. L., 1965, On the information content of umkehr observations. *Journal of the Atmospheric Sciences*, *22*, No. 4, 370–381.
Murakami, T., 1963, Analysis of various large scale disturbances and the associated zonal mean motions in the atmosphere. *Geofisica Pura e Applicata*, *54*, 119–165.
Obukhov, A. M., 1960, O statisticheski ortogonalnikh razlozheniyakh empiricheskikh funktsii. *Izvestiya Akademii Nauk SSSR, Seriya Geofizicheskaya*, No. 3, 432–439. (On statistically orthogonal expansions in empirical functions.)
Oort, A. H., 1964, On estimates of the atmospheric energy cycle. *Monthly Weather Review*, *92*, No. 11, 483–493.
Wiin-Nielsen, A. & Drake M., 1966, The contribution of divergent wind components to the energy exchange between the baroclinic and barotropic components. *Monthly Weather Review*, *94*, No. 1, 1–8.

О НЕСТАЦИОНАРНОЙ ЧАСТИ АТМОСФЕРНЫХ ПЛАНЕТАРНЫХ ВОЛН

Изучались структура и движение атмосферных планетарных волн с использованием простого временного фильтра и метода характерных потоков. Найдены три универсальных моды давления, независящих от зонального и меридионального волновых чисел и скорости волны: моды 1 и 3 имеют большое вертикальное среднее значение и движутся со скоростью, близкой и скорости волн Россби–Гаурвица для доминирующей меридиональной формы возмущения. Сильно дивергентная вертикальная мода 2 имеет среднюю скорость, близкую к скорости твердого вращения атмосферы в случае самых длинных волн, и близкую к скорости волн Россби–Гаурвица для коротких волн.

Четыре характерных меридиональных формы являются универсальными для самых быстрых волн, однако, индивидуальные меридиональные структуры имеют место для мод, возбуждаемых сезонным циклом и взаимодействием быстро движущихся мод.

Типичные периоды обмена энергией между модами намного меньше, чем типичные периоды флуктуаций в наблюдаемых фазовых скоростях.