# Probabilities of moving time averages of a meteorological variate

By IRVING I. GRINGORTEN, *Air Force Cambridge Research Laboratories, Bedford, Massachusetts*

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ABSTRACT

A meteorological element, such as the surface air temperature, averaged over a given time interval, can serve as a measure of the persistence or duration of the condition over that time interval. To study the probability distribution of moving time averages involving both common and extreme averages, it was found necessary to use a Monte Carlo simulation technique. A simple Markov process generating a sequence of values of a normally distributed variate, characterized by a constant hour-to-hour correlation, has worked fortuitously well as a model of duration on the selected examples, for the number of hours ranging from one to 768. Charts have been prepared to give the probability distribution of the highest $m$-hour average in $n$ hours, where $n$ is equal to the number of hours in one day, 8 days or 32 days, and $m$ is equal to 1, 3, 6, 12, 24, 48, 96, 192, 384 or 768 hours. The hour-to-hour correlation, in this study, ranges from zero to 0.999, with the usual value approximately 0.95.

## 1. Introduction

This study is the result of a search for a general model of the duration of weather events such as a rainy spell or a heat wave. Atmospheric processes suggest a Markov-type sequence of events. Cyclones, anti-cyclones, cold and warm fronts, that pass a station in cycles of two to ten days throughout the year, produce weather patterns that defy detailed analysis. But they lend support to the assumption that the events are part of a stochastic process that is stationary in time. A parameter, such as the hour-to-hour correlation of atmospheric surface temperature, can be estimated from regular hourly observations. The use of this single parameter, however, ignoring something as obvious as the diurnal cycle, is justified only by the practical results.

A simple Markov process, in which there is a constant correlation between successive regular observations of a variate, has proved applicable, making it highly desirable to examine the model purely as a statistical exercise. For the rest of this paper the key statistic is the $m$-period average of a normal variate, such as the $m$-hour average of temperature, $m$-day average of 24-hour rainfall, or the $m$-year average of annual precipitation. While there are other criteria of duration, such as a threshold value that is exceeded for a sustained period of time (Gringorten, 1966), the highest, or lowest, $m$-hour average serves us more effectively when the meteorological event is an integrated effect (Bennett, 1967). For example, a drought is recognizable by a deficiency of rainfall averaged over a period of several weeks, months or years.

## 2. The model

It is assumed that $y$ is a normal variate, observed hourly, with hourly serial correlation $\varrho$. Thus

$$E(y_i)=0, \quad E(y_i^2)=1 \quad (i \geqslant 0). \tag{1}$$

It has been shown (Feller, 1966) that, where $\eta_i$ is normally and randomly distributed,

$$y_0=\eta_0,$$
$$y_i=\varrho \cdot y_{i-1}+\sqrt{(1-\varrho^2)} \cdot \eta_i \quad (i \geqslant 1). \tag{2}$$

If $\varrho_{ij}$ denotes the correlation between the $i$th and $j$th hourly values of $y$, then

$$\varrho_{ij}=\varrho^{|i-j|}. \tag{3}$$

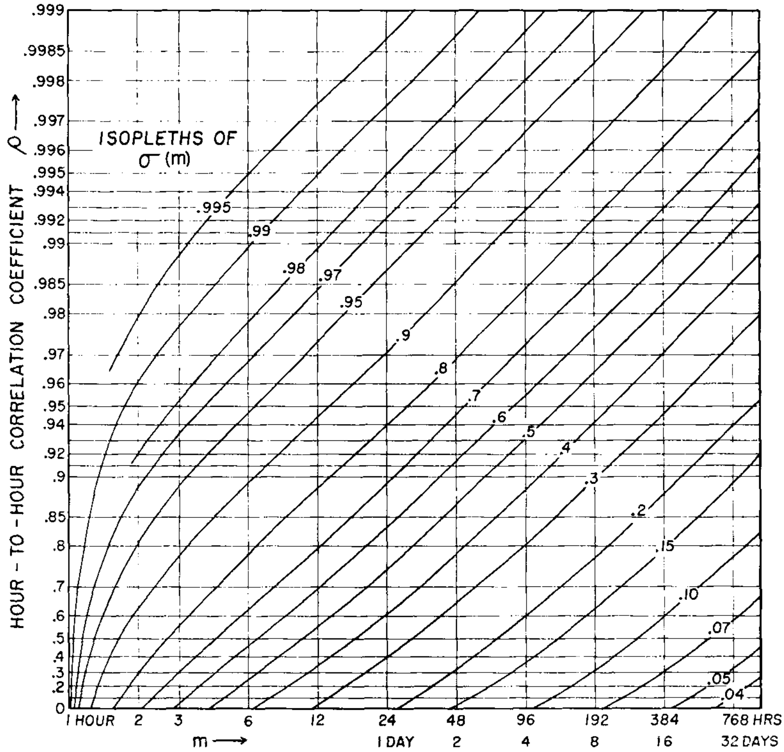



*Fig. 1.* Chart for determining the standard deviation $\sigma(m)$ of the $m$-hour average of the normal variate whose basic mean is zero and standard deviation 1.0, when there is an hour-to-hour correlation $\varrho$ in a Markov process.

Hence, it is not necessary to limit ourselves to hourly intervals. If observations are taken daily, then the day-to-day correlation would be $\varrho^{24}$.

The $m$-hour average $y(m)$ has an expected mean of zero, but its variance $\sigma^2(m)$ is given by (Godske, 1966; Brooks & Carruthers, 1953).

$$\sigma^2(m) = \frac{1}{m} + \frac{2\varrho}{m^2(1-\varrho)^2}[m(1-\varrho) - (1-\varrho^m)]. \qquad (4)$$

As a limiting case, when $\varrho = 0$ the variance becomes the familiar variance of the average of $m$ independent values. Fig. 1 has been prepared from a series of solutions of Equation (4) to give the standard deviation $\sigma(m)$ as a function of $m$ and $\varrho$. The probability that any $m$-hour average will not exceed a given value $y_{\mathrm{crit}}$ is given by

$$P(y(m) \leqslant y_{\mathrm{crit}}) = P(y \leqslant y_{\mathrm{crit}}/\sigma^{(m)}). \qquad (5)$$

This result is the simplest answer to the problem of duration. The number of hours ($m$) constitutes the period of duration, and the $m$-hour average is the statistic whose probability of being exceeded is the measure of risk to equipment or personnel.

But the practical questions related to duration require further study. An operation involving personnel or equipment usually is planned for a gross period of time, such as one day, one week or one month. It becomes important to determine if the average condition of, say, temperature, will exceed a threshold value for some fraction of the whole period. There is a need to know the probability distribution of the highest, or lowest, $m$-hour average in $n$ hours. For example, what is the probability that the highest 12-hour average temperature in the whole month of July will not exceed 90°F? Solution by direct mathematical analysis seems prohibitively involved. But a Monte Carlo technique, by computer, simulating the stochastic process by generating normal numbers

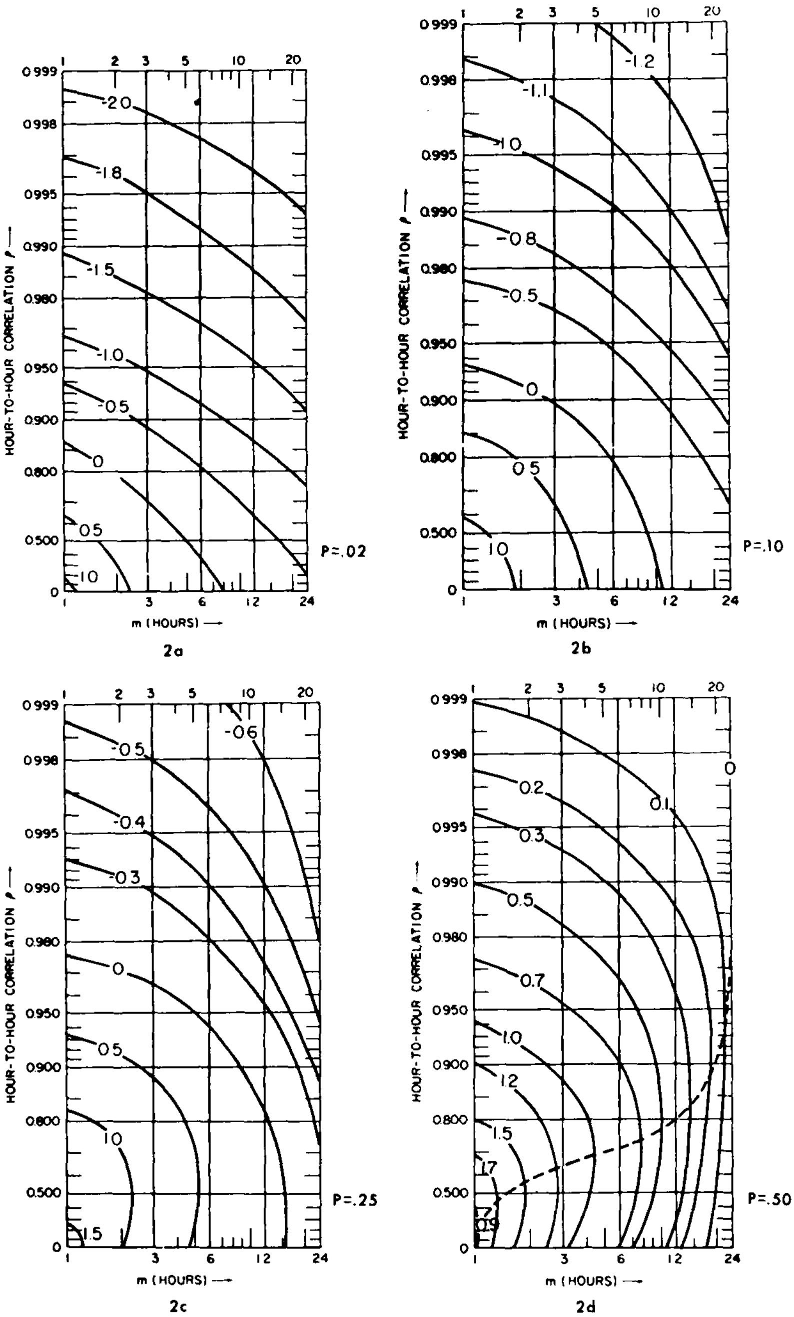
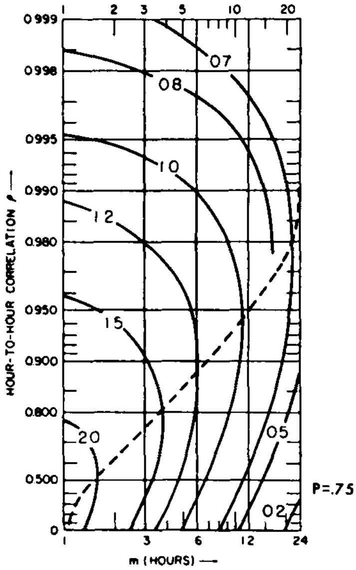



*Fig. 2.* The $P$-percentile of the highest $m$-hour average of $y$ in 24 hours, or the $(1-P)$-percentile of the lowest $m$-hour average of $-y$ in 24 hours. The basic variate $y$ is normally distributed with mean zero and variance 1.0.

$(y_i)$ with autocorrelation ($\varrho$) in a Markov Chain, provides the answers conveniently.

In a project with a 7090-computer, each "month" was composed of 32 days involving 768 hourly values of $y_i$ $(1 \leqslant i \leqslant 768)$. One hundred months yielded a total of 76,800 values of $y$. The value of $\varrho$ was made, in successive exercises,

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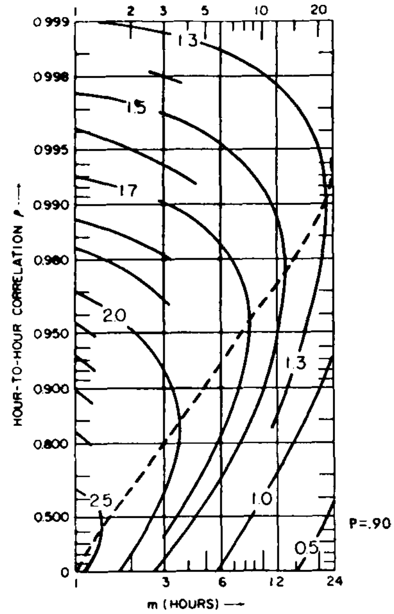


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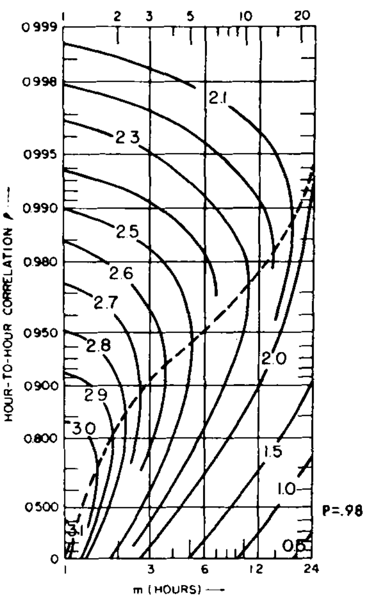


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$$\varrho = 0, .5, .8, .9, .95, .98, .99, .995, .998, .999.$$

In each month of 768 hours there were $(769 - m)$ $m$-hour averages, from which to choose the maximum. The highest $m$-hour average in $n$ hours ($n \leq 768$) was the highest of $(n - m + 1)$ successive $m$-hour averages, and the number of such highest $m$-hour averages per month, is equal to, or less than $(769 - m)/(n - m + 1)$.

The simulation exercise was performed for $m$ successively

$$m = 1, 3, 6, 12, 24, 48, 96, 192, 384, 768$$

and, to represent one day, 8 days approximating one week, and 32 days approximating one month, $n$ was made successively

$$n = 24, 192, 768.$$

The results of the simulation excercise were smoothed and presented in Figs. 2, 3 and 4.

Fig. 2 gives the $P$-percentile of the highest $m$-hour average of $y$ in 24 hours for

$$P = .02, .10, .25, .50, .75, .90, .98.$$

In each diagram the horizontal axis consists of $m$ from 1 to 24 hours. The vertical axis consists of the hour-to-hour correlation ($\varrho$) from zero to 0.999.

Fig. 3 gives the $P$-percentile of the highest $m$-hour average in 192 hours. Fig. 4 gives the

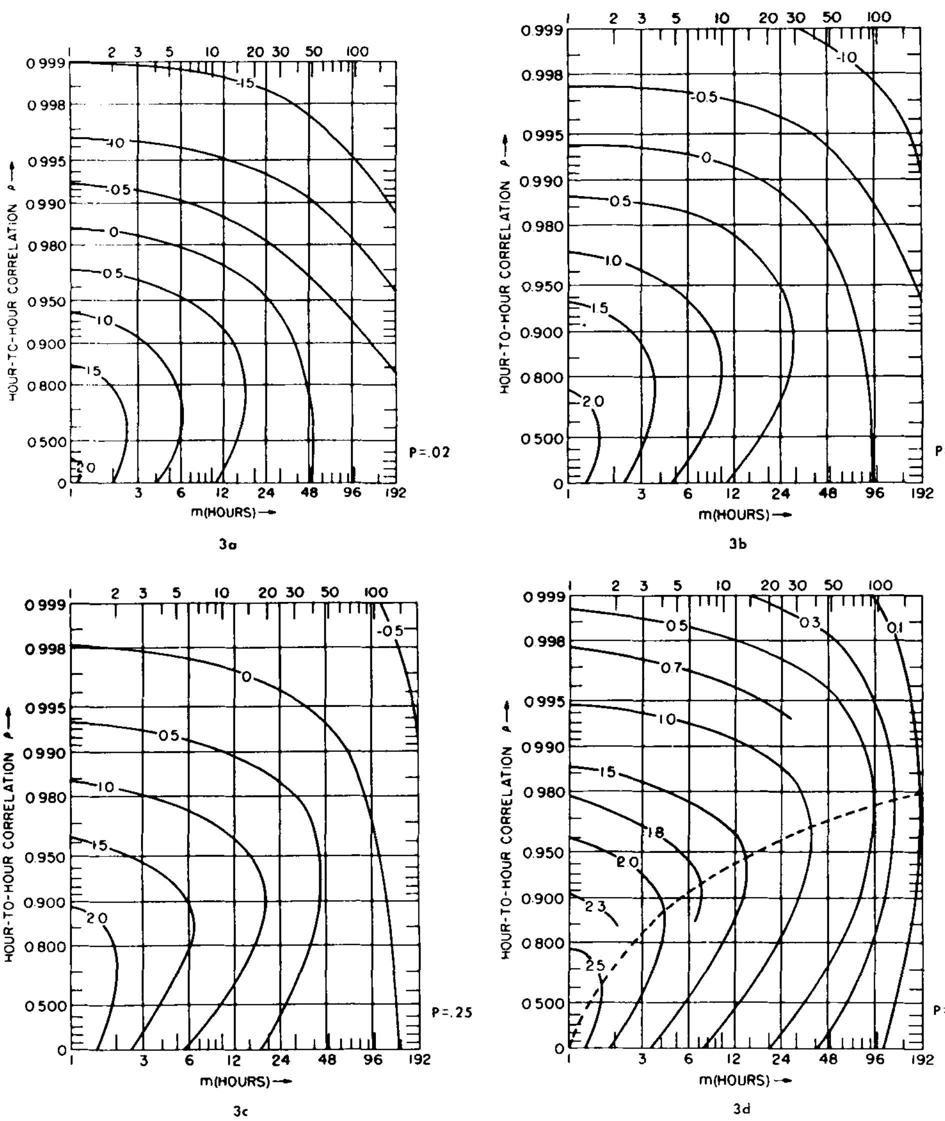
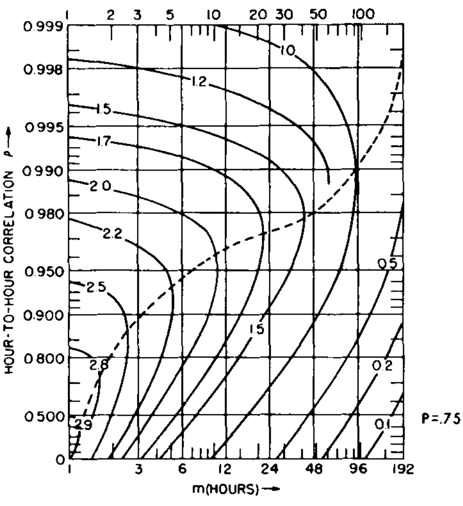



*Fig. 3.* The $P$-percentile of the highest $m$-hour average of $y$ in 192 hours, or the $(1-P)$-percentile of the lowest $m$-hour average of $-y$ in 192 hours.

$P$-percentile of the highest $m$-hour average in 768 hours.

Each of the figures (Figs. 2, 3 and 4) serves a dual purpose. The chart that gives the $P$-percentile of the highest $m$-hour average of $y$ in $n$ hours also gives the $(1-P)$-percentile of the lowest $m$-hour average of $-y$ in $n$ hours.

The broken lines in Figs. 2, 3 and 4 place a focus upon an interesting phenomenon. There is an optimum serial correlation that will maximize each percentile of the extreme $m$-hour average. For example, in Fig. 4*d*, for $\varrho = 0.93$, the median value of the highest 6-hour average in 32 days is read as $y = 2.4$. If the persistence of $y$ is greater, that is $\varrho > 0.93$ then, for any 6 hours within 32 days the $y$-value is not as likely to reach the extreme of 2.4 and the 6-hour average is not as likely to be as great

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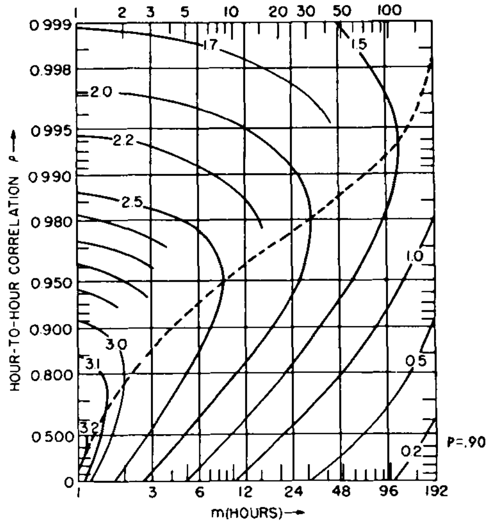


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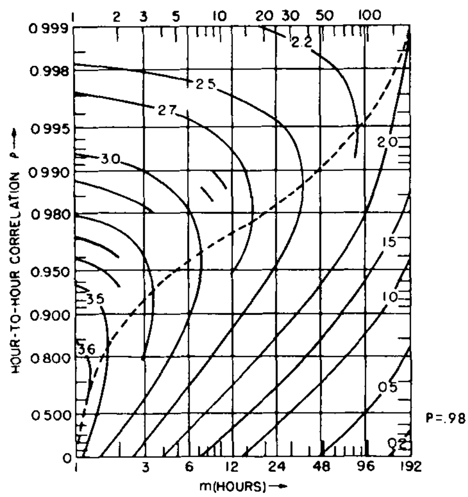


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as 2.4. On the other hand, if the persistence is not as great, that is $\varrho < 0.93$, then, while a single value of $y$ might more readily reach the value of 2.4, yet 6 consecutive values are not as likely to remain high enough to average 2.4.

Figs. 2, 3 and 4 are the essential product of the study and their presentation is the purpose of this paper.

## 3. Applications

Suppose $T$ is a normally distributed variable with mean $\bar{T}$ and standard deviation $s_T$. Then the estimate $\hat{T}$ can be given in terms of the corresponding estimate $\hat{y}$, as follows:

$$\hat{T} = \bar{T} + \hat{y} \cdot s_T. \tag{6}$$

That is, the estimate of $T$ is $\hat{y}$ standard deviations greater than its mean.

If $\hat{y}$ is the $P$-percentile of the $m$-hour average, then

$$\hat{y} = \sigma(m) \cdot y(P),$$

where $\sigma(m)$ is the standard deviation of the $m$-hour average (Fig. 1) and $y(P)$ is the $P$-percentile of the basically normally distributed variate. Hence the equation

$$\hat{T} = \bar{T} + s_T \cdot \sigma(m) \cdot y(P) \tag{7}$$

gives the estimate of the $P$-percentile of the $m$-hour average of $T$. The value of $y(P)$ is obtainable from almost any textbook in statistics that has a table of the normal distribution.

If $\hat{y}$ is the $P$-percentile of the highest $m$-hour average in $n$ hours, its value is to be found directly in Figs. 2, 3 and 4. The following examples are presented to illustrate the use of the charts (Figs. 1, 2, 3 and 4) to extend the climatological information on a variate that is approximately normally distributed.

*Example 1.* At Minneapolis, Minnesota a 10-year sample (1943–1952) yielded for July a

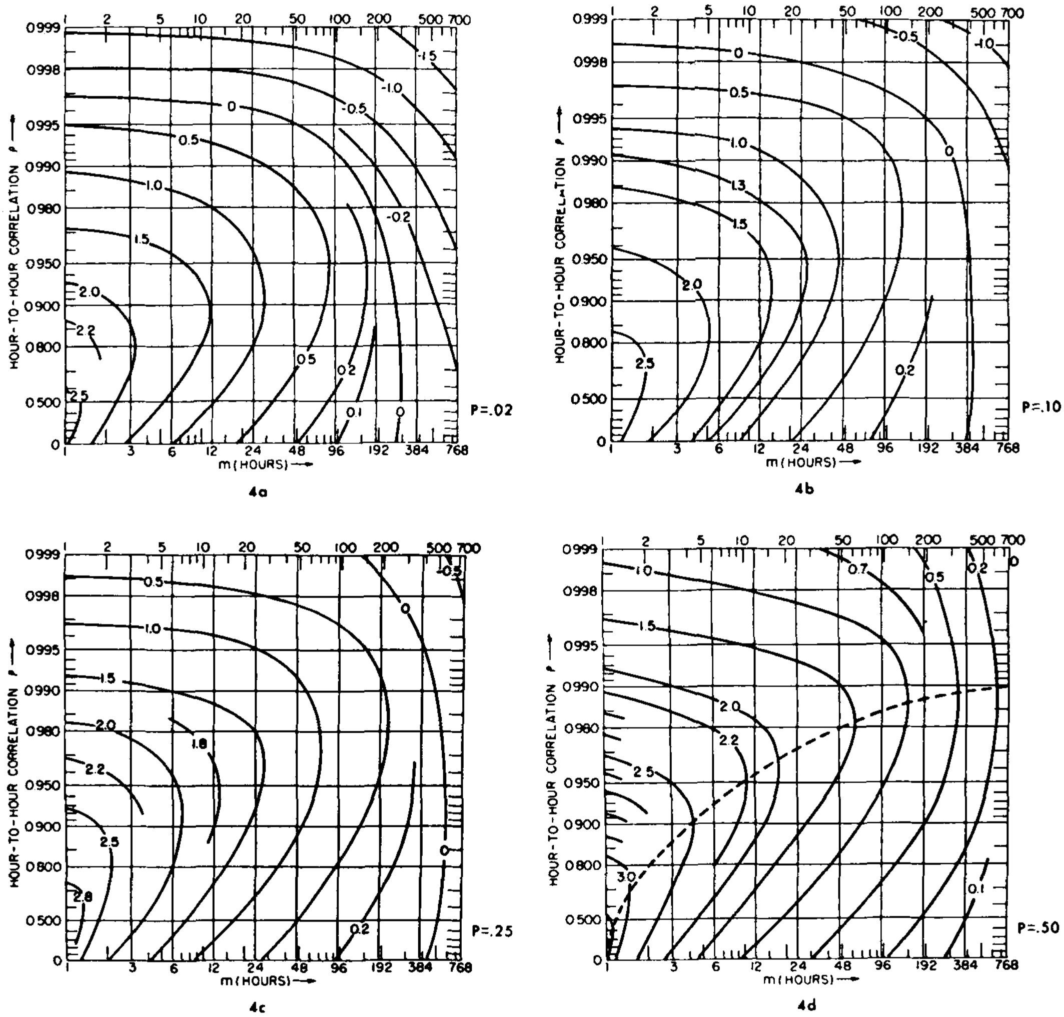
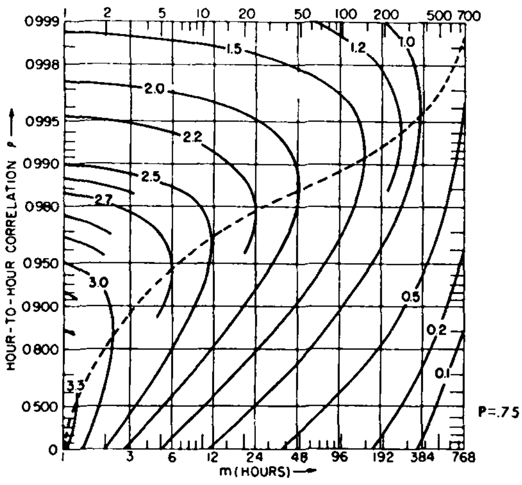



*Fig. 4.* The $P$-percentile of the highest $m$-hour average of $y$ in 768 hours, or the $(1-P)$-percentile of the lowest $m$-hour average of $-y$ in 768 hours.

mean temperature $\bar{T} = 72.9°F$, standard deviation $s_T = 8.9$ F°, and hour-to-hour correlation $\varrho = 0.928$. There is a significant diurnal amplitude of 8.4 F°, which inhibits the application of the model. Nevertheless, the results of applying Figs. 1, 2, 3 and 4 and Equation (6) and (7), as shown in Table 1, have a root mean square difference of only 2.0°F from the shaky estimates of direct sampling. The latter were not always possible to make, as indicated by empty spaces in Table 1.

*Example 2.* At Minneapolis, Minnesota a 10-year sample (1943–1952) of January temperature yielded a mean $\bar{T} = 13.6°F$, standard deviation 14.7°F, hour-to-hour correlation 0.982. But the diurnal amplitude was 4.3°F, which makes the application of the model more acceptable for estimation of duration of low temperature by time averages than for high temperatures in the previous example (Table 2). The root mean square difference of the model estimates from the shaky estimates by direct sampling, whenever the latter were possible, was 3.0°F. This value seems greater than it was for the July estimates, but relative to the standard deviation it is slightly smaller.

*Example 3.* At Birmingham, Alabama a 9-year sample (McGill University, 1960) revealed

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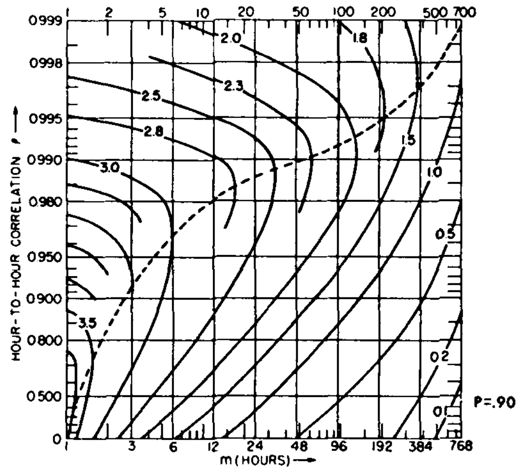


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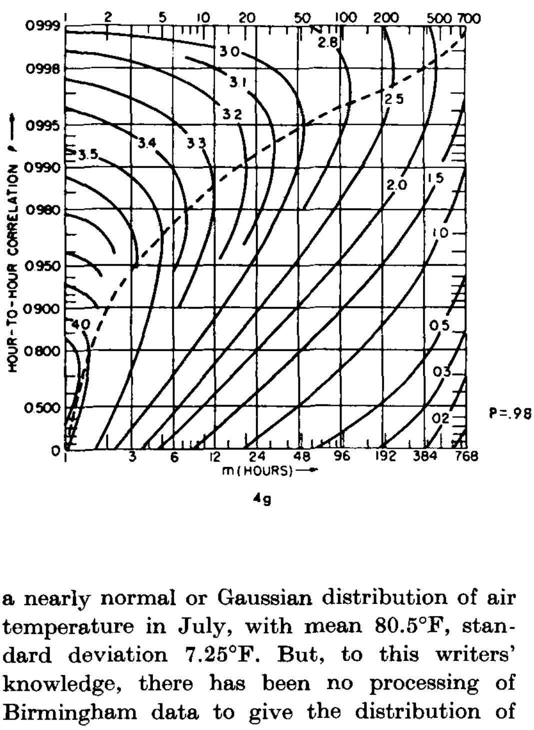


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a nearly normal or Gaussian distribution of air temperature in July, with mean 80.5°F, standard deviation 7.25°F. But, to this writers' knowledge, there has been no processing of Birmingham data to give the distribution of $m$-hour averages, except 24-hour and monthly averages. As this is being written the hour-to-hour correlation is not known, although it undoubtedly is greater than 0.9 and less than 0.99. In Figs. 2, 3 and 4 the broken lines join the upper limits of the highest $m$-hour averages of $y$ in 24, 192 and 768 hours. When these are transformed into values of temperature by the equation

$$T = 80.5 + 7.25\,y$$

the results are conservative estimates of duration, not likely to be exceeded (Table 3).

For comparison, the U.S. Weather Bureau records for Birmingham were consulted. They showed an extreme temperature of 107°F and an extreme monthly average of 90°F in 40 years, which corresponds to the estimate of $2\frac{1}{2}$% probability. The conclusion is that the estimates of Table 4 are high by no more than 3 F degrees.

*Example 4.* At Boston, Massachusetts a 37-year record (1930–1966) of annual precipitation yields a nearly normal or Gaussian distribution with mean 41.62 inches, standard deviation 8.88 inches. In 1966 the precipitation (36.01 inches) was 13% below the normal, in 1965 the precipitation (23.7 inches) was 43% below normal and the lowest of the 37-year record. In the four years 1963–1966 precipitation averaged 21% below normal, leading experts to believe that a drought was established and a secular trend was responsible.

Let us assume that one year's precipitation is independent of the precipitation in all previous years ($\varrho = 0$). Figs. 2 and 3 give the probability distribution of the lowest $m$-year average of $y$ in one quarter century (actually 24 years) and in two centuries (actually 192 years). By the equation

$$\hat{x} = 41.62 + 8.88\,y$$

Table 1. *Probability estimates of (a) the m-hour average of air temperature at Minneapolis, Minnesota, in July, (b) the highest m-hour average in n hours*

The 10-year mean is 72.7°F, standard deviation 8,9°F, diurnal amplitude 8.4°F, hour-to-hour correlation assumed to be 0.928. Estimates by model are in column A, by direct sampling in column B. When the 10-year sample was too small for a direct estimate, the figure in column B is missing

| | | Probability | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | .02 | | .10 | | .25 | | .50 | | .75 | | .90 | | .98 | |
| $n$ | $m$ | A | B | A | B | A | B | A | B | A | B | A | B | A | B |
| (*a*) *m*-hour average | | | | | | | | | | | | | | | |
| | 1 | 54 | 56 | 61 | 62 | 67 | 66 | 73 | 72 | 79 | 79 | 84 | 84 | 91 | 92 |
| | 3 | 55 | 55 | 62 | 62 | 67 | 67 | 73 | 72 | 78 | 79 | 84 | 84 | 90 | 92 |
| | 6 | 56 | 56 | 62 | 63 | 67 | 67 | 73 | 73 | 78 | 78 | 83 | 84 | 90 | 91 |
| | 12 | 57 | 57 | 63 | 63 | 68 | 68 | 73 | 73 | 78 | 78 | 83 | 83 | 89 | 89 |
| | 24 | 59 | 60 | 64 | 66 | 68 | 69 | 73 | 73 | 77 | 77 | 82 | 80 | 87 | 86 |
| | 48 | 61 | 62 | 66 | 66 | 69 | 70 | 73 | 73 | 76 | 76 | 80 | 80 | 84 | 84 |
| | 96 | 64 | 63 | 67 | 66 | 70 | 70 | 73 | 73 | 76 | 76 | 78 | 79 | 82 | 83 |
| | 192 | 66 | 65 | 69 | 68 | 70 | 70 | 73 | 73 | 75 | 75 | 77 | 76 | 79 | 81 |
| (*b*) Highest *m*-hour average in *n* hours | | | | | | | | | | | | | | | |
| 24 | 1 | 69 | 70 | 73 | 74 | 77 | 78 | 82 | 83 | 87 | 88 | 92 | 92 | 98 | |
| | 3 | 65 | 68 | 71 | 73 | 76 | 77 | 80 | 82 | 86 | 87 | 90 | 91 | 96 | 95 |
| | 6 | 63 | | 69 | 72 | 73 | 76 | 78 | 81 | 83 | 85 | 89 | 89 | 94 | 93 |
| | 12 | 61 | | 66 | 70 | 71 | 74 | 76 | 78 | 81 | 82 | 86 | 87 | 91 | 91 |
| 192 | 1 | 82 | | 86 | | 89 | 87 | 92 | 90 | 95 | 93 | 99 | 95 | 104 | |
| | 3 | 81 | | 84 | 84 | 87 | 87 | 90 | 89 | 94 | 92 | 98 | 94 | 102 | |
| | 6 | 79 | | 82 | 82 | 85 | 86 | 89 | 88 | 92 | 91 | 96 | 94 | 100 | |
| | 12 | 77 | | 80 | 78 | 83 | 82 | 87 | 85 | 90 | 87 | 92 | 89 | 96 | |
| | 24 | 74 | | 78 | 74 | 80 | 72 | 83 | 80 | 87 | 82 | 90 | 86 | 94 | |
| | 48 | 71 | | 74 | 72 | 77 | 74 | 80 | 77 | 83 | 80 | 86 | 82 | 90 | |
| | 96 | 68 | | 71 | 70 | 73 | 73 | 75 | 75 | 79 | 77 | 82 | 81 | 85 | |
| 768 | 1 | 91 | | 92 | | 95 | 92 | 97 | 94 | 100 | 97 | 103 | 100 | 107 | |
| | 3 | 89 | | 91 | 89 | 93 | 92 | 95 | 93 | 98 | 95 | 101 | 99 | 105 | |
| | 6 | 87 | | 89 | 88 | 91 | 90 | 94 | 92 | 97 | 94 | 99 | | 103 | |
| | 12 | 86 | | 87 | | 89 | 87 | 92 | 88 | 94 | 91 | 96 | 93 | 100 | |
| | 24 | 82 | | 84 | | 86 | | 88 | 82 | 91 | 86 | 93 | 89 | 97 | |
| | 48 | 79 | | 81 | | 82 | | 84 | 81 | 87 | 84 | 90 | 87 | 92 | |
| | 96 | 77 | | 78 | | 79 | 77 | 81 | 79 | 83 | 82 | 85 | | 88 | |
| | 192 | 76 | | 75 | | 76 | 75 | 78 | 77 | 80 | 80 | 81 | | 84 | |
| | 384 | 72 | | 73 | | 74 | 73 | 75 | 75 | 77 | 77 | 78 | | 80 | |

Table 2. *Probability estimates of (a) the m-hour average of air temperature at Minneapolis, Minnesota, in January, (b) the lowest m-hour average in n hours*

The 10-year mean is 13.6°F, standard deviation 14.7°F, diurnal amplitude 4.3°F, hour-to-hour correlation assumed to be 0.982. Estimates by model are in column A, by direct sampling in column B. When the 10-year sample was too small for even a shaky direct estimate the figure in column B is missing

| | | Probability | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | .02 | | .10 | | .25 | | .50 | | .75 | | .90 | | .98 | |
| $n$ | $m$ | A | B | A | B | A | B | A | B | A | B | A | B | A | B |
| (*a*) *m*-hour average | | | | | | | | | | | | | | | |
| | 1 | −16 | −17 | −5 | −7 | 4 | 2 | 14 | 15 | 24 | 24 | 32 | 32 | 44 | 37 |
| | 3 | −16 | −17 | −5 | −7 | 3 | 3 | 14 | 15 | 23 | 24 | 32 | 32 | 44 | 37 |
| | 6 | −16 | −16 | −5 | −6 | 4 | 4 | 14 | 15 | 23 | 24 | 32 | 32 | 44 | 37 |

Table 2, (*continued*)

| | | Probability | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | .02 | | .10 | | .25 | | .50 | | .75 | | .90 | | .98 | |
| $n$ | $m$ | A | B | A | B | A | B | A | B | A | B | A | B | A | B |
| | 12 | −15 | −15 | −4 | −6 | 4 | 4 | 14 | 15 | 23 | 24 | 32 | 32 | 43 | |
| | 24 | −14 | −14 | −4 | −5 | 4 | 4 | 14 | 15 | 23 | 24 | 31 | 31 | 42 | |
| | 48 | −13 | −13 | −3 | −3 | 5 | 4 | 14 | 15 | 22 | 23 | 30 | 29 | 40 | |
| | 96 | −10 | −8 | −1 | −1 | 6 | 6 | 14 | 14 | 21 | 21 | 28 | 27 | 37 | |
| | 192 | −6 | −8 | 1 | 3 | 7 | 7 | 14 | 14 | 20 | 21 | 26 | 26 | 34 | |
| | 384 | −2 | −3 | 4 | 4 | 9 | 8 | 14 | 14 | 19 | 19 | 23 | 24 | 29 | |
| | 768 | 2 | | 7 | 8 | 10 | 9 | 14 | 14 | 17 | 18 | 21 | 20 | 25 | |
| (*b*) Lowest $m$-hour average in $n$ hours | | | | | | | | | | | | | | | |
| 24 | 1 | −25 | | −14 | −13 | −6 | −6 | 4 | 6 | 14 | 17 | 23 | 24 | 32 | |
| | 3 | −23 | | −13 | −12 | −3 | −5 | 6 | 6 | 16 | 18 | 25 | 25 | 36 | |
| | 6 | −22 | | −12 | −11 | −2 | −4 | 8 | 8 | 18 | 19 | 27 | 27 | 38 | 35 |
| | 12 | −19 | | −9 | | 0 | −1 | 10 | 11 | 20 | 22 | 28 | 28 | 40 | |
| 192 | 1 | −33 | | −25 | −21 | −18 | −16 | −11 | −11 | −2 | −3 | 3 | 7 | 12 | 16 |
| | 3 | −32 | | −23 | −20 | −16 | −15 | −8 | −11 | 0 | −3 | 4 | 8 | 14 | 17 |
| | 6 | −30 | | −22 | −19 | −15 | −14 | −7 | −9 | 1 | −3 | 6 | 9 | 15 | |
| | 12 | −28 | −24 | −19 | −18 | −13 | −13 | −5 | −8 | 3 | −1 | 8 | 13 | 18 | 24 |
| | 24 | −25 | −21 | −17 | −13 | −11 | −8 | −2 | −3 | 5 | 5 | 11 | 16 | 21 | 26 |
| | 48 | −21 | −18 | −13 | −7 | −7 | −2 | 2 | 4 | 10 | 11 | 15 | 20 | 25 | 30 |
| | 96 | −16 | −11 | −8 | −1 | −1 | 4 | 7 | 9 | 15 | 15 | 20 | 23 | 28 | 31 |
| 768 | 1 | −40 | | −32 | −28 | −27 | −23 | −21 | −19 | −16 | −14 | −10 | −8 | −5 | −2 |
| | 3 | −39 | −31 | −31 | −28 | −25 | −23 | −19 | −18 | −15 | −14 | −8 | −8 | −4 | −2 |
| | 6 | −37 | | −30 | −27 | −24 | −21 | −18 | −17 | −13 | −12 | −7 | −6 | −3 | −5 |
| | 12 | −35 | | −28 | −21 | −22 | −18 | −16 | −15 | −11 | −11 | −5 | −5 | −1 | |
| | 24 | −32 | | −25 | −20 | −19 | −17 | −13 | −12 | −8 | −6 | −2 | 0 | 2 | 6 |
| | 48 | −30 | | −21 | −17 | −16 | −10 | −9 | −6 | −3 | 0 | 0 | 3 | 5 | 9 |
| | 96 | −23 | | −17 | −10 | −10 | −3 | −4 | 1 | 1 | 6 | 5 | 11 | 9 | |
| | 192 | −16 | | −9 | −7 | −4 | −2 | 1 | 4 | 5 | 11 | 9 | 13 | 14 | 22 |
| | 384 | −8 | | −1 | | −2 | | 8 | 8 | 11 | 03 | 14 | 18 | 19 | 22 |

Table 3. *Upper limit of the probability estimates, by the model, of the highest m-hour average in n hours of air temperature at Birmingham, Alabama, 1 July–1 August (incl.)*

Average is 80.5°F, standard deviation 7.25°F

| | | Probability of not exceeding | | | |
|---|---|---|---|---|---|
| $n$ | $m$ | .98 | .90 | .75 | .50 |
| 24 | 3 | 100°F | 96 | 92 | 89 |
| | 6 | 98 | 93 | 89 | 87 |
| | 12 | 97 | 92 | 87 | 83 |
| | 24 | 95 | 90 | 85 | 81 |
| 192 | 3 | 105 | 101 | 98 | 96 |
| | 6 | 103 | 99 | 96 | 94 |
| | 12 | 101 | 97 | 94 | 92 |
| | 24 | 99 | 95 | 93 | 89 |
| | 48 | 98 | 93 | 91 | 87 |
| | 96 | 96 | 92 | 88 | 84 |
| | 192 | 95 | 90 | 85 | 81 |

Table 3 (*continued*)

| | | Probability of not exceeding | | | |
|---|---|---|---|---|---|
| $n$ | $m$ | .98 | .90 | .75 | .50 |
| 768 | 3 | 107 | 104 | 102 | 100 |
| | 6 | 106 | 102 | 100 | 98 |
| | 12 | 104 | 101 | 98 | 96 |
| | 24 | 103 | 100 | 96 | 95 |
| | 48 | 102 | 98 | 95 | 92 |
| | 96 | 101 | 96 | 93 | 89 |
| | 192 | 99 | 94 | 90 | 87 |
| | 384 | 96 | 91 | 87 | 83 |
| | 768 | 93 | 88 | 84 | 81 |

it was possible to find the corresponding probabilities of annual precipitation (Table 4).

It is concluded from Table 4 that the observed lowest 1-year, 2-year, 4-year and 8-year averages in the 37-year record at Boston can be

Table 4. *The probability estimates, by model, of the lowest m-year average in 24 years and in 192 years of annual rainfall (inches) at Boston, Mass.*

The mean annual rainfall is 41.62 inches, standard deviation 8.88 inches

| | | Probability | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $n$ | $m$ | .02 | .10 | .25 | .50 | .75 | .90 | .98 |
| 24 | 1 | 14.0 | 18.5 | 21.6 | 25.6 | 28.0 | 29.8 | 32.5 |
| | 2 | 20.3 | 25.1 | 27.1 | 29.6 | 32.7 | 32.9 | 36.1 |
| | 4 | 26.9 | 30.5 | 31.9 | 33.5 | 36.3 | 36.7 | 39.0 |
| | 8 | 31.6 | 34.1 | 35.4 | 37.7 | 39.2 | 40.1 | 41.6 |
| 192 | 1 | 9.7 | 13.6 | 15.9 | 18.1 | 20.3 | 21.6 | 23.4 |
| | 2 | 17.6 | 20.3 | 22.5 | 24.3 | 26.5 | 27.1 | 28.3 |
| | 4 | 23.9 | 26.7 | 27.9 | 29.2 | 31.4 | 31.9 | 32.7 |
| | 8 | 29.0 | 31.2 | 31.9 | 33.0 | 33.9 | 35.7 | 35.9 |
| Actual lowest $m$-year average in 37 years | | | | | | | | |
| 37 | 1 | 1965 | | | 23.7 | | | |
| | 2 | 1964, 1965 | | | 29.8 | | | |
| | 4 | 1962–65 | | | 32.8 | | | |
| | 8 | 1939–46 | | | 37.1 | | | |

expected in a quarter-century as part of Nature's random process. Also these averages are nearly a certainty in two centuries.

## 4. Concluding remarks

A simple Markov process, operating on a sequence of normally distributed values with constant hour-to-hour correlation, yields realistic estimates of the probability of duration of the meteorological element. The use of the $m$-hour average, however, is admittedly an initial decision which has its limitations, because a variate must have a Gaussian, or nearly Gaussian, distribution before the model, as described herein, can be applied. It has proved effective on hourly temperatures and annual precipitation. To what extent the model will be applicable to other kinds of variables needs yet to be tested.

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# ВЕРОЯТНОСТИ СКОЛЬЗЯЩИХ СРЕДНИХ МЕТЕОРОЛОГИЧЕСКИХ ПЕРЕМЕННХЫХ

Метеорологический элемент, например приземная температура воздуха, осредненная по данному интервалу времени, может служить мерой сохранения или длительности определенных условий на этом интервале времени. Для изучения распределения вероятностей скользящих средних, как обычных так и экстремальных, оказалось необходимым использовать метод Монте Карло. Простая марковская последовательность нормально распределенных величин, характеризуемая постоянной межчасовой корреляцией, неожиданно оказалась подходящей моделью процесса в рассматриваемых примерах для периодов от одного до 768 часов. Были построены карты, дающие распределение вероятностей для максимального $m$-часового среднего в $n$-часовом интервале, где $n$ равно числу часов в сутках, 8 сутках или 32 сутках и $m$ равно 1, 3, 6, 12, 24, 48, 96, 192, 384 или 768 часов. Межчасовая корреляция изменялась от нуля до 0,999, причем обычно значение приближенно равнялось 0,95.