On the orientation of cloud bands

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(Manuscript received September 26, 1966)

ABSTRACT

The growth rate of line perturbations in a horizontally stratified saturated fluid is examined for the case of a vertical shear in the mean wind. Certain of these perturbations are unstable for Richardson numbers less than minus two, while perturbations along the direction of the wind shear vector are unstable so long as the static stability is less than zero. It is concluded that cloud bands will tend to be oriented in the wind shear direction.

Introduction

In recent years it has become increasingly evident that clouds often, if not usually, occur in a banded pattern. In early work, observations of small-scale cloud bands were commonplace (Mal, 1931; Walker, 1933) and such bands were considered mainly as indicators of certain meteorological conditions rather than as a phenomenon having a possible effect on larger-scale motions. The occurrence of "cloud streets" has been known for some time (Woodcock, 1942; Kuettner, 1949) but these have been treated as tertiary circulations with little significance to cyclone-scale motions or to the general circulation. Later, with the advent of radar, observations were made of squall lines and hurricane spiral bands (Wexler, 1947). Subsequently, high-level aircraft observations showed that convective clouds often occur in bands and finally, Tiros photographs revealed banded structure occurring much more generally than expected (Kuettner, 1962). From these observations it seems plausible that banded cloud structure may be of primary importance in the release of latent heat of condensation during cyclone development, transfer of moisture in the general circulation, and the radiation budget.

Three important related questions regarding cloud bands are: under what circumstances do they form, what are their dimensions, and what is their orientation? In this paper we will focus attention upon the *orientation* of cloud bands. As one might expect, attempts have been made to relate the orientation to the ambient wind field. Terada (1928) performed experiments with liquids in which the flow increased with height without changing direction. He found disturbances in the form of longitudinal rolls. Phillips & Walker (1932) and later, Graham (1934), showed by laboratory experiments in air, that rolls transverse to the direction of vertical wind shear can be formed when the shear is small. When the shear is very small, distorted cellular convection is found (Brunt, 1951). From the detailed observations in the free atmosphere by Malkus & Riehl (1964) it is found that the vertical wind shear determines cloud-band orientation. They conclude that the orientation of convection rows over the tropical oceans will be governed in the case of small cumulus by the wind shear between the convective layer and the surface, and in the case of deep convection by the shear above the uniform easterlies. Sometimes, both modes, referred to as a "parallel" and a "cross-wind" mode, respectively, may appear together. In Kuo's (1963) analytical treatment the growth rates of line perturbations are found and it is shown that with wind shear, line perturbations are preferred over cellular ones. In his analysis both the wind and wind shear are parallel so it can be concluded only that convection rolls are parallel to either the wind or the shear of the wind.

In this treatment the wind and wind shear are in different directions and we show that the governing factor in the orientation of line perturbations is the wind shear. The effect of wind shear transverse to a line perturbation is examined by the following method: perturbation equations are written for inviscid flow in conditionally unstable moist air or unstable dry air. We consider a line perturbation with no variations along some arbitrary direction in the horizontal plane and let this direction be the y-axis of an x, y, z coordinate system. Now we place a horizontal vector representing the wind shear in some particular direction, say northeast. By rotating the y-axis to the northeast there will be no component of wind shear perpendicular to the line perturbation, and by rotating the y-axis away from northeast some of the wind shear will be perpendicular to the line perturbation. Thus, by changing the orientation of the y-axis with respect to the vertical mean-wind shear vector we can inquire how growth rates of line perturbations are affected in the presence of wind shear transverse to the perturbation.

Perturbation equations

The equations used are: components of horizontal motion 1a, 1b, continuity 1c, the first law of thermodynamics 1d, and vertical acceleration 1e.

$$\frac{du^*}{dt} + \frac{1}{\varrho^*}\frac{\partial p^*}{\partial x} - fv^* = 0 \qquad (1\,\mathrm{a})$$

$$\frac{dv^*}{dt} + \frac{1}{\varrho^*}\frac{\partial p^*}{\partial y} + fu^* = 0 \qquad (1\,\mathrm{b})$$

$$\frac{d\varrho^*}{dt} + \varrho^*\left(\frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y} + \frac{\partial w^*}{\partial z}\right) = 0 \qquad (1\,\mathrm{c})$$

$$\frac{d\theta^*}{dt} = \frac{\dot{Q}}{Cp}\frac{\theta^*}{T^*} \qquad (1\,\mathrm{d})$$

$$\frac{dw^*}{dt} + \frac{1}{\varrho^*}\frac{\partial p^*}{\partial z} + g = 0 \qquad (1\,\mathrm{e})$$

The asterisk refers to the sum of the steady-state value and the perturbation. u^*, v^* and w^* are the components of motion in the x, y and z directions, respectively, ϱ^* is the density, p^* the pressure, θ^* the potential temperature, T^* the temperature, f the coriolis parameter, g the acceleration of gravity, and $\dot{Q}$ the rate of heating. For purposes of studying effects of wind shear on convection of the cumulus or strato-cumulus type we assume that the steady-state wind has no vertical component and varies only in the z direction, that second-order perturbations are negligible, that in the continuity equation density variations can be neglected, and that the coriolis effect on cumulus-scale perturbations is negligible. Strictly speaking the following analysis applies to strato-cumulus-type convection rather than cumulus, since the static stability will be kept constant. The resulting perturbation equations are:

$$\frac{\partial u}{\partial t} + U\frac{\partial u}{\partial x} + V\frac{\partial u}{\partial y} + w\frac{\partial U}{\partial z} + \frac{1}{\varrho_0}\frac{\partial p}{\partial x} = 0 \qquad (2\,\mathrm{a})$$

$$\frac{\partial v}{\partial t} + U\frac{\partial v}{\partial x} + V\frac{\partial v}{\partial y} + w\frac{\partial V}{\partial z} + \frac{1}{\varrho_0}\frac{\partial p}{\partial y} = 0 \qquad (2\,\mathrm{b})$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (2\,\mathrm{c})$$

$$\frac{\partial \theta}{\partial t} + U\frac{\partial \theta}{\partial x} + V\frac{\partial \theta}{\partial y} + w\sigma = 0 \qquad (2\,\mathrm{d})$$

$$\frac{\partial w}{\partial t} + U\frac{\partial w}{\partial x} + V\frac{\partial w}{\partial y} + \frac{1}{\varrho_0}\frac{\partial p}{\partial z} + \frac{g}{p_0}(1-k)p - \frac{g}{\theta_0}\theta = 0 \qquad (2\,\mathrm{e})$$

where

$$w\sigma = w\sigma_M = w\frac{\partial \theta_0}{\partial z} - \frac{\dot{Q}}{c_p}\frac{\theta_0}{T_0} \quad \text{for saturated air}$$

$$w\sigma = w\sigma_D = w\frac{\partial \theta_0}{\partial z} \quad \text{for unsaturated air}$$

and W is the x component of steady-state wind and V is the y component, and the subscript $_0$ refers also to steady-state values. Letting y variations be zero,

$$D = \frac{\partial}{\partial t} + U\frac{\partial}{\partial x}, \quad \text{and} \quad \nabla^2 w = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}$$

we find

$$D(u) + w\frac{\partial U}{\partial z} + \frac{1}{\varrho_0}\frac{\partial p}{\partial x} = 0 \qquad (3\,\mathrm{a})$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial x} = 0 \qquad (3\,\mathrm{b})$$

$$D(\theta) + w\sigma = 0 \qquad (3\,\mathrm{c})$$

$$D(w) + \frac{1}{\varrho_0}\frac{\partial p}{\partial z} - \frac{g}{\theta_0}\theta = 0 \qquad (3\,\mathrm{d})$$

Thus the wind and wind shear in the y direction have no effect on the form of the solution for the perturbations u, p, θ, or w. In the last equation $g/p_0(1-k)p$ is neglected. For the case of no steady wind, omission of this term gives perturbation amplitudes differing by a factor of $\exp\{-(1-k)g/RT_0Z\}$. So, for disturbances less than about 10 km height the results should not be much affected.

From Eqs. 3a through 3d we derive an expression for w. This is done first by eliminating the pressure perturbation, then with the continuity equation finding an expression relating θ and w, and finally, with the thermodynamic equation we obtain the desired expression given below.

$$D^2(\nabla^2 w) - \frac{\partial^2 U}{\partial z^2} D\left(\frac{\partial w}{\partial x}\right) + \frac{g\sigma}{\theta_0}\frac{\partial^2 w}{\partial x^2} = 0 \tag{4}$$

Case of no shear

If $U = U_0$ and we use a coordinate system moving at a velocity U_0 then equation (4) becomes

$$\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right) + \frac{g\sigma}{\theta_0}\frac{\partial^2 w}{\partial x^2} = 0 \tag{5}$$

This has solutions of the form

$$w = Ae^{qt}\sin k\times \sin\frac{\pi}{h}z$$

which leads to

$$q = \left(-\frac{g\sigma}{\theta_0}\right)^{\frac{1}{2}} F \tag{6}$$

where $$F = \frac{k}{\left(k^2 + \frac{\pi^2}{h^2}\right)^{\frac{1}{2}}}$$

Thus, we have a well-known relationship: when the static stability is negative q is positive and w will increase with time. In the case of moist air the release of latent heat is included in the static stability so perturbations are unstable when the lapse rate of temperature is less than the moist adiabatic.

Case of constant wind shear

If $$U = U_0 + \frac{\partial \overline{U}}{\partial z} z$$

where $(\partial\overline{U}/\partial z)$ is constant

and $$x = x' + U_0 t$$

then $$D = \frac{\partial}{\partial t} + U_0\frac{\partial}{\partial x} + \frac{\partial\overline{U}}{\partial z}z\frac{\partial}{\partial x} = \frac{\partial}{\partial t'} + \frac{\partial\overline{U}}{\partial z}z\frac{\partial}{\partial x'}$$

and

$$\left(\frac{\partial}{\partial t'} + \frac{\partial\overline{U}}{\partial z}z\frac{\partial}{\partial x'}\right)^2\left(\frac{\partial^2 w}{\partial x'^2} + \frac{\partial^2 w}{\partial z^2}\right) + \frac{g\sigma}{\theta_0}\frac{\partial^2 w}{\partial x'^2} = 0 \tag{7}$$

For convenience we drop the primes and treat the equivalent case of $U_0 = 0$.

We obtain an equation for w as a function of z by assuming solutions of the form

$$w = A\,e^{ik(x-ct)}\,W(z)$$

Thus

$$\left(c - \frac{\partial\overline{U}}{\partial z}z\right)^2 W''(z) - \left\{k^2\left(c - \frac{\partial\overline{U}}{\partial z}z\right)^2 - \frac{g\sigma}{\theta_0}\right\}W(z) = 0 \tag{8}$$

This equation can be put in a more suitable form by the transformation

$$s = c - \frac{\partial\overline{U}}{\partial z}z \qquad \frac{\partial\overline{U}}{\partial z} \neq 0$$

Hence

$$\frac{d^2W(s)}{ds^2} - \left(\frac{k^2}{\left(\frac{\partial\overline{U}}{\partial z}\right)^2} - \frac{g\sigma}{\theta_0\left(\frac{\partial\overline{U}}{\partial z}\right)^2 s^2}\right)W(s) = 0 \tag{9}$$

This is known as a Hamburger equation and has solutions of the form

$$W_1(s) = e^{k(\partial\overline{U}/\partial z)^{-1}s}\,(1 + c_1 s^{-1} + \cdots + c_m s^{-m}) \tag{10a}$$

and

$$W_2(s) = e^{-k(\partial\overline{U}/\partial z)^{-1}s}\,(1 - c_1 s^{-1} + \cdots \pm c_m s^{-m}) \tag{10b}$$

provided the equation

$$m(m+1) = -\frac{g\sigma}{\left(\dfrac{\overline{\partial U}}{\partial z}\right)^2 \theta_0} = -\text{ Richardson number} \tag{11}$$

has a positive integral root. Otherwise the series term in the solution for $W(s)$ does not converge. A positive root will be found in two conditions: when the air is saturated ($\sigma = \sigma_M$) and the temperature lapse rate is less than the moist adiabatic rate, and when the air is unsaturated ($\sigma = \sigma_D$), and the lapse rate is less than the dry adiabatic rate. (This latter condition is often found in the layer between the surface and a convective cloud base.) The value of m, then, depends upon the angle the line perturbation makes with the wind shear vector as shown in Fig. 1 for two orientations of the y axis, y_a and y_b, where V_2 is the wind at a lower level, and V_1 is the wind at a higher level. The total wind shear is the vector difference. The largest possible value of $\partial U/\partial z$, given the static stability, is found when $m = 1$. It is

$$\frac{\overline{\partial U}}{\partial z} = \left(-\frac{g\sigma}{2\theta_0}\right)^{\frac{1}{2}} \quad \sigma < 0 \tag{12}$$

We note here that for $m = 1$ the Richardson number is minus two. This is also the critical value since disturbances cannot exist for a more positive value of the Richardson number.

Now we find out how close the direction of the line perturbation must be to the wind shear vector. With $m = 1$

$$W_1(s) = e^{k(\overline{\partial V}/\partial z)^{-1}s}\,(1 + c_1 s^{-1}) \tag{13a}$$

$$W_2(s) = e^{-k(\overline{\partial V}/\partial z)^{-1}s}\,(1 - c_1 s^{-1}) \tag{13b}$$

where

$$c_1 = \frac{g\sigma}{2\theta_0 k \dfrac{\overline{\partial U}}{\partial z}} = -k^{-1}\frac{\overline{\partial U}}{\partial z}, \qquad m = 1, \quad \sigma < 0$$

Noting again $\quad s = c - \dfrac{\overline{\partial U}}{\partial z} z$

$$W(z) = a_1 W_1(z) + a_2 W_2(z) \tag{14}$$

The boundary conditions, for simplicity, are that $W(z) = 0$ at the lower boundary, $z = 0$, and at some level $z = h$. So

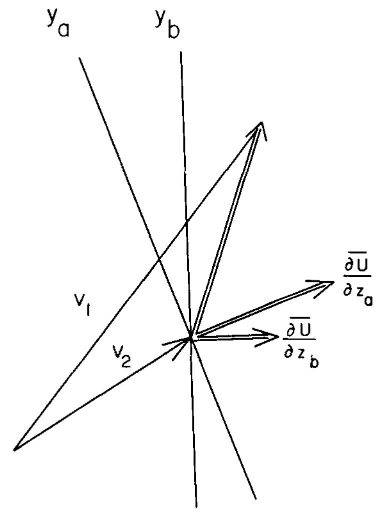

Fig. 1. Wind shear perpendicular to cloud band (y-axis) for two different orientations.

$$a_1 W_1(0) + a_2 W_2(0) = 0 \tag{15a}$$

and

$$a_1 W_1(h) + a_2 W_2(h) = 0 \tag{15b}$$

from which we get

$$c^2 - \frac{\overline{\partial U}}{\partial z} hc - k^{-2}\left(\frac{\overline{\partial U}}{\partial z}\right)^2 \left(1 - \frac{kh}{\tanh kh}\right) = 0 \tag{16}$$

or

$$c = \left(-\frac{g\sigma}{\theta_0}\right)^{\frac{1}{2}} \frac{1}{\sqrt{8}\,k} \times \left\{kh \pm \left[k^2h^2 + 4\left(1 - \frac{kh}{\tanh kh}\right)\right]^{\frac{1}{2}}\right\}, \quad \sigma < 0 \tag{17}$$

To obtain growth rates we consider the imaginary part of c. It may be written

$$c_{\mathrm{im}} = \pm i\left(-\frac{g\sigma}{\theta_0}\right)^{\frac{1}{2}} \frac{1}{k} G, \quad \sigma < 0 \tag{18}$$

where

$$G = -\frac{i}{\sqrt{8}}\left[k^2h^2 + 4\left(1 - \frac{kh}{\tanh kh}\right)\right]^{\frac{1}{2}}$$

For positive growth rate,

$$q = -ic_{\mathrm{im}} k = \left(-\frac{g\sigma}{\theta_0}\right)^{\frac{1}{2}} G, \quad \sigma < 0 \tag{19}$$

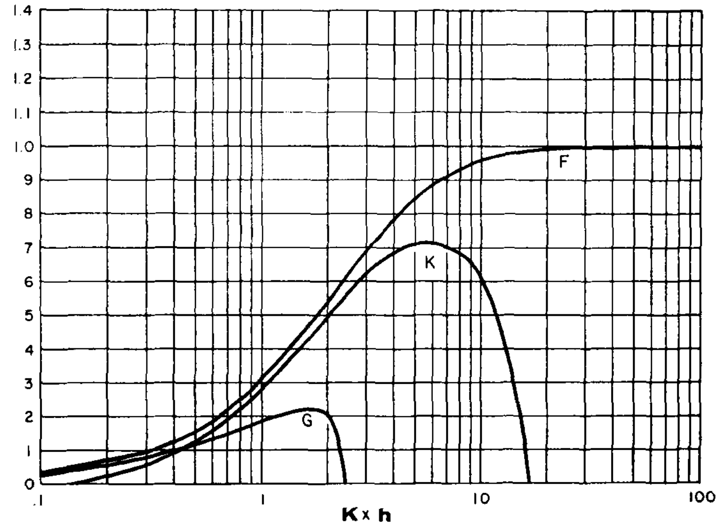

Fig. 2. Initial growth rate factor vs kh for convection in an atmosphere in which there is: no wind or viscosity (curve F); no wind, but is viscous (curve K); and wind shear, but no viscosity (curve G). Curve K derived from Kuo (1961).

We can compare cloud growth rates for a given static stability with and without wind shear by examining F and G. This is done for various scale sizes by varying the product kh, upon which F and G solely depend. Values of F and G are shown in Fig. 2 along with a curve derived from Kuo (1961) in which viscosity effects ($\nu = 10^3\ \mathrm{m}^2\ \mathrm{sec}^{-1}$) are included. We note that without wind shear or viscosity the growth rate factor, F, increases as kh increases and approaches unity as kh becomes large. That is, disturbances with small horizontal dimensions are the most unstable. With viscosity (curve K), however, the growth rate of such disturbances is greatly reduced, leaving a maximum at kh equal to about 5. With wind shear, but no viscosity (curve G), we find a similar variation of the growth rate factor as in the viscous case, except that the maximum occurs at kh equal to about 1.8, resulting in a horizontal dimension about three times greater than the viscous case, but a smaller growth rate.

As kh becomes small, the disparity between F and G becomes small and we conclude that as the disturbances become increasingly shallow the effect of wind shear is reduced. *Thus the effect of wind shear perpendicular to cloud lines is to greatly inhibit growth of deep ones and slightly inhibit growth of shallow ones.*

To the extent that surface friction can be neglected, these results also apply to statically unstable, unsaturated air, often in the region below developing cumulus clouds. In fact it has been reported (Plank, 1960) that cumulus clouds form in lines from their very first appearance. Such lines will tend to be along the wind shear vector, and in this case, the vector within the subcloud layer.

It should be noted that in this treatment $\partial^2 U/\partial z^2$ was set equal to zero. Thus, the results of Kuettner (1959) are not negated, but it does appear that the wind shear *itself* is sufficient to explain cloud-band orientation. When $\partial^2 U/\partial z^2$ becomes excessively great, then our results may be modified. Further investigation of Eq. 4 should clarify this.

REFERENCES

Brunt, D. 1951. Experimental cloud formation. *Compendium of Meteorology*, p. 1255. Baltimore, Md., Waverly Press, Inc.

Graham, A. 1934. Shear patterns in an unstable layer of air. *Phil. Trans. Roy. Soc.* (A) *232*, 285–296.

Kuettner, P. 1949. Der Segelflug in Aufwindstrassen. *Schweizer Aero Revue 24*, 480–482.

Kuettner, P. 1959. The band structure of the atmosphere. *Tellus 11*, 267–294.

Kuettner, P. 1962. Cloud bands in the earth's atmosphere as revealed by the TIROS satellites

and the mechanism of their formation. *Un. Geod. Geophys. Int. Mon.* 16 (Int. Conf. Cloud Phys., Canberra and Sydney, 1961) 11–12.
Kuo, H. 1961. Convection in conditionally unstable atmosphere. *Tellus 13*, 441–459.
Kuo, H. 1963. Perturbations of plane Coette flow in stratified fluid and origin of cloud streets. *The Physics of Fluids 6*, 195–211.
Mal, S. 1931. Forms of stratified clouds. *Beitr. Physik Freien Atmos. 17*, 40–68.
Malkus, J. S. & Riehl, H. 1964. *Cloud structure and distributions over the tropical Pacific Ocean*, Berkeley, Calif., Univ. of Calif. Press, 229.
Phillips, A. C. & Walker, G. T. 1932. The forms of stratified cloud. *Quart. Jour. Roy. Met. Soc. 58*, 23–30.
Plank, V. G. 1960. Cumulus convection over Florida. *Cumulus Dynamics*, 211 pp. New York, N.Y., Pergamon Press.
Tereda, T. 1928. *Some experiments on periodic columnar formation of vortices caused by convection.* Report Aeron. Res. Inst., Tokyo, Japan, 3.1–46.
Walker, G. 1933. Clouds and cells. *Quart. J. Roy. Met. Soc. 59*, 389–396.
Wexler, H. 1947. Structure of hurricanes as determined by radar. *Ann. New York Acad. Sci. 48*, 821.
Woodcock, A. 1942. Soaring over the open sea. *The Scientific Monthly 55*, 226.

ОБ ОРИЕНТАЦИИ ПОЛОС ОБЛАКОВ

Исследуется степень роста возмущений, задаваемых в виде линий, в горизонтально стратифицированной насыщенной жидкости при наличии вертикального поперечного градиента среднего ветра. Некоторые из этих возмущений неустойчивы для чисел Ричардсона, меньших -2, в то время как возмущения вдоль направления вектора градиента ветра неустойчивы до тех пор, пока параметр статической устойчивости меньше нуля. Делается вывод, что полосы облаков будут иметь тенденцию ориентироваться в направлении поперечного градиента ветра.