The Numerical Prediction Method for Upper Air Profiles Tried on some Regular Type Profiles

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Abstract

J. G. CHARNEY and A. ELIASSEN (1949) have proposed a numerical forecast-method for the height profile of the 500 mb-surface at a fixed latitude, and they have also calculated the constants necessary for 24 hour forecasts at 45° N. From those constants the author has derived a 48 hour formula, which has been tried on some simple and regular upper air pattern-types, such as sinusoidal profiles of different wave-length, and some types of regular but assymetric quasi-sinusoidal profiles, the purpose of these tests being to ascertain the nature of the circulation changes implied by the Charney-Eliassen method.

Introduction

The numerical forecast-method for the height profiles of the 500 mb-surface, as proposed by CHARNEY and ELIASSEN (1949), is a first attempt to replace the aerological forecasting procedure of to-day, i.e. a mainly qualitative and always somewhat subjective argumentation between factors of different kinds, with a quite objective, numerical method. The practical possibilities, however, of this pioneering way of forecasting are not quite clear from the two authors' mainly theoretical paper. Since the meteorological forcasters need an idea of the practical qualities of the method, it is necessary either to test the method against a series of actual situations, or to apply it to some simplified standard upper-air patterns. Synoptic tests of the prediction method are at present carried out at the University of Stockholm. Thus, we will in this paper deal only with the application of the forecast-method to some schematic height profiles. Thereby we will get a general idea of how the method works, and what kind of results we may expect in different cases.

To make the difference between the starting profile and the forecast profile more striking than it would be with 24 hour forecasts only, we prefer studying 48 hour forecasts. In addition to that, many of the routine upper-air forecasts of to-day are just 48 hour ones. For that reason we will at first derive a 48 hour modification of the Charney-Eliassen formula.

A 48 hour forecast formula

We may write the Charney-Eliassen formula for 24 hour forecasts in the following manner

$$z_{24}(x + U_{24}) = \sum_n B_n z_0 (x + 10\,n) \qquad (1)$$

where z_0 = height at $t = 0$,
z_{24} = height at $t = 24$ hours,
U_{24} = zonal speed in degrees of longitude per 24 hours,
B_n = numerical constants, the 45°-values to be found in Table I,
x = longitude,
n = 0, $\pm$ 1, $\pm$ 2, ...

On the right side of (1) we now find for each x, a forecast value z_{24}, which is to be plotted U_{24} degrees east of the point x. That means, that we can do the calculation in two steps: the first step is

Table I.

Numerical constants B_n in the 24 hour forecast formula (1).

n	$a^2 = 10$	$a^2 = 14$	$a^2 = 18$
−3	−0.070	−0.051	−0.037
−2	−0.100	−0.080	−0.064
−1	−0.506	−0.433	−0.382
0	0.788	0.818	0.839
1	0.305	0.314	0.316
2	0.119	0.112	0.102
3	0.177	0.149	0.123
4	0.065	0.051	0.038
5	0.095	0.065	0.047
6	0.034	0.022	0.014
7	0.049	0.030	0.016
8	0.017		
9	0.023		

to compute the z_{24}-values for each x, as was $U = 0$; the second one is to displace the resulting profile just U_{24} degrees to the east.

If we now insert the undisplaced values of z_{24} (as calculated for $U = 0$) in the formula again, we get

$$z_{48}(x) = \sum_n B_n z_{24}(x + 10\,n)$$

and displacing that profile $2\,U_{24}$ to the east, we have the 48 hour forecast. But (1) gives, when $U = 0$,

$$z_{24}(x + 10\,n) = \sum_m B_m z_0 (x + 10\,n + 10\,m)$$

Thus

$$z_{48}(x + 2\,U_{24}) = \sum_{n,m} B_n B_m z_0 (x + 10\,[n + m])$$

or

$$z_{48}(x + 2\,U_{24}) = \sum_n C_n z_0 (x + 10\,n) \qquad (2)$$

where C_n are numerical constants, easy to compute from the B_n-values, and to be found in Table II.

In this paper we have used the constants, which are valid when $a^2 = 14$.[1] If we want a more simple and

Table II.

Numerical constants C_n in the 48 hour forecast formula (2).

n	$a^2 = 10$	$a^2 = 14$	$a^2 = 18$	Approx.
−6	0.005	0.003	0.001	
−5	0.014	0.008	0.004	
−4	0.080	0.051	0.032	
−3	−0.008	−0.014	−0.014	
−2	0.056	0.025	0.014	
−1	−0.876	−0.770	−0.688	−0.8
0	0.265	0.364	0.438	0.4
1	0.314	0.388	0.434	0.4
2	0.075	0.138	0.170	0.1
3	0.262	0.257	0.232	0.3
4	0.102	0.125	0.112	0.1
5	0.186	0.148	0.116	0.2
6	0.101	0.084	0.065	0.1
7	0.122	0.089	0.054	0.1
8	0.078	0.046	0.025	0.1
9	0.082	0.020	0.012	
10	0.049	0.015	0.008	
11	0.024	0.006	0.004	
12	0.021	0.005	0.002	
13	0.010	0.001	0.000	
14	0.008	0.001	0.000	
15	0.004			
16	0.002			
17	0.000			
18	0.001			

less time-consuming formula than the complete equation (2), we may use the constants in the last column of Table II. That simplification is in fact a rather good approximation, as will be shown later on. Besides, that is clear from CHARNEY-ELIASSEN's statement, that there is very little difference in the forecasts made with $a^2 = 10$, $a^2 = 14$ and $a^2 = 18$.

Some general qualities of the numerical forecast method

As was already shown, when we deduced the 48 hour forecast formula, it is possible to split all changes due to the Charney-Eliassen method, in two components. *One is the deformation of the profile,* not at all depending on the zonal wind U[1]. *The other one is the displacement of the deformed profile.* That displacement depends only on U.

Of course, the method is not able to change the wave-length of a regular periodic profile.

If the deformation of the profile produces a

[1] According to CHARNEY-ELIASSEN, we have at 45° N, that

$$a^2 = 2.5 + \frac{2\pi}{U} - s^2$$

where U is the zonal wind in radians per day, and s is the number of stationary waves, encircling the globe. U is always of the order of 0.3. Then, for instance, $s = 3$ gives $a^2 = 14$.

[1] As far as the coefficients B_n and C_n do not depend on U, which is approximately true.

change in amplitude, that change is proportional to the amplitude value.

A forecast calculated from a profile, which is the sum of superimposed profiles, is equal to the sum of the forecasts calculated from each one of the individual profiles.

A study of sinusoidal profiles

Applying the Charney-Eliassen formula to regular, schematic profiles, we get a clear idea of what changes we may expect in different cases. Besides, it will then be much easier to discuss fundamentally, if the method is in agreement with experience, than by applying it to observed upper-air profiles only.

For that reason, we have tried the 48 hour formula (2) on sinus curves with wave-lengths between 10 and 180 degrees[1]. We are then only interested in the first or deforming step ($U = 0$), as the second step is nothing but a displacement due to the zonal speed. Fig. 1 shows an example, where $L = 60$ degrees of longitude. The full line is the starting profile, and the broken line gives the 48 hour forecast. Obviously the deformation step has moved the curve about 16 degrees to the west. So if the phase velocity, i.e. the speed of propagation of ridges or troughs, equals C_{24} per day, we get

$$C_{24} = U_{24} - \frac{16}{2}$$

or

$$U_{24} - C_{24} = 8$$

Now other wave-lengths give other values of $(U_{24} - C_{24})$, i.e. the difference between the zonal velocity and the phase velocity. The continuous curve in Fig. 2 gives those differences as a function of the wave-length L. The dotted line demonstrates the well-known formula

$$U - C = \frac{\beta L^2}{4 \pi^2}$$

found by Rossby (1939).

After studying how that formula fits empirical data, Namias & Clapp (1944) found, that it gave

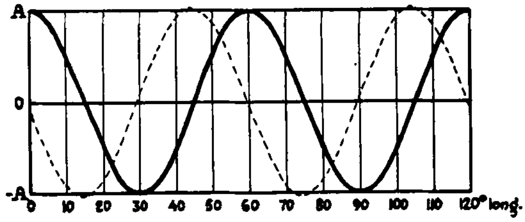


Fig. 1. Diagram showing a sinusoidal profile (full line) with the amplitude A and the wave-length $L = 60$ degrees of longitude, and the corresponding 48 hour forecast (broken line), according to the Charney-Eliassen method when $U = 0$. For other values of U, the forecast profile is to be moved $2\,U_{24}$ degrees to the right (to the east).

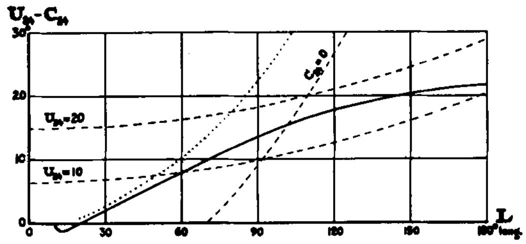


Fig. 2. The difference (U_{24}—C_{24}) between the zonal and the phase velocity in degrees of longitude per day as a function of the wave-length L. The full line shows the results of the Charney-Eliassen method. The dotted line represents the Rossby-formula, whereas the broken lines demonstrate the Namias-Clapp values for certain U_{24} and C_{24}.

too high values for large wave-lengths; so they proposed a corrected formula[1], namely

$$C_{24} = 0.156 \left(U_{24} - \frac{\beta L^2}{4 \pi^2} \right) + 2.11$$

The broken lines in Fig. 2 demonstrate the last-mentioned formula, when U_{24} equals 10 and 20 degrees per day, and when $C_{24} = 0$.

For short wave-lengths, the numerical forecast-method thus is in good agreement with the Rossby-formula. For wave-lengths between 90 and 150 degrees it just coincides with the Namias-Clapp formula for $U_{24} = 15$, which is probably a good average value of the zonal speed at 45° N.

In the example of fig. 1, where $L = 60$, we have

[1] We have done that in spite of the fact, that all those wave-lengths are not possible on the earth; to be so, the wave-lengths must of course be a factor of 360 degrees.

[1] It must be mentioned, that this formula was found by studying five-day mean maps. So we are not quite justified to state, that it is valid as well for the movements of individual troughs and ridges.

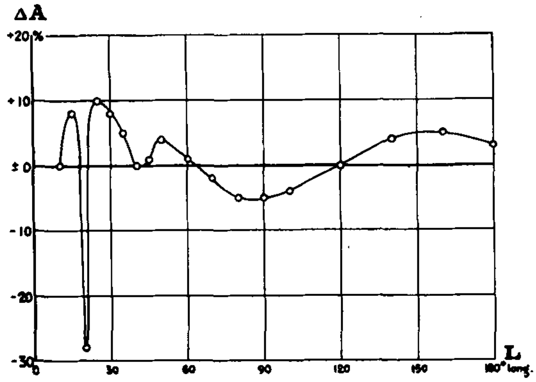


Fig. 3. Percental 48 hour amplitude-changes for wave-lengths between 10 and 180 degrees of longitude, according to the Charney-Eliassen formula.

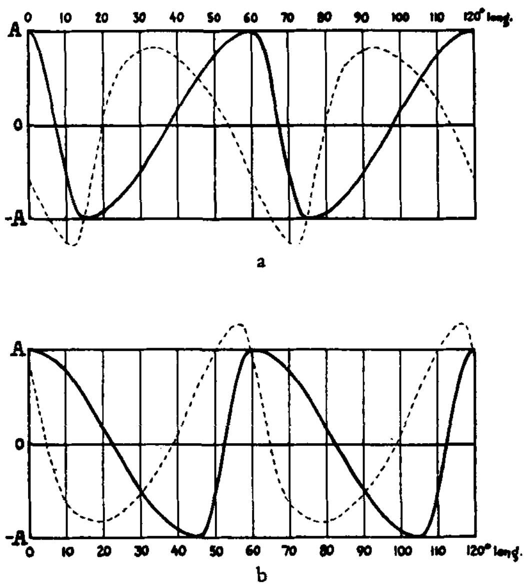


Fig. 4 a, b. Diagram showing two contrary types of askew quasi-sinusoidal profiles (full lines), and the corresponding 48 hour forecasts (broken lines), valid when $U = 0$.

got a slight increase of the amplitude (about 1 %). Other wave-lengths will give other amplitude changes, however. Fig. 3 shows how large those changes are. There is mainly an increase in amplitude for wave-lengths below 60 degrees and beyond 120 degrees; for wave-lengths between those values, however, we have a decrease of up to 5 %. The irregularities of the curve as for wave-lengths less than 30 degrees probably depend on the 10-degrees calculation intervals being too large here, compared with the wave-lengths themselves.

If we use the approximate constants of Table II (last column), we will get the same values of $(U_{24} - C_{24})$ as shown in Fig. 2, but the amplitude changes will differ somewhat from those of Fig. 3.

Quasi-sinusoidal profiles

We have also tried the forecast-method on regular but askew profiles such as given by the continuous curves in fig. 4. Looking at the dotted forecast profiles, we see that we have here in both cases got the contrary type. Having for instance, as in fig. 4 a, a trough, where the meridional wind is strongest on the back side, we evidently will get the strongest gradient on the front side 48 hours later, and vice versa. It is also seen, that we have in fig. 4 a got a deepening and sharpening of the troughs and a flattening of the ridges, but in fig. 4 b a rise and sharpening of the ridges and a filling of the troughs.

In fig. 5 a we have a series of ridges, sharp and flat alternately. Also here we obviously get a fore-

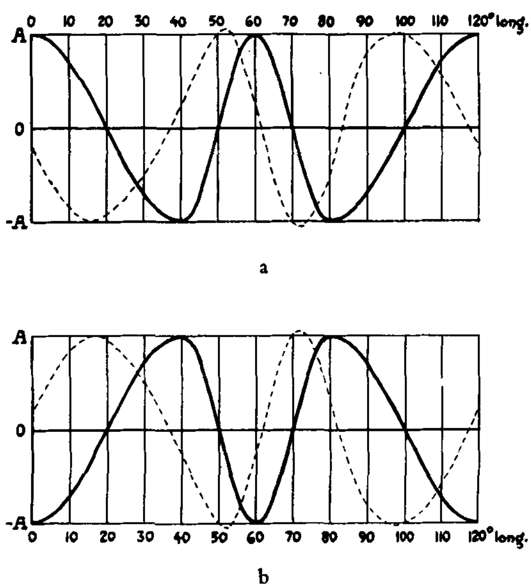


Fig. 5 a, b. Diagram showing two contrary types of quasi-sinusoidal profiles (full lines) with long and short wave-lengths alternately, and the corresponding 48 hour forecasts (broken lines), valid when $U = 0$.

cast, which looks similar to the reflected image (fig. 5 b) of the starting profile. Of course that is also true in the opposite case. In these two cases we do not get any remarkable changes in amplitude.

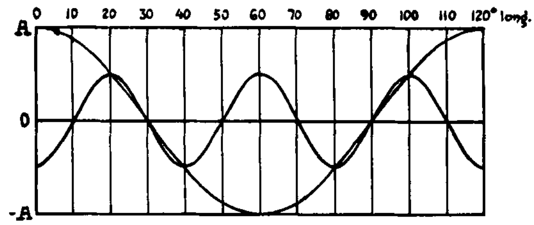
Superimposed profiles

In fig. 6 a, b and c we have given an example of two superimposed profiles. The sum of the two curves in 6 a is given by the continuous curve in 6 b. The dotted curve in that figure is the forecast curve. It is equal to the sum of the two individual forecasts, shown in 6 c. We now see from fig. 6 b, that if we have a double trough, the forecast-method will make the trough to the west deepen, and the eastern one fill. In the double ridge, on the other hand, the ridge to the west will rise, whereas the easternmost one will decrease extensively.

Summary

We have given some schematic examples of the consequences of the Charney-Eliassen method. It is seen from that, that we can get a trough deepening in three ways. Firstly certain wave-lengths always give deepening, i.e. an amplitude increase. Secondly, if we have a trough, where the meridional component of the wind is stronger on the back side than on the front side, there will also be a fall. Thirdly, a trough with a small ridge in it, will deepen in its western part.

Similar rules may be stated as to the ridges.



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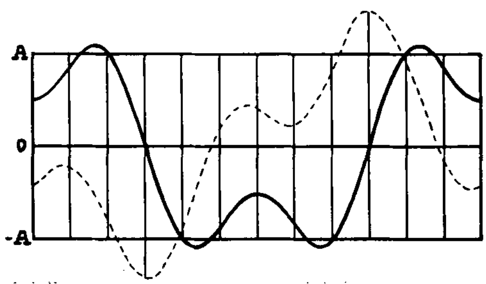

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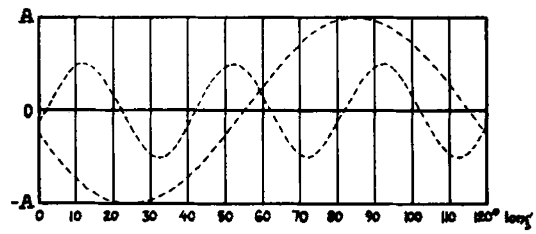

c

Fig. 6 a, b, c. Two sinusoidal profiles of different wave-length and amplitude as well as the result of adding them (full lines), and the corresponding 48 hour forecasts (broken lines), valid when $U = 0$.

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