On the annual variation and spectral distribution of atmospheric energy[1]

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ABSTRACT

A study is made of the annual variation of the six components, A_Z, A_E, K_E, K_Z, K_S, and K_M, of the energy in the atmosphere based on data from a complete year, February 1963–January 1964, and on eight vertical levels. The distribution of the energy with respect to pressure and time of the year is investigated with the result that all forms of energy have a maximum in the beginning of the year and a minimum during the month of July.

The energy conversions $C(A_Z, A_E)$, $C(K_E, K_Z)$ and $C(K_S, K_M)$ are also investigated during the period. The most remarkable result is the annual variation of $C(K_E, K_Z)$ showing a maximum during the fall, a phenomenon also observed during the previous year.

Fourier analysis of the data in time is employed to determine the amplitude and phase of the first few components. The first component contains a considerable fraction of the annual variations, but the second component is in certain cases, as for example $C(K_E, K_Z)$, very important in describing the annual behavior. A similar analysis is made of three years of data with essentially the same results. The latter set of energy data is used to determine the annual variation of the generation of available potential energy and the frictional dissipation using the energy equations, while the data for 1963 are used to calculate the energy conversion $C(A, K)$ and the generation $G(A)$ based on the energy equations and an assumption concerning the frictional dissipation.

The last section of the paper contains a description of the results of an investigation of the energy spectra showing that in the range of wave numbers $8 \leq n \leq 15$ we can approximate the form of the spectra by a power law: $E = a \cdot n^{-b}$. a and b are determined by regression analysis.

1. Introduction

During the past few years, the present author has been engaged in diagnostic calculations of atmospheric energy and energy conversions based on standard numerical analyses of routine meterorological data (Wiin-Nielsen, Brown & Drake, 1963, 1964; and Wiin-Nielsen, 1964). The calculations have been arranged to give information about the spectral distribution of the various forms of energy and energy conversions by calculating in the wave number regime up to and including longitudinal wave number 15. Most of the published results have been based on 5 pressure levels (85, 70, 50, 30 and 20 cb). Beginning in February 1963 the National Meteorological Center of the U.S. Weather Bureau began to analyse 8 levels on a daily basis adding the 100, 15, and 10 cb levels to those mentioned above. It was therefore decided to extend the calculations of the amounts of energy and some of the energy conversions to cover a complete year for which data existed at eight levels. The period selected was February 1963–January 1964.

The main purpose of the present paper is to report the results of the calculations and to make an analysis of the time variation of the amounts of energy and the energy conversions. We shall also investigate some properties of the spectra, in particular the functional relationship between the amount of energy and the wave number.

2. Annual variations

The following quantities have been computed for each day of the year, February 1963–January 1964.

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1. The zonal available potential energy, A_Z.
2. The eddy available potential energy, A_E.
3. The zonal kinetic energy, K_Z.
4. The eddy kinetic energy, K_E.
5. The kinetic energy in the vertical mean flow, K_M.
6. The kinetic energy in the vertical shear flow, K_S.
7. The energy conversion from zonal to eddy available potential energy, $C(A_Z, A_E)$.
8. The energy conversion from eddy to zonal kinetic energy, $C(K_E, K_Z)$.
9. The energy conversion from the vertical shear flow to the vertical mean flow, $C(K_S, K_M)$.

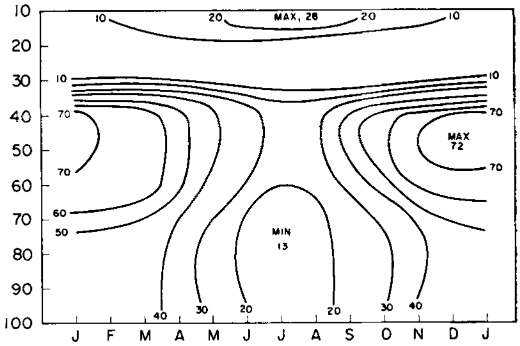

Fig. 1. The zonal available potential energy A_Z as a function of pressure and time of the year. Unit: $\text{kjm}^{-2}\ \text{cb}^{-1}$. Data: Feb. 1963–Jan. 1964.

The numerical procedures which have been used to calculate the quantities A_Z, A_E, K_Z and K_E in the wave number regime are described by Wiin-Nielsen (1964), while the procedures used in connection with the computations of K_S, K_M and $C(K_S, K_M)$ are given in two papers by Wiin-Nielsen & Drake (1965, 1966). The method of calculation used to obtain $C(A_Z, A_E)$ and $C(K_E, K_Z)$ has followed the description given by Wiin-Nielsen *et al.*, (1963, 1964). In all cases we have made the obvious changes due to the higher vertical resolution in the new data.

It is apparent from the list given above that important energy conversions such as $C(A, K)$ and all energy generations $G(A)$ and dissipations $D(K)$ have not been included in the present study. The reason is naturally that additional simplifying assumptions, such as adiabatic conditions, must be made to obtain $C(A, K)$ while other empirical assumptions are necessary to compute $G(A)$ and $D(K)$ although such calculations have been made. Brown (1964) extended an earlier study by Wiin-Nielsen & Brown (1960) to cover a nine-month period obtaining the atmospheric heat sources and the generations $G(A_Z)$ and $G(A_E)$ for each month. These calculations were made using only two levels of data (85 and 50 cb), and there is unfortunately no obvious way in which these calculations can be extended to the higher vertical resolution with sufficient accuracy although such calculations should be attempted in the future. Kung (1966) has attempted to calculate the frictional dissipation of kinetic energy $D(K)$ over the North American continent obtaining the dissipation $D(K)$ as a residual in the equation describing the rate of change of kinetic energy in an open volume. As pointed out by Kung (loc. cit.) it is very difficult, if not impossible, to extend his calculation to cover the northern hemisphere because of sparsity of data. We shall comment on the magnitudes and variations of $G(A)$ and $D(K)$ in a later section of this paper.

We are first going to consider the annual variations of the various forms of energy. The amounts of energy in the atmosphere have earlier been expressed as energy per unit area because of the limited geographical extent of the data region. The unit for energy amounts has therefore been kjm^{-2}. In the following figures we want to express the amounts of energy as a function of pressure and time. It is therefore natural to use the unit: $\text{kjm}^{-2}\text{cb}^{-1}$ which is the unit used in Figs. 1–7 of this paper. It may be instructive to express the amounts in the unit: energy per unit mass which has the dimension of a square of velocity. If this is desired, it is only necessary to multiply the numbers appearing in Figs. 1–7 by the acceleration of gravity or about 10 m sec^{-2}, in which case the energy unit becomes $\text{m}^2\ \text{sec}^{-2}$. The same procedures applied to any energy conversion, generation or dissipation as in Figs. 8–13 will result in the unit $\text{m}^2\ \text{sec}^{-3}$.

Fig. 1 shows the zonal available potential energy A_Z as a function of time and pressure for the period February 1963–January 1964. The monthly mean values of A_Z were used to prepare this figure, and the data for January 1964 has been replotted on the left side of the figure. It is seen that the maximum amounts of A_Z occur in winter while a minimum is found in the lower part of the troposphere during the

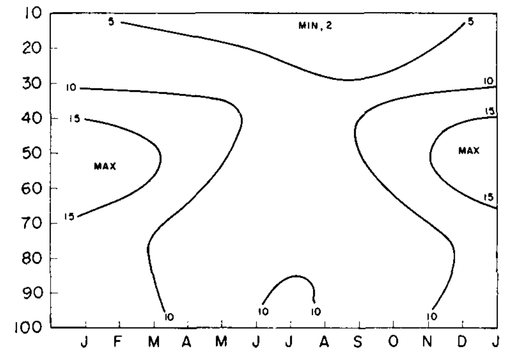

Fig. 2. The eddy available potential energy A_E as a function of pressure and time of the year.

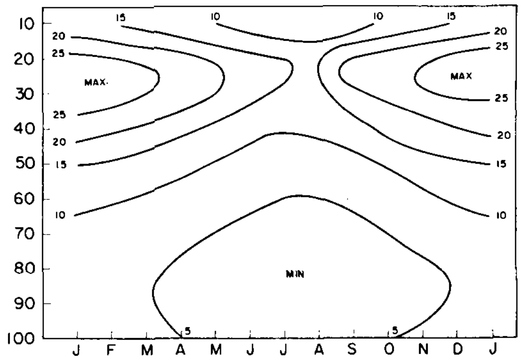

Fig. 4. The eddy kinetic energy K_E as a function of pressure and time of the year.

summer. The level of the primary maximum is found a little above 50 cb. There is a minimum in A_Z around 20 cb with a secondary maximum appearing between 10 and 15 cb during summer. During the month of July we find the greatest amount of A_Z at this level because the maximum in the lower stratosphere coincides with the minimum in the lower troposphere. The secondary maximum in the stratosphere is connected with the large temperature difference between pole and equator during summer with the warmer temperatures appearing at the high latitudes in these stratospheric layers.

Fig. 2 shows the distribution of eddy available potential energy as a function of time and pressure in an arrangement similar to Fig. 1. We notice the presence of a winter maximum at about 50 cb with a secondary maximum during the summer at 100 cb. Disregarding this level there is a minimum during the summer. There is in particular no tendency for a secondary maximum in the lower stratosphere corresponding to the maximum in A_Z. This question will be reconsidered in connection with the energy conversion $C(A_Z, A_E)$.

The annual variation of the zonal kinetic energy K_Z is shown in Fig. 3 as a function of pressure. We observe a well developed winter maximum at 20 cb with the largest values appearing in February. The zonal kinetic energy decreases rapidly in the lower stratosphere. A summer minimum appears at all pressure levels. The values of K_Z in July at 20 cb are only 20 % of the maximum values in February at the same level. At 100 cb we find very small values of K_Z amounting to only 2–3 % of the values at 20 cb.

The configuration of the isolines for the eddy kinetic energy K_E shown in Fig. 4 as a function of time and pressure is in many respects similar to the configuration in Fig. 3. The minimum values of K_E are found in the lower troposphere

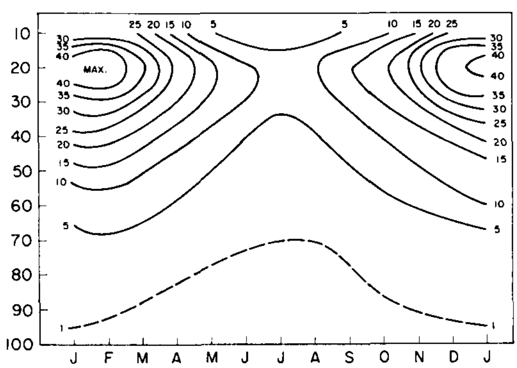

Fig. 3. The zonal kinetic energy K_Z as a function of pressure and time of the year.

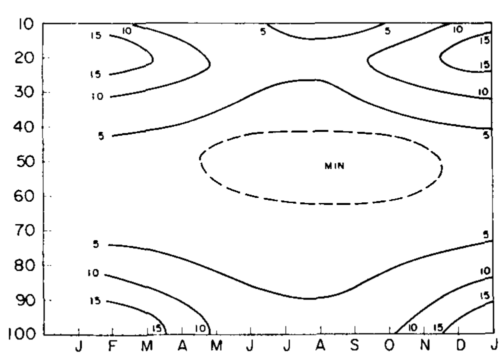

Fig. 5. The shear flow kinetic energy K_S as a function of pressure and time of the year.

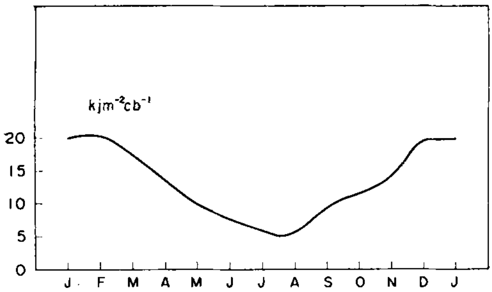

Fig. 6. The mean flow kinetic energy K_M as a function of time of the year.

during the summer. A comparison of Figs. 3 and 4 shows that $K_E < K_Z$ at 20 cb while the opposite relation holds in the lower half of the troposphere.

The kinetic energy K_S of the vertical shear flow which is defined as the deviation from the vertical average of the horizontal wind (Wiin-Nielsen, 1962) is shown in Fig. 5. The main feature of this figure is the well-pronounced minimum throughout the year at 50 cb. This minimum is due to the fact that the 50 cb flow is a reasonably good approximation to the vertical

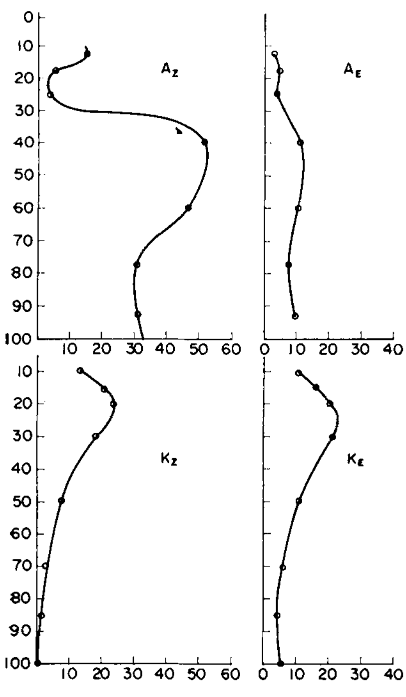

Fig. 7. Vertical profiles of the annual averages of A_Z, A_E, K_Z, and K_E as a function of pressure.

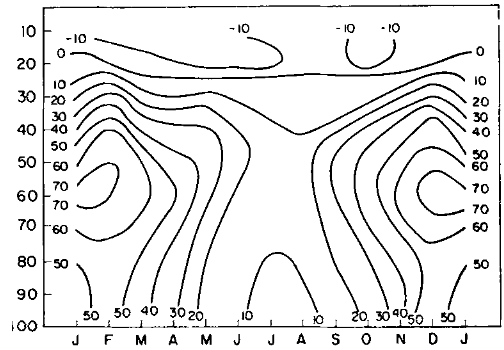

Fig. 8. The energy conversion, $C(A_Z, A_E)$ as a function of pressure and time of the year. Unit: 10^{-6} kjm^{-2} sec^{-1} cb^{-1}.

mean flow, and that the shear flow at 50 cb therefore is weak. In each vertical column we find therefore maxima at the ground (100 cb) and at 20 cb. The annual variation is characterized by winter maxima and summer minima in the lower and upper troposphere.

The last component of the kinetic energy which we consider in this study is the kinetic energy of the vertical mean flow K_M. This energy component is not a function of pressure, and its annual variation can therefore be shown as a function of time as given in Fig. 6. K_M is at almost all times of the year and at all pressure levels larger than the corresponding value of K_S.

The different energy components as a function of pressure are shown in Fig. 7 which was constructed on the basis of the annual mean values of A_Z, A_E, K_Z, and K_E. Fig. 7 shows good agreement with similar curves computed from four months of data during the year 1962 (Wiin-Nielsen, 1964). We notice in particular that the secondary maximum of A_Z in the lower stratosphere (10–15 cb) is well established, that there is a slight tendency for a secondary maximum at the same level in A_E, and that the vertical variation in K_E below 60 cb is rather small.

The next subject of this section will be a description of the annual variation of the energy conversions $C(A_Z, A_E)$, $C(K_E, K_Z)$ and $C(K_S, K_M)$ as computed from the data for the period February 1963–January 1964. In Fig. 8 $C(A_Z, A_E)$ is shown as a function of time and pressure. It is first of all seen that the maximum value of this energy conversion is found in winter at about 60 cb. The minimum value during summer at the same level is less than 20 % of the

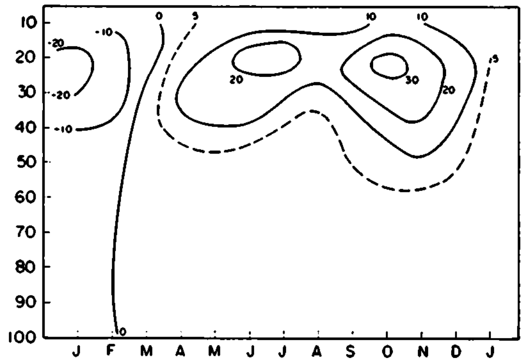

Fig. 9. The energy conversion $C(K_E, K_Z)$ as a function of pressure and time of the year.

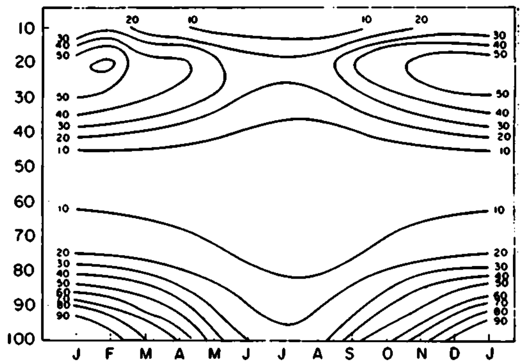

Fig. 10. The energy conversion $C(K_S, K_M)$ as a function of pressure and time of the year.

maximum value during winter. We notice next that the contributions to $C(A_Z, A_E)$ from layers above 50 cb decrease rapidly and become zero at about 20 cb during most of the year. The contribution from the layers above 20 cb is negative with the numerically largest values appearing in spring and fall. The negative contributions to $C(A_Z, A_E)$ are consistent with the secondary maximum found in the annual distribution of A_Z in the lower stratosphere (see Fig. 1) during summer. We may in fact say that the negative values of $C(A_Z, A_E)$ helps to produce the secondary maximum in A_Z. The reversal of the energy conversion $C(A_Z, A_E)$ in the lower stratosphere as compared to the troposphere is mainly due to the fact that the higher temperatures at these levels are found at the high latitudes and the lower temperatures at low latitudes compared with the observation that the heat transport is from south to north (White, 1954; Oort, 1964).

In contrast to $C(A_Z, A_E)$ shown in Fig. 8 we find that the energy conversion $C(K_E, K_Z)$ tends to have its maximum value in the upper part of the troposphere at about 20 cb as seen in Fig. 9. This figure shows also a number of other noticeable features. During February and March we find negative values of $C(K_E, K_Z)$ at least at the upper levels. The energy conversion $C(K_E, K_Z)$ has been investigated earlier for January 1963, the month preceding the present period. $C(K_E, K_Z)$ was negative at all levels during this month. We notice next that $C(K_E, K_Z)$ has its maximum positive values during summer and fall. In comparison with $C(A_Z, A_E)$ there is a marked difference in the time of the maximum values. One might think that the summer and fall maxima of $C(K_E, K_Z)$ seen in Fig. 9 could be due to a small sample of only one year of data. Although our records of $C(K_E, K_Z)$ indeed are very short, so that we can hardly speak of a climatological average, it is pertinent to mention that the data from January, April, July and October 1962 also show a maximum during the fall season (Wiin-Nielsen *et al.*, 1964). It is consequently of interest to calculate $C(K_E, K_Z)$ from meteorological data covering many years in order to establish whether or not there in general is a maximum of $C(K_E, K_Z)$ in summer and fall. We notice finally from Fig. 9 that there are relatively minor contributions to $C(K_E, K_Z)$ from levels below 50 cb throughout the year. Notice that the data for January 1963 have been reproduced at the left side of Fig. 9 because of the anomalous behavior of this winter month, although the calculation for this month is based on only 5 levels.

Fig. 10 shows the energy conversion between the vertical shear flow and the vertical mean flow $C(K_S, K_M)$ as a function of time and pressure. $C(K_S, K_M)$ has a minimum in the middle troposphere just as the kinetic energy K_S. We find maxima in the upper and lower troposphere during winter with relative minima during summer. There is a high degree of similarity between the distributions of K_S and $C(K_S, K_M)$.

As a supplement to Figs. 8, 9, and 10 we have prepared figures showing the vertical variations of the three energy conversions $C(A_Z, A_E)$, $C(K_E, K_Z)$ and $C(K_S, K_M)$. These curves were computed from the annual averages for the period February 1963–January 1964. Fig. 11 shows the energy conversion $C(A_Z A_E)$, while Figs. 12 and 13 illustrate the vertical variation

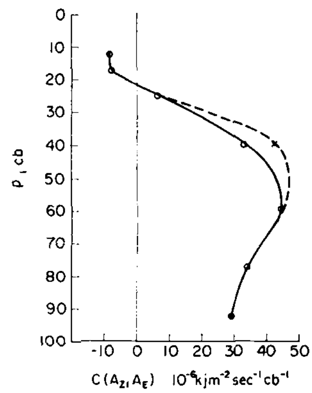

Fig. 11. Vertical profile of the annual mean of $C(A_Z, A_E)$. Dashed curve based on data from 1962.

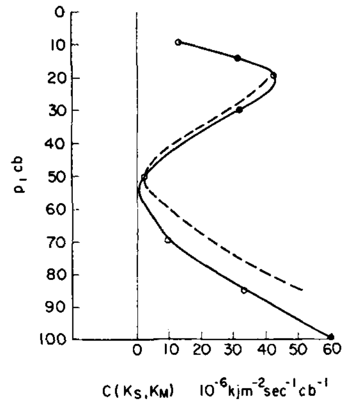

Fig. 13. Vertical profile of the annual mean of $C(K_S, K_M)$. Dashed curve based on data from 1962.

of the two kinetic energy conversions $C(K_E, K_Z)$ and $C(K_S, K_M)$. These curves clearly demonstrate that a considerable vertical resolution is needed in the data in order to obtain reliable estimates of all the conversions. Many earlier estimates have been based on data from only one or two isobaric levels. Fig. 11 should be compared with the corresponding figure appearing in Krueger, Winston & Haines (1965). It is seen that the extrapolation performed by these authors to obtain values above 20 cb is erroneous because they failed to recognize the negative values of $C(A_Z, A_E)$ at those levels. In Figs. 11, 12 and 13 we have furthermore entered curves showing the estimated vertical variations of the energy conversions during 1962 to illustrate variations from one year to another. The data for 1962 consist of the already published results for January, April, July and October (Wiin-Nielsen *et al.*, 1964) combined with additional calculations carried out for the months of November and December. The annual mean value was obtained as a weighted sum of the monthly mean values. All calculations for 1962 are obtained from five levels of data and are restricted to the interval between 85 and 20 cb.

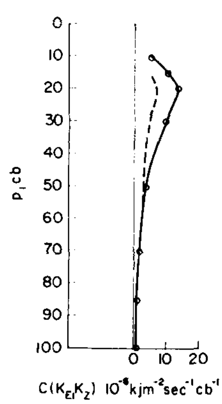

Fig. 12. Vertical profile of the annual mean of $C(K_E, K_Z)$. Dashed curve based on data from 1962.

3. Fourier analysis of energy data

The diagrams presented in the preceding section give a first impression of the variations of various forms of energy and energy conversions during a year. However, the details of the variations are difficult to see using the diagrams. It was therefore found to be of interest to compute the first few Fourier components using one year as the basic period. We have in other words computed the coefficients FC_n and FS_n defined by the expression

$$F(t) = F_0 + \sum_{n=1}^{N} \left[FC_n \cos\left(n \frac{2\pi}{T} t\right) + FS_n \sin\left(n \frac{2\pi}{T} t\right)\right], \qquad (3.1)$$

where F_0 is the annual mean value, while FC_n and FS_n are determined from the integrals

$$FC_n = \frac{2}{T} \int_0^T F(t) \cos\left(n \frac{2\pi}{T} t\right) dt, \qquad (3.2)$$

$$FS_n = \frac{2}{T} \int_0^T F(t) \sin\left(n \frac{2\pi}{T} t\right) dt. \qquad (3.3)$$

Table 1. *Energy and energy conversions, Feb. 1963–Jan. 1964*

Units: Energy in kjm^{-2}, energy conversion 10^{-4} kjm^{-2} sec^{-1}.

	A_Z	A_E	K_E	K_Z	K_S	K_M	$C(A_Z, A_E)$	$C(K_E, K_Z)$	$C(K_S, K_M)$
Jan.	4077	1191	1365	1442	815	1992	37.0	2.91	32.9
Feb.	3968	1112	1299	1541	816	2024	42.1	−5.73	33.0
Mar.	4262	886	1224	1238	717	1745	32.7	0.14	27.3
Apr.	3385	754	1102	829	574	1357	26.3	4.10	24.7
May	2594	705	912	515	452	976	15.5	5.30	16.8
June	2185	627	776	367	374	769	6.81	6.59	11.7
July	1786	654	652	269	330	592	5.84	6.60	9.50
Aug.	1967	542	618	293	312	599	6.13	3.69	9.20
Sept.	2669	609	876	476	402	950	14.2	7.29	17.0
Oct.	3104	774	1001	640	473	1168	23.2	10.40	22.3
Nov.	3957	780	1033	982	573	1442	31.8	9.95	27.4
Dec.	4024	1132	1345	1396	773	1968	43.0	6.15	32.9

In the expressions above, T corresponds to one year. Due to the variations in the energy and energy conversion values from one year to another we should naturally desire to have several years of data in order to determine a climatological value. Lacking this information it is nevertheless interesting to consider the single year of data available at this time. The Fourier coefficients were in all cases computed from the monthly mean values of the total energy or energy conversion.

The data which have been used for the Fourier analysis are summarized in Table 1 in which the energy is given in the unit kjm^{-2} while the conversions are in the unit 10^{-4} kjm^{-2} sec^{-1}.

The results of the Fourier analysis of the data in Table 1 are given in Table 2 for the first four components. Replacing the coefficients FC_n and FS_n, we have preferred to give the amplitude and phase angle, F_n and γ_n defined by the relations:

$$F_n^2 = FC_n^2 + FS_n^2 \tag{3.4}$$

and

$$\tan\left(n\frac{2\pi}{T}\gamma_n\right) = \frac{FS_n}{FC_n}. \tag{3.5}$$

It is easily verified that γ_n gives the time of the maximum of the component. γ_n as presented in Table 2 is in the unit of days where we have used 365 days in the year.

The period February 1963–January 1964 is at the moment the only year for which calculations exist for every month based on a high vertical resolution. The only other extensive calculation of energies and energy conversions is presented by Krueger *et al.* (1965). It covers a much longer period, but is mainly based on data from the levels 85 and 50 cb. We shall later compare our results with this calculation. For the year 1962 calculations have been made for the months: January, April, July, October, November and December, although only the results from the first four months have been published (Wiin-Nielsen *et al.*, 1964; Wiin-Nielsen, 1964; and Wiin-Nielsen & Drake, 1965). Fourier analysis has not been made on these data, but we have estimated the annual mean value as a weighted sum of the results for the six months. These values have been entered in Table 2 in the last column. Comparing the firsl and last column in Table 2 we find in general very good agreement. One notices that the totat available potential energy is slightly smaller in 1962 as compared to 1963, while the total kinetic energy is a little larger in 1962 than in 1963. These changes in the energy levels are due to corresponding changes in the zonal form of the energy, while the eddy energies show changes opposite to the zonal form. Consistent with the changes in the energy levels we find that $C(A_Z, A_E)$ is smaller while $C(K_E, K_Z)$ is larger in 1963 than in 1962.

Each of the energy components has considerable amplitude in the first Fourier component. F_1 is in no case less than one third of F_0, and in some cases we find a ratio of more than one half between F_1 and F_0. The phase angle γ_1 shows that the maximum in the first

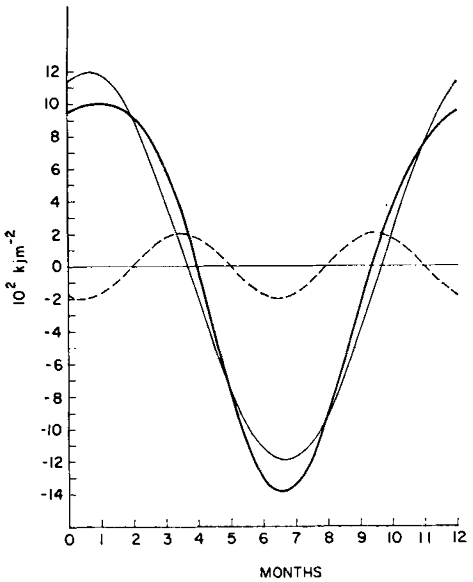

Fig. 14. The first and second Fourier component and their sum for A_Z.

Fourier component is found in the latter part of January with one important exception. The maximum in the first Fourier component of $C(K_E, K_Z)$ is found to be on the 256th day, or in other words on September 13.

It has been noted by Krueger *et al.* (1965) that there are significant differences between the springtime decline and the autumn buildup in several of the energy quantities, in particular A_Z, for which they noted that the springtime decline is smaller than the increase in fall. We can easily test this observation by comparing F_1 and γ_1 with F_2 and γ_2. For A_Z we find first of all that F_2 is a considerable fraction of F_1. The maximum in the first component occurs on January 20, while the maximum in the second component is found on April 15, or almost one quarter of a year later. Superimposing the second Fourier component on the first we thus find the same asymmetry as noted by the authors mentioned above. The same kind of asymmetry is also found in A and K_E, although the amplitude F_2 in K_E is quite small. For A_E and K_Z we find, on the other hand, that there is a tendency for the first maxima in the first and second Fourier component to coincide. This means that there is a tendency for symmetry around the middle of the year, but the summer minimum will be less pronounced than the winter maximum for these energy quantities.

Amplitudes of the third and fourth Fourier components are in general relatively small compared with F_1 and are therefore of smaller importance for the annual variation. The first and second Fourier component together with their sum for A_Z, A_E, K_Z, and K_E are shown in Figs. 14, 15, 16 and 17 illustrating the arguments used above.

It is also of interest to consider the first two Fourier components of the energy conversions. Figs. 18 and 19 show the first and second Fourier components and their sum for $C(A_Z, A_E)$ and $C(K_S, K_M)$. The first Fourier components have by far the larger amplitude for both energy conversions with the maximum in January. The second Fourier component is also very similar for the two conversions with the first maximum occurring about May 1 resulting in a less pronounced winter maximum and a deeper summer minimum. In contrast to $C(A_Z, A_E)$ and $C(K_S, K_M)$ we find that the first Fourier component of $C(K_E, K_Z)$ has its maximum in September. The second component has an amplitude which is almost as large as F_1, but with a maximum in May. The combined effect of F_1

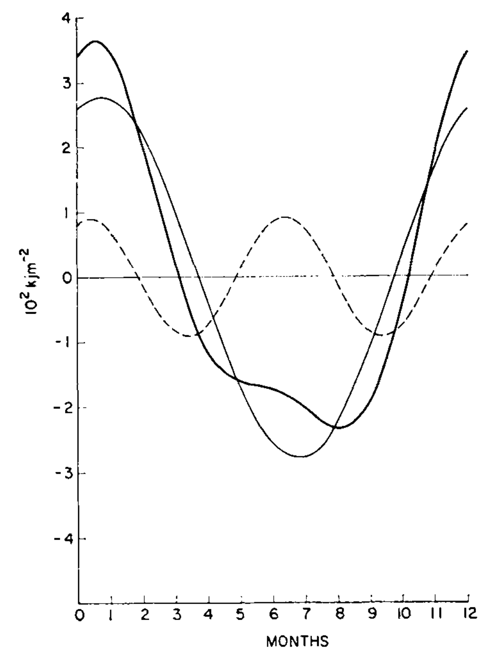

Fig. 15. The first and second Fourier component and their sum for A_E.

and F_2 produced a minimum in February and a maximum in November. An analysis of the data from 1962 published by Wiin-Nielsen *et al.* (1964) shows a first Fourier component with an amplitude of 4.25×10^{-4} kjm^{-2} sec^{-1} and a phase angle of 268 days (Sept. 25), and a second Fourier component with an amplitude of 2.10×10^{-4} kjm^{-2} sec^{-1} and a phase angle of 110 days (April 20). There is therefore qualitative agreement between the results for 1962 regarding the annual variation of $C(K_E, K_Z)$.

In view of the fact that the results described above are for a single year only, it becomes of importance to make similar analyses for a number of years in order to establish if the results obtained for 1963 are peculiar for this year, or if they represent the typical variation of energy through the year. It is unfortunately a very large (and expensive) project to create long records of energetical quantities because they require not only a collection of daily data from many levels, but also extensive computations. The only long record available at the present time is the calculations, mentioned earlier, by Krueger *et al.* (1965). These computations were based on two levels of data, 50 and 85 cb, and will therefore not represent the total atmosphere without certain corrections. We can, however,

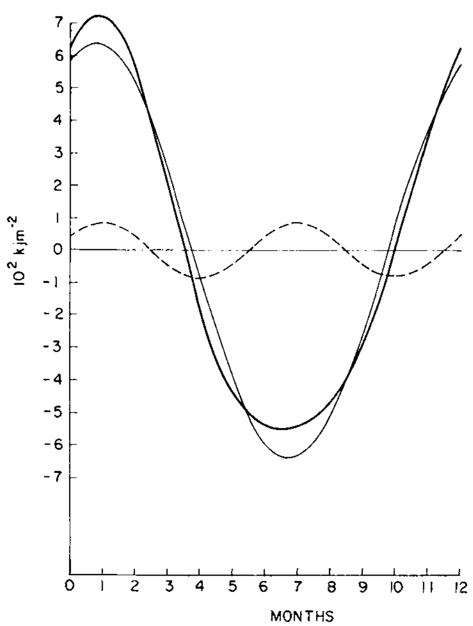

Fig. 16. The first and second Fourier component and their sum for K_Z.

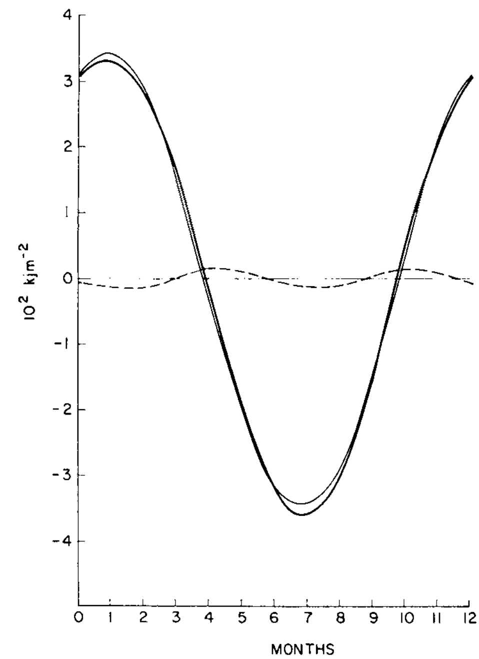

Fig. 17. The first and second Fourier component and their sum for K_E.

use a method of corrections based on the following principle. The ratio of the energy in the total depth of the atmosphere for 1963 to the contribution from the layer 85–50 cb was calculated for each month of the year. Under the assumption that these ratios can be applied to the corresponding months for other years it is possible to convert the data for the 85–50 cb layer to apply to the whole atmosphere. Such a conversion has been applied to Krueger, Winston and Haines' data for the years 1960, 1961, and 1962. The corrected data were then analyzed to obtain the first Fourier components to be compared with the components listed in Table 2. The results of these computations are given in Table 3 in which we have listed the amplitudes and phase angles of components 0, 1 and 2 only. The figures listed in Table 3 for the different years should be compared with each other and with the corresponding figures in Table 2. We notice considerable agreement between the corresponding numbers. The favorable comparison between Table 2 and 3 increases the confidence in the computed numbers in Table 2 although they are based on data from one year only. The only quantities on which no compari-

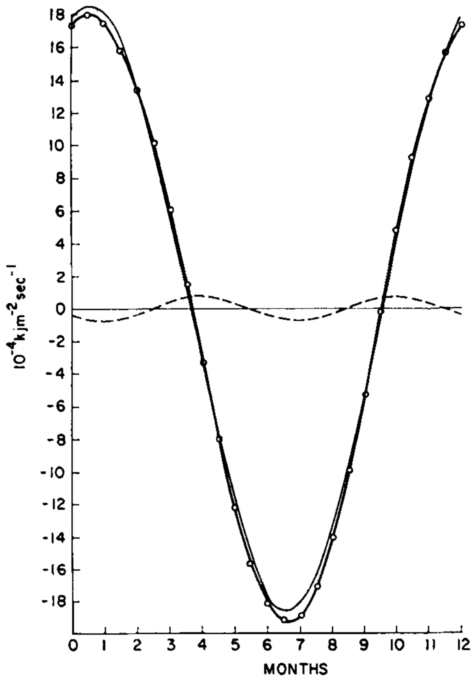

Fig. 18. The first and second Fourier component and their sum for $C(A_Z, A_E)$.

son can be made are K_S, K_M, $C(K_E, K_Z)$ and $C(K_S, K_M)$ which were not computed in the study by Krueger *et al.* (1965) who, on the other hand, included $C(A, K)$ $C(A_E, K_E)$ and $C(A_Z, K_Z)$. With regard to the last conversion we find considerable variations from year to year, while $C(A, K)$ and $C(A_E, K_E)$ show a maximum of the first component in late December or early January, while the first maximum of the second component occurs some time in late April or early May, giving a tendency for a pronounced summer minimum and a rather flat winter maximum.

4. Indirect calculations of energy generation and dissipation

Any estimate of the basic energy generation, energy conversion or dissipation from atmospheric data is very difficult. The generation of available potential energy requires a knowledge of the heat sources; the energy conversion from available potential energy to kinetic energy depends on the vertical velocity or, alternately, a knowledge of the ageostrophic winds, and the dissipation of kinetic energy requires a knowledge of the frictional forces. It is pertinent to point out that if we know one of these quantities we may compute the other two from the energy equations and a knowledge of the time variation of the amounts of available potential energy A and the kinetic energy K according to the equations

$$\frac{dA}{dt} = G - C, \tag{4.1}$$

$$\frac{dK}{dt} = C - D, \tag{4.2}$$

where $G = G(A)$, $C = C(A, K)$ and $D = D(K)$. Equation (4.1) was, for example, used by Krueger *et al.* (1965) to calculate G from a known C and $A = A(t)$ estimating dA/dt by finite differences.

It is interesting to use this approach in somewhat greater detail by using the Fourier analysis technique. As our first example we shall use the data reproduced in Table 3 for A, K and $C(A, K)$ to calculate the resulting distributions of G and D. Using a representation of the type (3.1) for all quantities we obtain by substitution in (4.1) and (4.2) that

$$G_0 = D_0 = C_0 \tag{4.3}$$

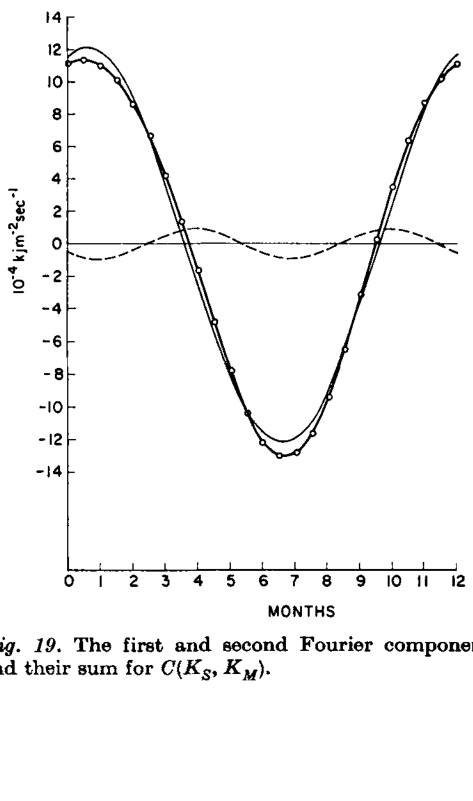

Fig. 19. The first and second Fourier component and their sum for $C(K_S, K_M)$.

Table 2. *Amplitudes and phase angles of Fourier components of energy and energy conversions*
Last column contains averaged values for 1962.

	0	1		2		3		4		0
n	F_0	F_1	γ_1	F_2	γ_2	F_3	γ_3	F_4	γ_4	F_0
A	3978	1474	20	111	108	72	81	130	79	3813
K	1849	982	24	72	28	38	16	83	80	1959
A_Z	3164	1196	20	203	105	107	81	126	75	2915
A_E	814	278	22	93	11	36	22	38	7	898
K_E	1017	346	25	14	126	44	15	58	88	1180
K_Z	832	636	24	86	29	6	69	43	70	779
K_S	551	255	28	31	25	10	14	19	83	648
K_M	1298	727	23	42	31	28	17	64	80	1311
$C(A_Z, A_E)$	23.7	18.6	18	0.7	120	0.8	94	0.6	68	29.9
$C(K_E, K_Z)$	4.8	4.1	256	3.7	137	0.5	98	1.6	5	3.6
$C(K_S, K_M)$	22.1	12.2	18	0.9	119	0.5	14	0.6	2	29.3

and

$$\left.\begin{aligned} -n\cdot\frac{2\pi}{T}\,AC_n &= +GS_n - CS_n, \\ +n\cdot\frac{2\pi}{T}\,AS_n &= GC_n - CC_n, \\ -n\cdot\frac{2\pi}{T}\,KC_n &= CS_n - DS_n, \\ +n\cdot\frac{2\pi}{T}\,KS_n &= CC_n - DC_n. \end{aligned}\right\} \qquad (4.4)$$

We can use (4.3) and (4.4) to calculate G_0, D_0, GC_n, GS_n, DC_n, and DS_n knowing the Fourier coefficients of A, K and C. Such a calculation has been carried out using the averaged distribution of A, K and C for the years 1960, 1961, and 1962 from Table 3. Expressing the data in amplitude and phase angle we obtain the results given in Table 4 for $n = 0$, 1 and 2. It is seen that the maximum value of G for $n = 1$ occurs at $\gamma_1 = 354$ days (December 20), while the maximum values of D for $n = 1$ is found on January 17. There is thus a lag of almost a month between the maximum generation and dissipation as measured by the first Fourier component. We note further that the maximum dissipation for $n = 1$ coincides nearly with the maximum kinetic energy. The values of the amplitudes of C, G

Table 3. *Amplitudes and phase angles of Fourier components of energy and energy conversions during 1960, 1961, and 1962*

Year ...	1960					1961					1962				
	0	1		2		0	1		2		0	1		2	
n	F_0	F_1	γ_1	F_2	γ_2	F_0	F_1	γ_1	F_2	γ_2	F_0	F_1	γ_1	F_2	γ_2
A	4817	1881	15	364	107	4875	1935	16	280	119	5044	1915	16	161	113
K	1384	828	17	99	23	1412	811	14	74	4	1468	801	18	107	32
A_Z	3923	1437	16	504	102	3887	1509	17	361	111	4010	1412	17	297	103
A_E	895	445	14	158	2	992	431	11	119	1	1035	504	14	155	2
K_E	678	303	13	223	158	733	320	4	52	134	813	363	16	17	33
K_Z	706	527	20	102	29	679	499	20	99	20	655	438	20	90	32
$C(A_Z, A_E)$	27.1	23.4	11	1.8	158	27.2	21.9	13	1.2	115	29.0	22.6	15	0.8	95
$C(A_E, K_E)$	16.7	10.3	12	2.4	126	19.4	11.0	11	4.2	118	23.4	15.5	3	4.5	132
$C(A_Z, K_Z)$	−0.9	1.0	210	0.7	66	−3.1	3.9	238	1.5	144	−5.4	4.4	142	2.1	49
$C(A, K)$	15.8	9.3	11	2.2	118	16.3	8.6	357	5.3	125	18.0	12.6	16	2.5	126

Table 4. *Estimation of $G(A)$ and $D(K)$ from known values of A, K and $C(A, K)$. Averaged data from 1960, 1961, and 1962*

	0	1		2	
n	F_0	F_1	γ_1	F_2	γ_2
A	4912	1910	16	264	111
K	1421	813	16	86	21
$C(A, K)$	16.7	10.0	8	3.3	122
$G(A)$	16.7	11.1	354	3.1	133
$D(K)$	16.7	9.8	17	3.2	119

and D for $n = 2$ are of the same order of magnitude and the maximum of the components occurs in late April and early May. We may compare the results for $G(A)$ in Table 4 with an analogous analysis of the estimates of this quantity made by Brown (1964) for the months of January, April, July and October which enable us to compute at least the first Fourier component. The results are $G_0 = 9.5 \times 10^{-4}$, $G_1 = 5.2 \times 10^{-4}$, $\gamma_1 = 334$ for the year 1962. Although the amplitude and the annual mean value are somewhat lower than those found in Table 4, there is at least qualitative agreement in the phase angle. If we, on the other hand, combine all the values given by Brown (loc. cit.) we find $G_0 = 10.0 \times 10^{-4}$, $G_1 = 5.8 \times 10^{-4}$ and $\gamma_1 = 336$ in close agreement with results for 1962. From the available data we conclude that G_1 has a maximum in December, while D_1 tends to have its maximum about a month later.

The general procedure which was applied in the preceding paragraphs can be extended to cover a more detailed energy conversion scheme. We shall first consider a partition of the total energy in the energy forms: available potential energy A, shear flow kinetic energy K_S, and mean flow kinetic energy K_M. The energy equations are:

$$\left.\begin{aligned} \frac{dA}{dt} &= G(A) - C(A, K), \\ \frac{dK_S}{dt} &= C(A, K) - C(K_S, K_M) - D(K_S) \\ \frac{dK_M}{dt} &= C(K_S, K_M) - D(K_M). \end{aligned}\right\} \quad (4.5)$$

The energy conversion $C(K_S, K_M)$ is only known for an extended period of time during the year 1963 during which we also have calculations of A, K_S, and K_M. In order to solve the equations (4.5) it is necessary to know either $G(A)$, $C(A, K)$ or $D(K) = D(K_S) + D(K_M)$. None of these quantities are known for the period. It is thus necessary to obtain one of them indirectly. In spite of our limited knowledge of the frictional processes in the atmosphere, $D(K)$ is nevertheless the only quantity which we can relate to the quantities which are known for the period. It is seen from the last equation in (4.5) that $D(K_M)$ can be computed from our knowledge of $C(K_S, K_M)$ and K_M. If we therefore can estimate the dissipation $D(K)$ we can compute $D(K_S) = D(K) - D(K_M)$. The middle equation in (4.5) will then give us $C(A, K)$, while $G(A)$ can be computed from the first equation in (4.5).

The energy dissipation in the atmospheric boundary layer may be estimated using the method designed by Lettau (1962) and Kung (1963). The boundary layer dissipation $D_b(K)$ is calculated from the formula:

$$D_b = \varrho C_g V_{g,0}^3, \quad (4.6)$$

where ϱ is density, C_g the geostrophic drag coefficient and $V_{g,0}$ the geostrophic wind in the atmospheric boundary layer. Using the data from the period February 1963 to January 1964 for which 100 cb analyses were available we can compute ϱ and $V_{g,0}^3$, while C_g was taken from Kung (1963). The results of this calculation are listed in Table 5.

It has commonly been assumed that the greater part of the atmospheric kinetic energy dissipation is accounted for by the boundary layer dissipation. The results obtained by Kung (1966) show, on the other hand, a considerable dissipation in the so-called free at-

Table 5. *Boundary layer dissipation,* D_b

	ϱ, 10^3 tm^{-3}	10^3 C_g	$V^3_{g,0}$, m^3 sec^{-3}	D_b, 10^{-4} kjm^{-2} sec^{-1}
Jan.	1.268	0.878	2335	25.99
Feb.	1.262	0.900	2127	24.14
Mar.	1.246	0.928	1617	18.60
Apr.	1.226	0.940	1293	14.91
May	1.205	0.990	1153	13.77
June	1.191	1.050	1209	15.13
July	1.185	1.090	1002	12.95
Aug.	1.191	1.068	1170	14.89
Sep.	1.207	1.010	1864	22.72
Oct.	1.228	0.984	2098	25.34
Nov.	1.248	0.946	2484	29.62
Dec.	1.263	0.916	2949	34.10

mosphere. If we assume that the total dissipation $D = D_b$ we get from (4.5) assuming that the annual mean is a steady state that $D(K_M) = C(K_S, K_M) = 22 \times 10^{-4}$ kjm^{-2} sec^{-1}, $D(K_S) = -1 \times 10^{-4}$ kjm^{-2} sec^{-1} and $C(A, K_S) = G(A) = 21 \times 10^{-4}$ kjm^{-2} sec^{-1}. Kung (1966) finds $D(K) = 63.8 \times 10^{-4}$ kjm^{-2} sec^{-1}, $D_b = 18.7 \times 10^{-4}$ kjm^{-2} sec^{-1} and $D_f = 45.1 \times 10^{-4}$ kjm^{-2} sec^{-1} where D_f is the dissipation outside the boundary layer. These values are probably high because they were obtained over North America. However, if we assume the same ratio between D_f and D_b as found by Kung (loc. cit.) we get $D_f = 50.6 \times 10^{-4}$ kjm^{-2} sec^{-1} and a total dissipation $D(K) = 71.6 \times 10^{-4}$ kjm^{-2} sec^{-1} which in turn leads to a value of $D(K_S) = 49.6 \times 10^{-4}$ kjm^{-2} sec^{-1}. Using again the assumption of steady state conditions for the annual mean we find $G(A) = C(A, K) = 71.6 \times 10^{-4}$ kjm^{-2} sec^{-1}. Such values of $G(A)$ and $C(A, K)$ are several times larger than independent estimates of these quantities. The residence times of A and K_S using these values of $G(A)$ and $C(A, K)$ would be 6.4 days and 0.9 days. Although considerable uncertainty exists with respect to numerical values of D_f, $D(K_M)$ and $D(K_S)$ it is unlikely that they are as large as estimated by Kung (loc. cit.) and we shall therefore restrict our calculations to smaller values.

Assuming that $D = D_b$ we can compute the values of the annual mean and the amplitudes and phases of the Fourier components of $D(K_M)$, $D(K_S)$, $C(A, K)$, and $G(A)$ from the known values of $C(K_S, K_M)$, A, K_S and K_M using the equations (4.5). The results of such a calculation are given in Table 6 including only the first Fourier component under heading A. These values were computed using the values of D listed in Table 5. In view of the fact that the horizontal variation of ϱ and C_g in (4.6) is considerably smaller than that of $V_{g,0}$ it is interesting to make a calculation in which C_g

Table 6. *Calculation of mean values and first component in three cases using different values of dissipation*

	A			B			C		
	0	1		0	1		0	1	
n	F_0	F_1	γ_1	F_0	F_1	γ_1	F_0	F_1	γ_1
$G(A)$	21.0	12.8	328	37.2	16.5	2	18.6	9.3	354
$C(A, K)$	21.0	10.4	336	37.2	15.9	12	18.6	8.2	5
$C(K_S, K_M)$	22.1	12.2	18	22.1	12.2	18	22.1	12.2	18
$D(K_s)$	−1.1	8.8	252	15.1	3.8	1	−3.5	4.4	214
$D(K_M)$	22.1	12.2	24	22.1	12.2	24	22.1	12.2	24

and ϱ are constant. Calculation B listed in Table 6 was made using $\varrho = 1.3 \times 10^{-3}$ tm^{-3} and $C_g = 3 \times 10^{-3}$ simulating the values used by Phillips (1956), while calculation C was made using a total dissipation half the value in calculation B.

We shall next compare the values appearing in Table 6 with independent calculations of $C(A, K)$ and $G(A)$. Calculation B is inconsistent with the results obtained by Krueger *et al.* (1965) because the mean value of $G(A, K)$ is too large. Calculation C, on the other hand, is in reasonably good agreement with these results (see Table 4) because both the annual mean value and amplitude and phase of the first component are of the same order of magnitude. The agreement between the results in calculation A and the results from observational studies is less satisfactory with respect to $C(A, K)$ mainly because of the phase angle γ_1. Comparing the values of $G(A)$ obtained by Brown (1964) giving $G_0 = 9.5 \times 10^{-4}$ kjm^{-2} sec^{-1}, $G_1 = 5.2 \times 10^{-4}$ kjm^{-2} sec^{-1} and $\gamma_1 = 334$ days we find somewhat better agreement with calculation A than with calculation C again judging on γ_1, but it should be realized that the values based on observations are obtained for four months only. In short, the calculations A and C compare about equally well with observational studies in most respects.

Concluding this section we shall finally mention that analogous calculations can be made when we subdivide in the four energy reservoirs A_Z, A_E, K_Z and K_E. In this case we must again estimate the kinetic energy dissipations $D(K_Z)$ and $D(K_E)$ from empirical formulas and use the observational studies of $C(A_Z, A_E)$ and $C(K_E, K_Z)$. We are then able to compute the annual mean values and the Fourier components of $C(A_Z, K_Z)$, $C(A_E, K_E)$, $G(A)$ and $G(A_E)$. Such a study is presently in progress.

5. Energy spectra

Several investigations have been made of the energy spectra for the large-scale, geostrophic flow in the atmosphere. Ogura (1958) found that the momentum transport $\overline{u_E v_E}$ where u_E and v_E are the deviations from the zonal averages u_Z and v_Z, is small compared with $\overline{u_E^2}$ and $\overline{v_E^2}$ for sufficiently high wave numbers. This conclusion is supported by the results obtained by Wiin-Nielsen *et al.* (1963) who computed the momentum transport as a function of wave number.

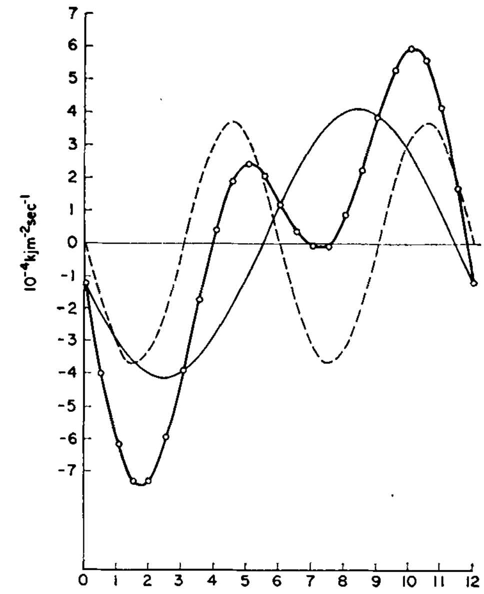

Fig. 20. The first and second Fourier component and their sum for $C(K_E, K_Z)$.

Horn & Bryson (1963) investigated the relation between the eddy kinetic energy K_E and the wave number n and found that the spectrum of K_E can be approximated by a relation $K_E = a \cdot n^{-b}$ where a and b are constants determined by regression calculations from the data. The relation $K_E = a \cdot n^{-b}$ was fitted to the data after the energy spectrum was subdivided into two classes, one containing the smaller wave numbers and the other the larger wave numbers. Two different regression lines were calculated, one for each group of wave numbers, and it was found that the power law approximates the data quite well for the group containing the larger wave numbers. The values of b determined by Horn & Bryson (1963) range from 2.3 to 2.9 with an average value of slightly less than 8/3. They found furthermore that the values of b in general are larger in the upper troposphere (30 cb) than at lower elevations (50 and 70 cb). The investigations made so far have been restricted to data samples of less than one year, and all investigators suggest the desirability of further investigations. In this section we shall describe the results obtained by an analysis of the spectra for the period February 1963–January 1964. This period is not only longer than those represented by earlier investigators but we have also a higher vertical resolution using eight data levels.

We shall first investigate the spectrum of eddy kinetic energy K_E as a function of longitudinal wave number n. The spectrum $K_E(n)$ was computed for each day of the period February 1963–January 1964. The mean spectrum for the year was obtained by averaging the daily spectra. The procedures used in the calculations are described in Wiin-Nielsen & Drake (1965). In view of the fact that we want to fit the empirical relation to the data we write the relation in the form

$$\log K_E = \log a - b \log n, \qquad (5.1)$$

log a and b may then be determined by simple linear regression techniques.

When log K_E is plotted as a function of log n it is clearly seen that the portion corresponding to large values of n can be approximated by a straight line. Fig. 21 shows that the partitioning in the two regimes takes place around wave number 7. The straight line on the right hand

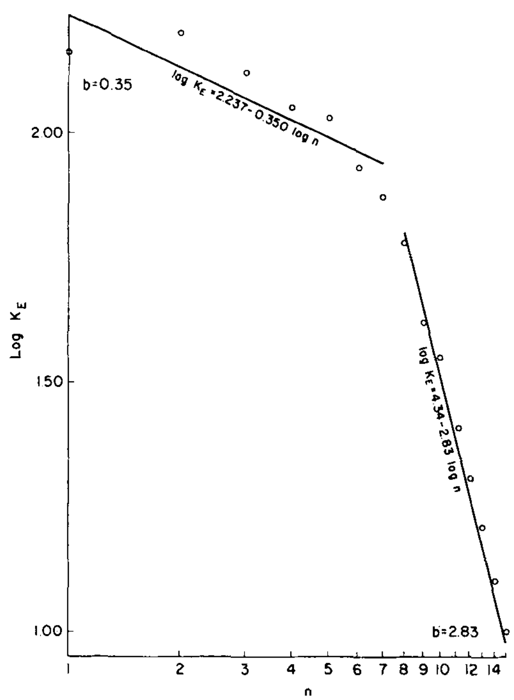

Fig. 21. The eddy kinetic energy K_E for the total atmosphere as a function of wave number n plotted on logarithmic scales.

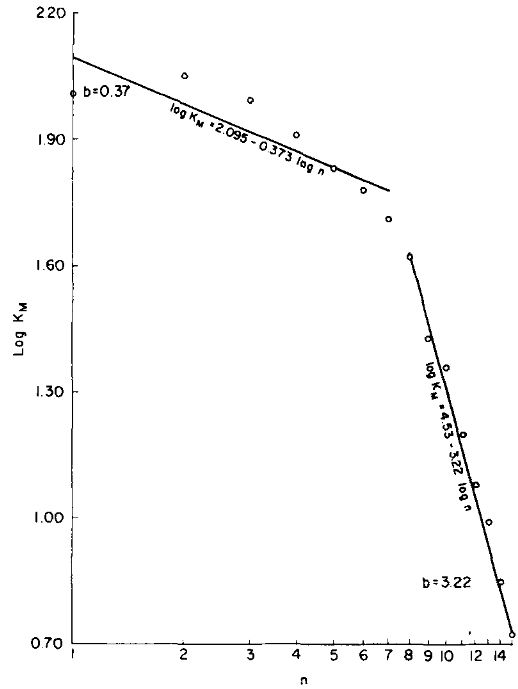

Fig. 22. The mean flow kinetic energy K_M for the total atmosphere as a function of wave number n plotted on logarithmic scales.

side of Fig. 21 was obtained using the data $8 \leq n \leq 15$ and has a slope of $b = 2.8$ which is in good agreement with the mean value obtained by Horn & Bryson (1963). The regression line fitted to the wave number group $1 \leq n \leq 7$ is not intended to show any linear relation in this range, but shows merely the change in the relations in the two groups. Even if the regression line for the group $8 \leq n \leq 15$ is a good approximation to the observed data it should be realized that errors will occur if we want to use the regression line to estimate the energy. However, no error is larger than 10 % of the observed kinetic energy $K_E(n)$.

It is of interest to investigate if the spectral relation found for K_E also holds for the barotropic and baroclinic components of the atmospheric flow. The barotropic component is defined as the vertical mean flow, while the baroclinic component is the deviation from the vertical mean flow (Wiin-Nielsen, 1962). The data for the kinetic energy in the wave number regime of K_M and K_S were treated as described above for K_E, and the data are presented in Figs. 22 and 23, showing that the spectra

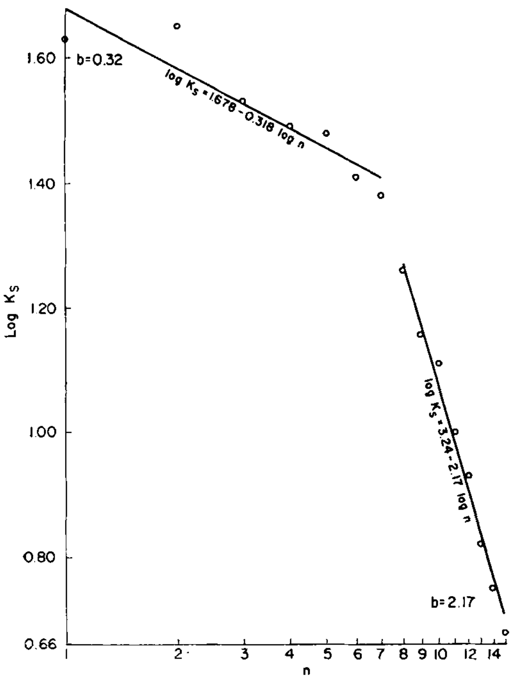

Fig. 23. The shear flow kinetic energy K_S for the total atmosphere as a function of wave number n plotted on logarithmic scales.

$K_M(n)$ and $K_S(n)$ can be approximated with good accuracy using relations of the type (5.1). We find that the regression line for K_M has a larger slope than the line representing K_E, while the slope for K_S is smaller than for K_E. The effect of forming the vertical average is therefore as one would expect that relatively small amounts of energy are left on the smaller scales of motion. The smaller slope for K_S is most likely due to the influence of the smaller scale motion which is present in the lower part of the troposphere. It is, however, quite clear from the figures that the partitioning in all three spectra is of the same nature with the power law applying in the range $8 \leqslant n \leqslant 15$.

The results described in the preceding paragraphs are in agreement with the study made by Horn & Bryson (1963) both with respect to the slope of the regression line for K_E in the range $8 \leqslant n \leqslant 15$ and with respect to the wave number which divides the spectrum into two portions. It is not evident at the present time why the partitioning takes place and why the slopes have the observed numerical values. The fact that the atmosphere is baroclinically unstable with a maximum instability around $n = 6$ is undoubtedly related to the form of the spectra. From a two level quasi-geostrophic model one finds that all waves are stable provided the wave number n is sufficiently large. The critical wave number n_c is determined from the relation

$$n_c = a \cos \varphi \frac{f}{P} \sqrt{\frac{2}{\sigma}}, \tag{5.2}$$

where a is the radius of the earth, f the Coriolis parameter $P = 50$ cb and $\sigma = -\alpha\ \partial \ln\theta / \partial p$ the static stability. For $\varphi = 45°$ N, $f = 10^{-4}$ sec^{-1} and $\sigma = 2.5$ MTS-units we get $n_c = 8$ indicating that the waves $8 \leqslant n \leqslant 15$ correspond to the stable waves in the atmosphere. It is thus tempting to use the analogy that this range of wave numbers is similar to the equilibrium range in homogeneous turbulence in which there is no direct conversion from available potential to kinetic energy, and where the form of the spectrum is determined by the frictional dissipation and non-linear exchange processes. However, a much more detailed study of these exchange processes combined with a deeper understanding of frictional dissipation is needed before such an analogy can be established. The analogy is furthermore not complete because

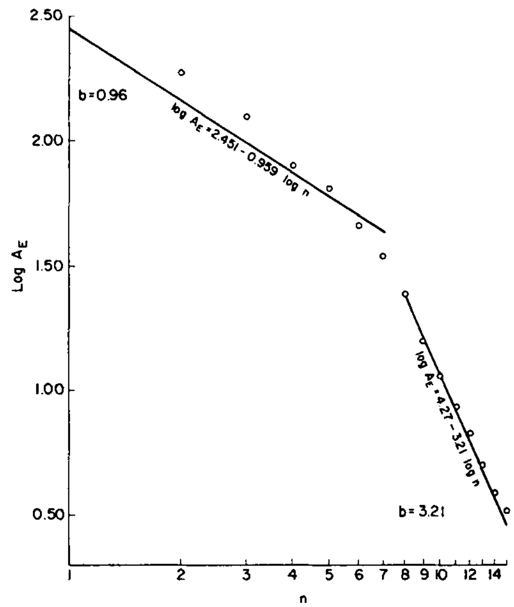

Fig. 24. The available potential energy A for the total atmosphere as a function of wave number n plotted on logarithmic scales.

Table 7. *Values of the slope b as a function of pressure for the eddy kinetic energy K_E*

p ...	20	30	50	70	85	100
b	+ 2.8	+ 3.1	+ 2.9	+ 2.6	+ 2.4	+ 1.9

the homogeneous turbulence theory gives $b = +1.7$, a value smaller than found in these studies of the large scale motion.

We note finally that the eddy available potential energy A_E should display similar partitioning of the energy if the description given above is correct. Fig. 24 shows the data for A_E and the regression lines fitted to the observed data. We find a regression line for $8 \leq n \leq 15$ with a value of $b = 3.2$.

It has been noted by Horn & Bryson (1963) that the slope b tends to increase with decreasing pressure. They investigated the three levels 70, 50 and 30 cb at three latitudes 25, 45 and 65° N. Although our data are averaged over all latitudes from 20° N to the Pole, it is nevertheless possible and of interest to investigate the dependence on pressure. The values of b are given in Table 7 for the various pressure levels. It is seen that b in general increases with decreasing pressure confirming the earlier results and indicating that we in general find relatively more energy on the smaller scales in the lower part of the troposphere. We may also investigate the variation of b with pressure for A_E. The results of this investigation are given in Table 8. We find again a general increase of b with decreasing pressure in agreement with the results for K_E. The relatively small value of b in the layer 20–30 cb is related to the relatively small amounts of A_E in this layer.

It has been pointed out by Ogura (1957) that the spectral distribution of energy is influenced by the finite difference techniques which are employed when the geostrophic winds are calculated from the height field. Although the spectral distributions in this study were obtained by a one-dimensional Fourier analysis of the streamfunction, along the latitude circles we are nevertheless using finite difference techniques in the meridional direction, and our energy data should therefore be corrected for this effect. Due to the complexity of the numerical technique which has been applied, it is not possible to find a simple influence function as the one used by Ogura (1958) in his study of the spectra at 30 cb. It can, however, be seen from Ogura's study (loc. cit.) that the correctioı will tend to decrease the value of the slope b.

We should furthermore stress that our energy data are obtained from numerically analysed maps which have been smoothed using various smoothing operators, Shuman (1957). Since the smoothing operators are designed to eliminate small scale irregularities in the analysed fields, and since it is extremely difficult to construct filters which leave the longer waves unaltered, it is possible that the smoothing will change the spectral distribution of energy for the high wave numbers. It is the purpose of the following paragraphs to illustrate this effect by a simple example.

Let us consider an elementary one-dimensional operator of the form:

$$\bar{\psi} = \psi + \alpha \frac{\partial^2 \psi}{\partial \lambda^2}, \tag{5.3}$$

where ψ is the streamfunction, λ is longitude and α is a constant. Let the streamfunction ψ at a certain latitude and pressure have a Fourier expansion

Table 8. *Values of the slope b as a function of pressure for eddy available potential energy A_E*

Layer ...	10–15	15–20	20–30	30–50	50–70	70–85	85–100
b	3.3	3.5	2.7	3.5	3.3	2.8	2.4

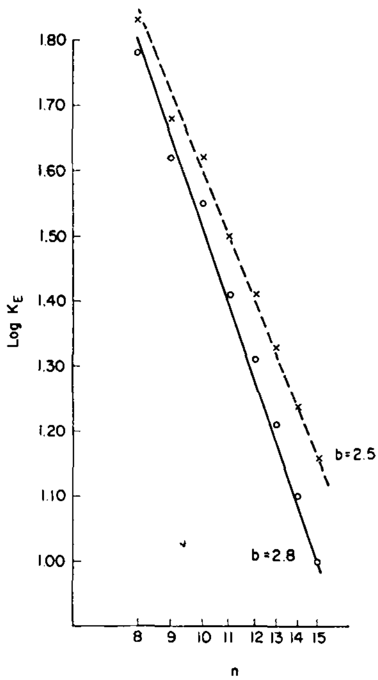

Fig. 25. Solid line and circles reproduced from Fig. 21. Dashed line and crosses are data of kinetic energy corrected for smoothing by a simple smoothing operator.

$$\psi = \psi_0 + \sum_{n=1}^{\infty} [A_n \cos(n\lambda) + B_n \sin(n\lambda)]. \quad (5.4)$$

Substituting (5.4) in (5.3) we find that the Fourier expansion of ψ is

$$\bar{\psi} = \psi_0 + \sum_{n=1}^{\infty} [(1 - \alpha n^2)(A_n \cos(n\lambda) + B_n \sin(n\lambda))]. \quad (5.5)$$

α is determined in such a way that the amplitude of a certain wave number $n = n_*$ is reduced to zero. We find

$$\alpha = \frac{1}{n_*^2}. \quad (5.6)$$

The kinetic energy of the meridional component of the wind per unit area and per unit pressure interval is

$$K = \frac{1}{8\pi a^2 \cos^2\varphi} \cdot \frac{1}{g} \sum_{n=1}^{\infty} [n^2(A_n^2 + B_n^2)] \quad (5.7)$$

while the corresponding kinetic energy $\bar{K}$ calculated from the smoothed streamfunction is

$$K = \frac{1}{8\pi a^2 \cos^2\varphi} \cdot \frac{1}{g} \sum_{n=1}^{\infty} \left[\left(1 - \frac{n^2}{n_*^2}\right)^2 n^2 (A_n^2 + B_n^2)\right]. \quad (5.8)$$

We find thus that

$$\bar{K}_n = \left(1 - \frac{n^2}{n_*^2}\right)^2 K_n. \quad (5.9)$$

If the operator (5.3) was applied to the "true" analysis we would thus find values of the kinetic energy $\bar{K}_n$ smaller than the "true" values K_n. Assuming that we use (5.3) we can correct the calculated values of $\bar{K}_n$ by multiplying these values by $(1 - (n^2/n_*^2))^{-2}$. To see how much this can influence the regression line fitted to the data $8 \leq n \leq 15$ we have corrected the computed values for K_E in Fig. 21 and determined the regression line for the corrected data using $n_* = 36$. The original data, denoted by circles, and the corrected data, denoted by crosses, are entered on Fig. 25 together with the two regression lines. The slope changes from $b = 2.8$ to $b = 2.5$. Although the example illustrates the influence of a simple smoothing operator, it is likely that the more complex operators used on the analysed fields will have a similar effect. Due to the special care which is taken to avoid a significant reduction of the amplitude of the larger scales it is, however, likely that the effect will be smaller than the change found in our example.

6. Concluding remarks

It has been attempted in this study to make a rather detailed analysis of the variation of energetical quantities through one year and to estimate the energy conversions, generations and dissipations consistent with the computed quantities. The study may be considered as a first step in the direction of obtaining a climatology of atmospheric energetics. A comparison with other available studies based on longer time periods, but smaller vertical resolution adds confidence to the results obtained for a single year.

Any observational study will normally focus the attention on a number of problems which require a solution. One of the most interesting problems presented by the results of this study is the general behavior of the energies and energy conversions through the year, in particular the asymmetries relative to the solar cycle. It is evident from Table 2 that the maxima in all forms of energy occur about one month after the winter solstice. A first step to reproduce the annual variation from a numerical experiment has been made by Kraus & Lorenz (1966), who were able to obtain a lag of the right order of magnitude from a relatively simple two level model including seasonal variations in the forcing function. One of the more surprising aspects of the energy conversions is the tendency for a maximum in $C(K_E, K_Z)$ during the fall. This phenomenon which was present in both 1962 and 1963 has not been explained by the numerical experiments, but it should be possible to reproduce it using a model similar to the one employed by Kraus & Lorenz (loc. cit.)

The indirect calculations presented in section 4 leave much to be desired. The most important problem is to relate the frictional dissipation to the atmospheric flow, and to obtain better knowledge of frictional dissipation in the so-called free atmosphere as compared to the atmospheric boundary layer. It is characteristic for studies of the atmospheric energetics that our knowledge is still very limited concerning the most basic part of the atmosphere: the intensity of the general circulation, which could be measured by $G(A)$, $C(A, K)$ or $D(K)$. None of these quantities are known with great accuracy including the yearly mean values and the annual variation.

The results of the empirical studies of the energy spectra seem to indicate that the range of wave numbers $8 \leqslant n \leqslant 15$ forms a group for which a simple power law gives a good approximation to the spectra. These wave numbers coincide with the waves which are either baroclinically stable or only slightly unstable according to stability theories. Although there are reasons to believe that the partitioning of the energy spectra is caused by the stability properties of the atmospheric flow, it has not yet been possible to account for the numerical value of the exponent in the power law.

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О ГОДОВЫХ ВАРИАЦИЯХ И СПЕКТРАЛЬНОМ РАСПРЕДЕЛЕНИИ ЭНЕРГИИ АТМОСФЕРЫ

Проведено изучение годовых изменений шести компонент энергии атмосферы: A_Z, A_E, K_E, K_Z, K_S, K_M на основе данных по восьми вертикальным уровням за целый год (февраль 1963 — январь 1964). Исследовано распределение энергии относительно давления и времени года, получено, что все формы энергии имеют максимум в начале года и минимум в течение июля месяца.

Для этого периода также были исследованы преобразования энергии $C(A_Z, A_E)$, $C(K_E, K_Z)$ и $C(K_S, K_M)$. Наиболее замечательным результатом является годовое изменение $C(K_E, K_Z)$, показывающее максимум в течение осени, явление наблюдавшееся также во время предшествующих лет.

Был выполнен анализ Фурье данных по времени с целью определения амплитуды и фазы первых нескольких компонент. Первая компонента содержит значительную долю годовых изменений, но вторая компонента в определенных случаях, как например для $C(K_E, K_Z)$, очень важна сля описания годового поведения. Подобный анализ проделан также для трехлетних данных и показал существенно те же самые результаты. Последний ряд данных об энергии использовался с целью определить годичные изменение генерации доступной потенциальной энергии за счет сил трения при использовании энергетических уравнений, тогда как данные за 1963 год использовались для подсчетов преобразований энергии $C(A, K)$ и генерации $G(A)$ на основе уравнений энергии и некоторого предположения о характере диссипации за счет трения.

Последняя часть статьи содержит описание результатов исследования спектра энергии, показывающего, что в интервале волновых чисел $8 \leqslant n \leqslant 15$ форму спектра можно аппроксимировать степенным законом: $E = an^{-b}$. Величины a и b определены с помощью регрессионного анализа.