Mean meridional circulation and large-scale eddies

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ABSTRACT

The mean meridional circulation is maintained by the eddy terms, $\overline{v'(\partial u'/\partial y)}$ and $\overline{w'(\partial u'/\partial z)}$. The magnitude of these terms in different latitudes is discussed.

Introduction

The mean meridional motion should satisfy the equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - fv + gw$$

$$-\frac{uv}{a}\tan\varphi + \frac{uw}{a} = -\frac{1}{\varrho}\frac{\partial p}{\partial x} + F_x \qquad (1)$$

averaged along latitude circle and over long time interval. X and Y are curvilinear distances along latitude and longitude circles. a is the earth's radius. F_x represents frictional forces. $g = 2\,\omega\cos\varphi$. uw/a can be neglected in comparison with other terms.

The averaged equation is

$$\left\{\bar{\varrho}\left(\bar{v}\frac{\partial\bar{u}}{\partial y} - f\bar{v} + \overline{w}\frac{\partial\bar{u}}{\partial z} + g\overline{w} - \frac{\bar{u}\bar{v}}{a}\tan\varphi\right)\right\}$$

$$+\left[\varrho', \frac{\partial u'}{\partial t}\right] + \left[\varrho', \frac{\partial u^{2'}/2}{\partial x}\right] + \left[\varrho', v', \frac{\partial u'}{\partial y}\right]$$

$$-f[\varrho', v'] + g[\varrho', w'] + \left[\varrho', w', \frac{\partial u'}{\partial z}\right]$$

$$-\frac{\tan\varphi}{a}[\varrho', u', v'] = \bar{\varrho}\overline{F}_x. \qquad (2)$$

Bar indicates average along latitude circle and over time. Primed quantities are departures from the average. The terms in the square brackets are the means of the terms involving the departures and arise from large scale eddies. If these eddy terms and the mean frictional force are insignificant, the first term (hereafter referred to as $\bar{\varrho}A$) will define the mean meridional motion in relation to the zonal motion.

Data

The mean meridional motion has been mapped directly from observational data only recently by Tucker (1959) and Palmen & Vuorela (1963). The data of latter authors, though for the winter period only, are more complete, particularly for the highest levels. Using the values of $\bar{u}$ from Heastie & Stephenson (1960) and $\bar{v}$ and $\overline{w}$ from Palmen & Vuorela (1963), calculated values of A for Winter are shown in Table 1. Most of the values are substantial, showing that the large-scale eddy terms are important for balancing the mean meridional circulation. This is in conformity with the conclusion of several earlier studies that the mean circulation is maintained through the eddies. M. S. V. Rao (1963) computed average values of A for the whole year using for $\bar{v}$ and $\overline{w}$ weighted means from the results of Mintz and Tucker. Rao's (1963) values are broadly in agreement with those in Table 1. In equation (2), the term $\bar{\varrho}(\partial\bar{u}/\partial t)$, has been taken as zero. This may be justified for the period December to February, as the change in the mean zonal winds from the beginning to the end of the period is small. For the present we shall neglect the frictional term and consider its magnitude later.

Magnitude of eddy terms

It is possible to estimate the upper limit of the eddy terms from the standard deviations of the quantities. If m' and n' represent the deviation of two sets of quantities from their means, $|\overline{m'\,n'}|$ cannot be greater than $\sigma_m\,\sigma_n$. Similarly in the case of three sets of quantities (m,n,p), $|\overline{m'\,n'\,p'}|$ cannot be greater than $\sigma_m\,\sigma_n\,\sigma_p$.

The term A in equation (1) has to be com-

Table 1. *Values of* $A = (\bar v(\partial\bar u/\partial y) - \bar v f + \overline{w}(\partial\bar u/\partial z) + g\overline{w} - (\tan\varphi/a)\bar u\bar v)$

Latitude: North. Unit: 10^{-4} cm $\sec^{-2}$.

Pressure (mb.)	70°	65°	60°	50°	40°	35°	30°	25°	20°	15°	10°	5°
150	14	28	26	0	0	6	−15	−33	−32	−23	−28	−11
200	28	42	66	11	0	0	−15	−44	−58	−49	−37	−8
250	28	56	80	23	0	−3	−8	−29	−43	−54	−35	3
300	28	56	66	37	11	−6	−8	−8	−31	−54	−28	6
400	28	44	40	20	8	−3	−8	−14	−31	−36	−8	8
500	28	28	25	20	8	−3	−6	−19	−31	−28	−6	0
600	28	−14	14	8	8	−3	−6	−19	−33	−22	−3	0
700	14	−14	−14	−3	0	−3	−14	−28	−39	−17	6	6
800	−56	−67	−50	−3	−22	−39	−36	−44	−19	8	25	19
850	−70	−81	−78	−3	−36	−50	36	−36	−11	42	58	19
900	−70	−81	−103	−44	0	−3	36	64	103	108	86	28
950	−10	−81	−103	−89	19	25	75	108	141	130	92	22

pared with $1/\bar\varrho$ of the eddy terms. Hence the terms to be evaluated are

$$\frac{1}{\bar\varrho}\left|\left[\varrho', \frac{\partial u'}{\partial t}\right]\right| \ngtr \frac{\sigma_\varrho}{\bar\varrho}\sigma_{\partial u/\partial t} \approx \frac{\sigma_T}{\bar T}\sigma_{\partial u/\partial t},$$

$$\frac{1}{\bar\varrho}\left|\left[\varrho', \frac{\partial u'^2/2}{\partial x}\right]\right| \ngtr \frac{\sigma_\varrho}{\bar\varrho}\sigma_{(\partial u^2/2)/\partial x} \approx \frac{\sigma_T}{\bar T}\sigma_{(\partial u^2/2)/\partial x},$$

$$\frac{1}{\bar\varrho}\left|\left[\varrho', v', \frac{\partial u'}{\partial y}\right]\right| \ngtr \sigma_v\sigma_{\partial u/\partial y} + \frac{\sigma_\varrho}{\bar\varrho}\bar v\sigma_{\partial u/\partial y} + \frac{\sigma_\varrho}{\bar\varrho}\frac{\partial\bar u}{\partial y}\sigma_v + \frac{\sigma_\varrho}{\bar\varrho}\sigma_v\sigma_{\partial u/\partial y}$$

$$\approx \sigma_v\sigma_{\partial u/\partial y} + \frac{\sigma_T}{\bar T}\bar v\sigma_{\partial u/\partial y} + \frac{\sigma_T}{\bar T}\frac{\partial\bar u}{\partial y}\sigma_v + \frac{\sigma_T}{\bar T}\sigma_v\sigma_{\partial u/\partial y},$$

$$\frac{f}{\bar\varrho}\left|[\varrho', v']\right| \ngtr f\frac{\sigma_\varrho}{\bar\varrho}\sigma_v \approx f\frac{\sigma_T}{\bar T}\sigma_v,$$

$$\frac{g}{\bar\varrho}\left|[\varrho', w']\right| \ngtr g\frac{\sigma_\varrho}{\bar\varrho}\sigma_w \approx g\frac{\sigma_T}{\bar T}\sigma_w,$$

$$\frac{1}{\bar\varrho}\left|\left[\varrho', w', \frac{\partial u'}{\partial z}\right]\right| \ngtr \sigma_w\sigma_{\partial u/\partial z} + \frac{\sigma_\varrho}{\bar\varrho}\overline{w}\sigma_{\partial u/\partial z} + \frac{\sigma_\varrho}{\bar\varrho}\frac{\partial\bar u}{\partial z}\sigma_w + \frac{\sigma_\varrho}{\bar\varrho}\sigma_w\sigma_{\partial u/\partial z}$$

$$\approx \sigma_w\sigma_{\partial u/\partial z} + \frac{\sigma_T}{\bar T}\overline{w}\sigma_{\partial u/\partial z} + \frac{\sigma_T}{\bar T}\frac{\partial\bar u}{\partial z}\sigma_w + \frac{\sigma_T}{\bar T}\sigma_w\sigma_{\partial u/\partial z},$$

$$\frac{\tan\varphi}{a}\left|[\varrho', u', v']\right| \ngtr \frac{\tan\varphi}{a}\left(\sigma_u\sigma_v + \frac{\sigma_\varrho}{\bar\varrho}\bar u\sigma_v + \frac{\sigma_\varrho}{\bar\varrho}\bar v\sigma_u + \frac{\sigma_\varrho}{\bar\varrho}\sigma_u\sigma_v\right)$$

$$\approx \frac{\tan\varphi}{a}\left(\sigma_u\sigma_v + \frac{\sigma_T}{\bar T}\bar u\sigma_v + \frac{\sigma_T}{\bar T}\bar v\sigma_u + \frac{\sigma_T}{\bar T}\sigma_u\sigma_v\right).$$

In the above equations $\sigma_\varrho/\bar\varrho$ has been approximated by $\sigma_T/\bar T$, thereby neglecting the changes in density due to pressure variations at a constant height level, in comparison with temperature fluctuations, which may be justified. Crossley (1950) has pointed out that standard deviation of temperature at constant pressure is smaller than that at constant height. Hence values of standard deviation from Goldie, Moore & Austin (1958) have been increased by 10 per cent and adopted as σ_T for the present computations.

For obtaining $\sigma_{\partial u/\partial y}$ between points Y_1 and Y_2, it can be shown that

$$\frac{\sigma_{u_2}+\sigma_{u_1}}{y_2-y_1} > \sigma_{\partial u/\partial y} > \frac{\sigma_{u_2}-\sigma_{u_1}}{y_2-y_1}.$$

A similar procedure is adopted to evaluate the limits of $\sigma_{\partial u/\partial z}$. Assuming that σ_u data available are from daily observations, it is taken that

Table 2. *Mean values and standard deviations*

	Position			
	250 mb (60° N)	900 mb (60° N)	200 mb (20° N)	900 mb (15° N)
$\overline{T}$ (°A)	217	255	219	293
$\bar{u}$ (m sec^{-1})	13.9	2.1	24.7	−5.1
$\bar{v}$ (m sec^{-1})	−0.6	0.8	1.5	−3.0
$\bar{w}$ (cm sec^{-1})	0.0	0.0	−0.5	0.0
σ_T (°A)	5.0	6.0	3.0	2.0
σ_u (m sec^{-1})	15.0	10.0	16.0	6.0
σ_v (m sec^{-1})	15.0	10.0	16.0	6.0
$\sigma_{\partial u/\partial y}$ (10^{-6} sec^{-1})	27 to 3	17 to 1	29 to 1	10 to 1
$\sigma_{\partial u/\partial z}$ (10^{-3} sec^{-1})	10 to 1	24 to 0	9 to 0	23 to 0
$\sigma_{\partial u/\partial t}$ (10^{-2} cm sec^{-2})	3.5 to 0	2.3 to 0	3.7 to 0	1.4 to 0
$\sigma_{(\partial u^2/2)/\partial x}$ (10^{-2} cm sec^{-2})	4.7 to 0	1.3 to 0	7.9 to 0	0.7 to 0
$\left\lvert \frac{\partial \bar{u}}{\partial y} \right\rvert$ (10^{-6} sec^{-1})	4.6	1.4	13.4	0.5
$\left\lvert \frac{\partial \bar{u}}{\partial z} \right\rvert$ (10^{-3} sec^{-1}	1.0	0	0.5	0

$$\frac{2\sigma_u}{1\ \text{day}} > \sigma_{\partial u/\partial t} > 0.$$

Assuming normal distribution of a quantity m, it can be shown that

$$\sigma^2_{m^2} = 2\sigma^2_m(\sigma^2_m + 2\overline{m}^2).$$

Hence $$\sigma^2_{u^2/2} = \tfrac{1}{2}\sigma^2_u(\sigma^2_u + 2\bar{u}^2)$$

and $$\frac{2\sigma_{u^2/2}}{\Delta x} > \frac{\sigma_{\partial u^2/2}}{\partial x} > 0$$

where Δx is appropriate length.

Values of $\overline{T}$ and σ_T have been taken from Goldie *et al.* (1958). Values of σ_u and σ_v have been taken from Brooks *et al.* (1950), assuming that either is $1/\sqrt{2}$ of the standard vector deviation. In these publications, the standard deviations of temperature and standard vector deviations are given for 700 mb and aloft. Values for 700 mb may be assumed to hold for lower levels. Starr & White (1954) have given σ_u and σ_v for the whole year 1950 at latitudes 13, 31, 42.5, 55 and 70° N. At 13 and 31°, σ_v is less than σ_u but equal at higher latitudes. σ_w data are not available.

It is considered sufficient to evaluate the eddy terms at the points where A is high. Four points have been selected for the purpose, viz. 250 mb 60° N, 900 mb 60° N, 200 mb 20° N and 900 mb 15° N. Mean values and standard deviations at these points are given in Table 2. In Table 3 are given the values of the various eddy terms, excepting those involving σ_w. Table 4 gives the value of σ_w required to make the concerned eddy terms (involving σ_w) equal to A.

Some explanation is necessary regarding the evaluation of upper limit of standard deviations such as $\sigma_{\partial u/\partial y}$. It may seem that by making $(Y_2 - Y_1)$ sufficiently small, any high value can be obtained for the upper limit. But this is fallacious. The upper limit for $\sigma_{\partial u/\partial y}$ is based upon the assumption that u values at Y_2 and Y_1 are perfectly negatively correlated while the lower limit is on the basis that they are perfectly positively correlated. When $(Y_2 - Y_1)$ is made very small, the positive correlation between u_1 and u_2 will increase and σ_{u1} will tend to σ_{u2} so that $\sigma_{\partial u/\partial y}$ will tend to zero. 10° interval

Table 3. *Magnitudes of components of eddy terms*

All values less than 0.1 are shown as zero. Unit: 10^{-4} cm sec^{-2}.

	Position			
Eddy term	250 mb (60° N)	900 mb (60° N)	200 mb (20° N)	900 mb (15° N)
$\frac{\sigma_T}{\bar{T}}\sigma_{\partial u/\partial t}$	8 to 0	5.4 to 0	5.1 to 0	1.0 to 0
$\frac{\sigma_T}{\bar{T}}\sigma_{(\partial u^2/2)/\partial x}$	11 to 0	3.1 to 0	10.9 to 0	0.5 to 0
$\sigma_v\,\sigma_{\partial u/\partial y}$	405 to 45	170 to 10	464 to 16	60 to 6
$\frac{\sigma_T}{\bar{T}}\bar{v}\,\sigma_{\partial u/\partial y}$	0.4 to 0	0.3 to 0	0.6 to 0	0.2 to 0
$\frac{\sigma_T}{\bar{T}}\frac{\partial \bar{u}}{\partial y}\sigma_v$	1.6	0.3	2.9	0.0
$\frac{\sigma_T}{\bar{T}}\sigma_v\,\sigma_{\partial u/\partial y}$	9 to 1	4 to 0.2	6 to 0.2	0.4 to 0
$f\frac{\sigma_T}{\bar{T}}\sigma_v$	44	30	11	2
$\frac{\sigma_T}{\bar{T}}\bar{w}\,\sigma_{\partial u/\partial z}$	0	0	0.6 to 0	0
$\frac{\tan\varphi}{a}\sigma_u\,\sigma_v$	61	27	15	2
$\frac{\tan\varphi}{a}\frac{\sigma_T}{\bar{T}}\bar{u}\,\sigma_v$	1.3	5.7	0.3	0.0
$\frac{\tan\varphi}{a}\frac{\sigma_T}{\bar{T}}\bar{v}\,\sigma_u$	0.0	0.1	0.0	0.0
$\frac{\tan\varphi}{a}\frac{\sigma_T}{\bar{T}}\sigma_u\,\sigma_v$	1.4	0.6	0.2	0.0
A	78	103	56	109

for ΔY, 10° interval for Δx, height interval corresponding to 100 mb for ΔZ, and time interval of one day for Δt have been used in the computations in Table 2.

It is clear from Tables 3 and 4 that only two eddy terms may possibly be of the magnitude of A. They are $\sigma_v\,\sigma_{\partial u/\partial y}$ of $(1/\bar{\varrho})\,[\varrho', v', \partial u'/\partial y]$ and $\sigma_w\,\sigma_{\partial u/\partial z}$ of $(1/\bar{\varrho})\,[\varrho', w', \partial u'/\partial z]$. The maximum possible magnitude of $\sigma_v\,\sigma_{\partial u/\partial y}$ exceeds A at 60° N at both the levels and at 20° N at 200 mb. At 15° N and 900 mb it is about 55 per cent of A. The standard deviations of w required to make $\sigma_w\,\sigma_{\partial u/\partial z}$ equal to A, assuming the maximum value of $\sigma_{\partial u/\partial z}$, is very small and seems probable. In the next section the magnitudes of $\overline{v'(\partial u'/\partial y)}$ and $\overline{w'(\partial u'/\partial z)}$, of which $\sigma_v\,\sigma_{\partial u/\partial y}$ and $\sigma_w\,\sigma_{\partial u/\partial z}$ are respectively the possible upper limits, are further examined from actual data.

Table 4. *Magnitude of σ_w required to make the eddy term equal to A*
(Unit: Cm Sec^{-1}.)

Eddy term	Position 250 mb (60° N)	900 mb (60° N)	200 mb (20° N)	900 mb (15° N)
$\sigma_w \, \sigma_{\partial u/\partial z}$	0.78 to 7.8	0.43 to ∞	0.63 to ∞	0.47 to ∞
$\frac{\sigma_T}{\overline{T}} \frac{\sigma \bar{u}}{\partial z} \sigma_w$	339	∞	816	∞
$\frac{\sigma_T}{\overline{T}} \partial_w \, \sigma_{\partial u/\partial z}$	33 to 335	18 to ∞	46 to ∞	69 to ∞
$g \frac{\sigma_T}{\overline{T}} \sigma_w$	4460	5760	2980	11300

From Table 4 it may seem that the values of σ_w required to make $(\sigma_T/\overline{T})\, \sigma_w \, \sigma_{\partial u/\partial z}$ equal to A, assuming maximum $\sigma_{\partial u/\partial z}$, are within plausible limits. But the rather high value of this term arises from the product $\sigma_w \, \sigma_{\partial u/\partial z}$ and in that case the contribution from $\sigma_w \, \sigma_{\partial u/\partial y}$ will far outweigh that from $(\sigma_T/\overline{T})\, \sigma_w \, \sigma_{\partial u/\partial z}$.

Values of $(\tan \varphi/a)\sigma_u \sigma_v$ at 60° N in Table 3 may suggest that $(\tan \varphi/a)\, \overline{u'\, v'}$ may make some contribution to the value of A at high latitudes. Studies of Starr & White (1954) show that the correlation coefficient between u' and v' does not exceed 0.2 and is quite often of the order of 0.1. Hence its contribution will not be more than one fifth of the value indicated in Table 3. The term $f(\sigma_T/\overline{T})\, \sigma_v$ also seems large. But $\overline{T'\, v'}$ has been evaluated in studies on eddy transport of sensible heat and has not been found to exceed 10^3 °A cm sec^{-1}. Hence $(f/\bar{\varrho})\, \overline{\varrho'\, v'} \approx (f/\overline{T})\; \overline{T'\, v'}$ may not exceed 5×10^{-4} cm sec^{-2}.

Evaluation of $\overline{v'(\partial u'/\partial y)}$ and $\overline{w'(\partial u'/\partial z)}$ from actual data

$\overline{v'(\Delta u'/\Delta y)}$ has been calculated from the data of Lerwick (01°11′ W, 60°08′ N) and Shanwell (02°52′ W, 56°26′ N). Data of January for the three years 1962, 1963 and 1964 have been used. Values of $\overline{v'(\Delta u'/\Delta y)}$ which would correspond to the mean latitude of 58° N are shown in Table 5. In this table are also given the mean value of A at 60° N and the expected total of the eddy terms which should be equal and opposite to A. $\overline{v'(\Delta u'/\Delta y)}$ is negative from 900 to 100 mb while the expected total of the eddy terms is positive from 900 to 700 mb and negative aloft. The expected total is the average along the latitude circle while the calculated value refers to a point. However, we proceed on the assumption that the calculated values from Lerwick and Shanwell may be broadly representative of the latitude circle. The expected value of the eddy terms is practically the sum $\overline{v'(\partial u'/\partial y)} + \overline{w'(\partial u'/\partial z)}$. Hence $\overline{w'(\partial u'/\partial z)}$ may be expected to be positive from 900 to 600 mb and negative from 300 to 200 mb. At 500, 400 and 150 mb its contribution may be small. Direct evaluation from observations of $\overline{w'(\partial u'/\partial z)}$ is not possible as w values corresponding to u observations are not known. The sign of the term $\overline{w'(\partial u'/\partial z)}$ can however be deduced indirectly. When sky is clear or lightly clouded w is either negative or a small positive value. Higher positive values of w will be associated with heavy cloudiness. Hence $\Delta\bar{u}/\Delta z$ has been evaluated for Lerwick separately for occasions when clouding is 3/8 or less and when it is more. Between 900 and 250 mb (except between 400 and 500 mb) $\Delta\bar{u}/\Delta z$ is more with greater clouding, suggesting positive correlation between w and $\Delta u/\Delta z$ which would lead to positive values of $\overline{w'(\partial u'/\partial z)}$. Between 250 and 150 mb $\Delta\bar{u}/\Delta z$ is less on occasions of more clouds and the contribution of $\overline{w'(\partial u'/\partial z)}$ will be negative. This change from greater wind shear on more cloudy occasions in the lower troposphere to lesser wind shear in the

Table 5. *Magnitude of $\overline{v'(\Delta u'/\Delta y)}$ and $\Delta\bar{u}/\Delta z$ from data of Lerwick (60° N) and Shanwell (56°N)*

Pressure (mb)	A at 60° N (10^{-4} cm sec^{-2})	Expected total of eddy terms (10^{-4} cm sec^{-2})	$\overline{v'(\Delta u'/\Delta y)}$ at 58° N (10^{-4} cm sec^{-2})	$(\Delta\bar{u}/\Delta z)$ at Lerwick, 10^{-4} sec^{-1}		$\sigma_{\Delta u/\Delta z}$ (10^{-4} sec^{-1})
				Clouding $\frac{3}{8}$ or less	Clouding $> \frac{3}{8}$	
900	−103	103	−57			
				1.3	14.8	59
850	−78	78	−56			
				0.0	10.8	42
800	−50	50	−57			
				6.0	6.0	29
700	−14	14	−59			
				7.1	9.7	29
600	14	−14	−57			
				5.8	11.7	29
500	25	−25	−39			
				16.2	9.9	33
400	39	−39	−30			
				8.1	12.4	35
300	64	−64	−11			
				2.1	15.6	34
250	78	−78	−23			
				10.1	6.3	42
200	64	−64	−36			
				17.0	9.1	39
150	25	−25	−40			
				15.8	16.0	21
100			−9			

upper troposphere fitting broadly with the requirements in the values of $\overline{w'(\partial u'/\partial z)}$ is itself an interesting feature. To fit the present data of expected totals of the eddy terms and values of $\overline{v'(\Delta u'/\Delta y)}$, it would have been better if the change in the sign of correlation between w and $\Delta u/\Delta z$ had taken place at about 500 mb instead of 250 mb as observed from present data of Lerwick. This difference may arise out of considering data of one station only instead of several stations along the latitude circle. The order of magnitude of σ_w required can also be estimated on some simple assumptions. Between 900 and 850 mb, the expected total of the eddy terms is about $(103+78)/2 \approx 91 \times 10^{-4}$ cm sec^{-1}. As $\overline{v'(\Delta u'/\Delta y)}$ is $-(56+57)/2 \approx -57 \times 10^{-4}$ cm sec^{-1}, $w'(\partial u'/\partial z)$ should be $+148 \times 10^{-4}$ cm sec^{-1}. $\overline{w'(\partial u'/\partial z)}$ may be written as $r\,\sigma_w\,\sigma_{\partial u/\partial z}$ where r is the correlation coefficient. Assuming $r = +0.1$ and taking $\sigma_{\Delta u/\Delta z} = 59 \times 10^{-4}$ sec^{-1} calculated from Lerwick data, σ_w will come to 25 cm sec^{-1}. Similarly between 250 and 200 mb, assuming negative correlation coefficient of 0.1, σ_w comes to 10 cm. These are plausible values.

For comparison with the values of A in low latitudes, Madras (80°11′ E, 13°00′ N) has been chosen. Combining Madras data with Visakhapatnam, it is found that the contribution of $\overline{v'(\partial u'/\partial y)}$ is insignificant. For assessing $\overline{w'(\partial u'/\partial z)}$, $\Delta\bar{u}/\Delta z$ has been computed separately for instances of 3/8 or less cloudiness and when cloudiness is more. Data of January for the three years 1962 to 1964 have been used. Table 6 presents the results. Up to about 3 Km the expected value of eddy terms is negative. To satisfy this, $\Delta\bar{u}/\Delta z$ under more cloudy conditions should be less than with clear or little cloudiness. Madras data show this to be the case between 0.6 and 2.1 Km. Above 3 Km eddy terms are expected to be positive and consequently $\Delta\bar{u}/\Delta z$ greater with more cloudiness. Madras data show this trend between 2.1 and 7.2 Km (except in the layer 5.4 to 6.0 Km). Though this difference with cloudiness should continue aloft up to the tropopause, Madras data show between 7.2 and 12 Km lesser values of $\Delta\bar{u}/\Delta z$ with greater cloudiness. In the layers 7.2 to 9.0 Km and 10.5 to 12.0 Km the differ-

Table 6. *Magnitude of $\Delta\bar{u}/\Delta z$ at Madras (13° N)*

Height (Km)	A at 13° N (10^{-4} cm sec^{-2})	Expected total of eddy terms (10^{-4} cm sec^{-2})	$\Delta\bar{u}/\Delta z$ (10^{-4} sec^{-1}) Clouding $\frac{3}{8}$ or less	Clouding $> \frac{3}{8}$	$\sigma_{\Delta u/\Delta z}$ (10^{-4} sec^{-1})
0.3					
			−2.4	−1.0	60
0.6	112	−112			
			−3.4	−10.4	50
0.9	103	−103			
			7.3	5.8	37
1.5	48	−48			
			20.4	15.2	47
2.1	20	−20			
			12.1	16.5	33
3.0	−5	5			
			3.1	6.1	24
4.5	−14	14			
			1.5	2.8	31
5.4	−18	18			
			8.6	−6.2	43
6.0	−20	20			
			6.5	10.1	29
7.2	−21	21			
			18.6	6.1	23
9.0	−37	37			
			7.1	6.5	28
10.5	−45	45			
			9.5	0.1	27
12.0	−44	44			
			−4.8	−3.0	28
14.1	−26	26			
			−29.4	−0.1	38
16.0					

ence in wind shear with cloudiness seems very significant. The total number of observations is 186 at 1 Km., 159 at 9 Km., 85 at 14 Km and 36 at 16 Km.

Equation in isobaric coordinates

The averaged equation in isobaric coordinates will be

$$\left\{\bar{v}\frac{\partial\bar{u}}{\partial y} - f\bar{v} + \bar{\omega}\frac{\partial\bar{u}}{\partial p} + g\bar{w} - \frac{\bar{u}\bar{v}\tan\varphi}{a}\right\} + \left[v', \frac{\partial u'}{\partial y}\right] + \left[\omega', \frac{\partial u'}{\partial p}\right] - \frac{\tan\varphi}{a}[u', v'] = \bar{F}_x. \tag{3}$$

The eddy terms are only three. If however, ω is expanded as

$$\omega = \frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z},$$

it is seen that

$$\bar{\omega}\frac{\partial\bar{u}}{\partial p} = \left(\bar{w} + \frac{1}{g}f\bar{u}\bar{v}\right)\frac{\partial\bar{u}}{\partial z} + \left(\overline{u'\frac{\partial p'}{\partial x}} + \overline{v'\frac{\partial p'}{\partial y}} - \overline{w'g\varrho'}\right)\frac{\partial\bar{u}}{\partial p}$$

and

$$\left[\omega', \frac{\partial u'}{\partial p}\right] = \left[\frac{\partial p'}{\partial t}, \frac{\partial u'}{\partial p}\right] + \left[\left(u\frac{\partial p}{\partial x}\right)', \frac{\partial u'}{\partial p}\right] + \left[\left(v\frac{\partial p}{\partial y}\right)', \frac{\partial u'}{\partial p}\right] + \left[\left(w\frac{\partial p}{\partial z}\right)', \frac{\partial u'}{\partial p}\right].$$

Much simplification can be achieved by putting $\omega \approx w(\partial p/\partial z)$ but as eddy terms are the result of small correlations between component terms, it is better to consider the other terms in ω and show that their contribution to eddy terms is negligible. In that case, the number of eddy terms is about the same.

Equation (3) can also be written as

$$\frac{\partial}{\partial y}(\bar{u}\bar{v}) + \frac{\partial}{\partial p}(\bar{u}\bar{\omega}) - f\bar{v} + g\bar{w} + \frac{\partial}{\partial y}(\overline{u'v'}) + \frac{\partial}{\partial p}(\overline{u'\omega'}) = \bar{F}_x. \quad (4)$$

Hence

$$\frac{\partial}{\partial y}(\overline{u'v'}) + \frac{\partial}{\partial p}(\overline{u'\omega'}) \approx \overline{v'\frac{\partial u'}{\partial y}} + \overline{\omega'\frac{\partial u'}{\partial p}} \approx \overline{v'\frac{\partial u'}{\partial y}} + \overline{\omega'\frac{\partial u'}{\partial z}}.$$

$\overline{u'v'}$ has been computed by many workers studying the angular momentum balance of the atmosphere and $\overline{u'w'}$ has been estimated as a residual required to maintain balance. In the present study attempt has been made to evaluate $\overline{v'(\partial u'/\partial y)}$ and $\overline{w'(\partial u'/\partial z)}$. These provide information on the correlation between vertical velocity and vertical shear and also between meridional wind component and vertical vorticity due to zonal winds which are characteristics of synoptic disturbances.

Frictional term

$\bar{F}_x$ may be written as $K(\partial^2\bar{u}/\partial z^2)$. The magnitude of k in the free atmosphere is not known. If $\bar{u}_1$, $\bar{u}_2$ and $\bar{u}_3$ are the mean winds at two successive height intervals of Δz,

$$\frac{\partial^2 \bar{u}}{\partial z^2} = \frac{\bar{u}_3 + \bar{u}_1 - 2\bar{u}_2}{(\Delta z)^2}.$$

Taking $\Delta z = 1$ Km, $K = 10^4$ cm^2 sec^{-1} and $\bar{u}_3 + \bar{u}_1 - 2\bar{u}_2 = 20\ Kt$, $K(\partial^2\bar{u}/\partial z^2) = 10^{-3}$ cm sec^{-2}. The mean wind profiles do not show this degree of curvature even near the jet stream. With the observed mean wind profiles, the maximum magnitude of $K(\partial^2\bar{u}/\partial z^2)$ is found to be 20×10^{-4} cm sec^{-2} with $K = 10^5$ cm^2 sec^{-1} which is near the sub-tropical jet stream. It appears reasonable to conclude that the frictional term may not appreciably alter the contribution of $\overline{v'(\partial u'/\partial y)}$ and $\overline{w'(\partial u'/\partial z)}$ to A in the free atmosphere.

Conclusions

From the above study the following conclusions may be drawn regarding the eddy terms principally contributing to the mean meridional circulation in winter:

(i) $\overline{v'(\partial u'/\partial y)}$ and $\overline{w'(\partial u'/\partial z)}$ are of the same order magnitude to the north of 50° N. In low latitudes (below 20°) $\overline{v'(\partial u'/\partial y)}$ is insignificant and only $w'(\partial u'\partial z)$ matters.

(ii) At 60° N $\overline{v'(\partial u'/\partial y)}$ seems negative upto 100 mb. In most of the troposphere $\overline{w'(\partial u'/\partial z)}$ is positive and negative aloft. It is the contribution of $\overline{w'(\partial u'/\partial z)}$ that makes the total of the eddy terms positive as required in the lower troposphere.

(iii) In low latitudes $\overline{w'(\partial u'/\partial z)}$ is negative in the lower troposphere and positive aloft. This is opposite to that at 60° N. Vertical wind shear as a characteristic of tropical disturbances, seems worth more investigation.

(iv) Between 30° and 40° latitudes, total of the eddy terms should be small. This may be due to the change in the sign of $\overline{w'(\partial u'/\partial z)}$ from very high latitudes to low latitudes. $\overline{v'(\partial u'/\partial y)}$ may be small between 30° and 40° as in low latitudes.

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СРЕДНЯЯ МЕРИДИОНАЛЬНАЯ ЦИРКУЛЯЦИЯ И КРУПНОМАСШТАБНАЯ ТУРБУЛЕНТНОСТЬ

Средняя меридиональная циркуляция поддерживается напряжениями Рейнольдса, выражениями вида $v'(\partial u'/\partial y)$ и $w'(\partial u'/\partial z)$. Обсуждается величина таких членов уравнений для различных широт.