On the theory of the mean temperature of the Earth's surface

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ABSTRACT

A formula for the normal temperature of the earth's surface is derived from a simple semi-empirical representation of the surface heat balance. Computations from the formula, for January and July conditions along 45° N Latitude, indicate the usefulness of such a representation as an interfacial sea–air–land boundary condition for dynamical models of macroclimate.

1. Introduction

Figure 1 shows an estimate of the observed mean surface temperature of the earth around 45° N latitude, reduced to sea level over continents, for January and July. The oceanic data for this figure are derived from mean ocean surface temperature maps, and the continental data are taken from mean sea-level air temperature maps on the assumption that the mean ground temperature of the continents is close to that of the overlying air. In the near future, it should be possible to obtain a more accurate description of the surface temperature by means of satellite radiometer measurements, e.g., Fritz (1963). We shall suppose that the largest scale aspects of the distribution shown in Fig. 1 will not turn out to be significantly in error.

Some well-known climatological facts are apparent in this figure, the most evident of which being the close relation between the surface temperature and the distribution of continents and oceans. In winter, the amplitude of the temperature variation around the latitude circle is larger than in summer, with warmer ocean temperatures and cooler continents, and there is also a tendency for an eastward phase shift of the temperature variation relative to the oceans and continents. In summer, the mean temperature contrast between the ocean and continent surfaces is reduced and reversed in sign from the winter. The well-known "continentality" effect of a larger seasonal temperature range over land than oceans is clearly shown.

While these features are generally accepted as plausible from qualitative physical reasoning, it appears that relatively little in the way of a quantitative theory for them has yet been formulated. For example, it is by no means obvious that the ocean-continent temperature difference should reverse between summer and winter to the extent that it does. It is our purpose here to make an attempt at a quantitative discussion of these phenomena based on a highly simplified semi-empirical representation of the energy budget of the surface layer.

The ulterior aim of this study is to suggest a procedure for including the surface ground temperature as a free dependent variable in the three-dimensional theory of the macroclimate of the earth-air-ocean system, cf. Saltzman (1966). Up to now it has been common to prescribe this quantity as a fixed function of geography, e.g., Mintz (1958), Döös (1962), Sankar-Rao (1965). It would appear, also, that a quantitative understanding of the normal surface temperature would be a natural antecedent of a physical theory of the departures-from-normal which are so intimately connected to the problem of long-range forecasting, cf. Namias (1965).

2. The normal energy budget at the Earth's surface

Let us consider an infinitesimal layer encompassing the interface between the atmosphere, and the hydrosphere, and lithosphere (including organic and man-made surfaces).

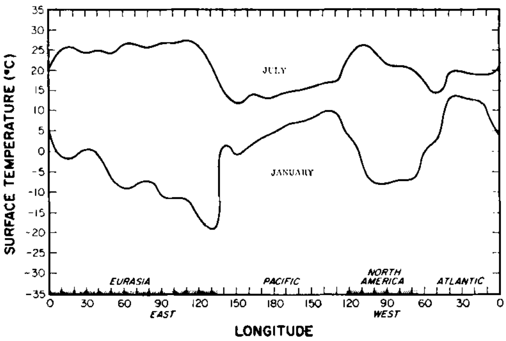

Fig. 1. Observed distribution of the surface temperature around 45° N latitude for January and July.

Since in the limit, this infinitesimal layer has zero heat capacity, the normal heat flux into it can be expressed formally by the balance relation

$$\sum_{n=1}^{6} \bar{H}^{(n)} = 0, \tag{1}$$

where $H^{(n)}$ are the rates of heat flux in g cal/cm² day (i.e., ly/day) due to the six groupings of effects listed in Table 1. The bar denotes an average over a time or ensemble period Δt, i.e.,

$$(\bar{\ }) \equiv \frac{1}{\Delta t}\int_0^{\Delta t} (\ )\,dt = (\ ) - (\ )'. \tag{2}$$

In this study, Δt will generally be taken to be the ensemble of days included in a given month for several continuous years (i.e., $(\bar{\ })$ denotes a monthly normal).

Each of the forms of heat transfer described in Table 1 represents a complicated physical process, the instantaneous measurement of which requires a detailed knowledge of the state of the ground or ocean or the atmosphere above. For the ensemble average conditions we are considering, however, it is possible to bypass the high-frequency micrometeorological details involved in an instantaneous description, in favor of a much cruder, but still physically plausible, representation. For example, in the case of radiative heating, many workers have proposed useful semi-empirical formulas which have a satisfactory basis in the exact physical theory, e.g., Kondrat'yev (1965). In a similar manner, for the cases of convective transfer of sensible and latent heat between the surface and the air above, we shall propose semi-empirical formulas in the belief that they can portray an integrated effect of the micrometeorological processes taking place over the long time periods and large spatial scales in which we are interested.

Table 1. *Forms of heat transfer*[1]

n	$H^{(n)}$
1	Short wave (solar) radiation
2	Effective long wave radiation
3	Conduction and convection of heat from the atmosphere
4	Water phase transformations at the surface
5	Conduction and convection of heat from below
6	Heat addition due to hydrometeors, viscous dissipation, and organic chemical processes

[1] The horizontal advection of heat by air and ocean currents enter the surface heat balance through their effects on vertical fluxes, $H^{(3)}$ and $H^{(5)}$, cf. Budyko (1956), p. 7.

We now proceed with a very brief discussion of these representations for each of the heating forms shown in Table 1, in the order they are given in the Table.

1. *Short-wave (solar) radiation*

Good discussions of semi-empirical formulas for the average short-wave radiation are given by Budyko (1956) and Laevastu (1962). The formula we have selected for use here is that due mainly to Kimball (1928) and Mosby (1936), i.e.,

$$\bar{H}^{(1)} = (1-\bar{r})(1-\chi\bar{C})\,R_0(t,\phi), \tag{3}$$

where

- r = reflectivity (albedo) of the ground,
- χ = a constant, given as 0.66 by Budyko (1956) for latitude $\phi = 45°$,
- C = cloud cover (in fraction of the whole sky), and
- R_0 = intensity of total (direct plus diffuse) clear-sky solar radiation incident at the surface during one day for a given time of the year and latitude (see Table 1 of Budyko, 1956).

2. *Effective long-wave radiation*

This quantity, which represents the difference between the incoming long-wave radiation from the atmosphere and the outgoing, nearly black-body, radiation from the surface, is a complicated function of the state of the atmo-

Table 2. *Regression coefficients and per cent reductions (P.R.) of variance for* $\bar{H}^{(3)}$ *[eq. (5)], and* $\bar{H}^{(4)}$ *[eq. (9)], for a combined January and July ensemble of normal values at every 10° longitude along 45° N*

a $\frac{\text{ly}}{\text{day deg}}$	b $\frac{10^6 \text{ ly sec}}{\text{day deg}}$	c $\frac{\text{ly}}{\text{day}}$	P.R. $[\bar{H}^{(3)}]$	e	f $\frac{\text{ly}}{\text{day}}$	P.R. $[\bar{H}^{(4)}]$
8.311	0.693	−71.765	61.54	1.265	−80.117	64.52

sphere, e.g., Kondrat'yev (1965). For this reason, numerous attempts have been made in the past to formulate simpler statistical expressions giving the approximate average effects. We shall here adopt a relationship derived from a form originally proposed by Brunt (1932) using the additional assumption that the normal surface temperature is much less than 273°C:

$$\bar{H}^{(2)} = -[\alpha + (\beta - \lambda \bar{V})\theta - \gamma \bar{V}] \cdot (1 - \delta \bar{C}), \quad (4)$$

where

θ = the normal temperature of the surface (°C),

$\bar{V} = \sqrt{\bar{p}_v}$ (p_v = the vapor pressure in mm),

and α, β, γ, δ, and λ are empirical constants. Using the results of M. E. & T. G. Berlyand (1952), we take these constants to be

α = 252.00 (ly/day),
β = 3.72 (ly/day),
γ = 37.70 (ly/day mm$^{\frac{1}{2}}$),
δ = 0.70,
λ = 0.55 (ly/day deg mm$^{\frac{1}{2}}$).

3. *Conduction and convection of sensible heat to the surface from the atmosphere*

Discussions of this form of heat transfer (and also of the related water vapor transfer) have been given by many investigators (see, e.g., Sutton, 1953, Priestley, 1959). It is sufficient here to say that the process is complex, depending sensitively on the structure of the atmosphere above the ground. For our long-time average considerations, however, we may be justified in proposing a rather rough form of parameterization in which we express the fact that convection is favored in regions of relatively high positive temperature difference between the surface and free atmosphere, and in regions where cold advection of surface air normally takes place. Thus, let us write

$$\bar{H}^{(3)} = -[a(\theta - \bar{T}_7) + bN + c], \quad (5)$$

where

$\bar{T}_7$ = mean temperature at 700 mb (deg C),
$N = (\mathbf{V} \cdot \nabla \bar{T})_s$ = surface temperature advection (deg/sec),

and a, b, and c are regression coefficients to be determined empirically.

Using the approximate values of $\bar{H}^{(3)}$ along 45° N presented by Jacobs (1950) and Clapp (1961), and the corresponding observed fields of θ, $\bar{T}_s$, $\bar{T}_7$, and $\bar{V}_s$ given by Haurwitz & Austin (1944) and the Normal Weather Charts of the U.S. Weather Bureau (1952), we have derived the values of the coefficients given in Table 2, for a combined January and July ensemble. It would be worth while in the future to attempt to relate $\bar{H}^{(3)}$ to the normal dependent variables in a more physical manner than we have done here, but for present purposes, let us adopt (5) in the hope that it can adequately portray the gross statistical effects of the daily convective process.

4. *Heating due to water phase transformations at the surface*

This form of heating can be expressed as

$$\bar{H}^{(4)} = \bar{H}^{(4)}_M + \bar{H}^{(4)}_E, \quad (6)$$

where

$\bar{H}^{(4)}_M = -\varrho_i L_s \bar{M}$,
$\bar{H}^{(4)}_E = -\varrho_w L_e \bar{E}$,
ϱ_i = density of snow or ice,
ϱ_w = density of water,
L_s = heat of sublimation,
L_e = heat of evaporation,
M = rate of melting (cm/sec),
E = rate of evaporation (cm/sec).

Let us now set

$$E = wE^*, \quad (7)$$

where E^* is the rate of evaporation which could be realized under the prevailing atmospheric and surface temperature conditions providing that an unlimited supply of surface water were available (i.e., *potential evaporation*). The quantity w is therefore a measure of the *availability* of water, which, in principle, can be determined by controlled measurements for any geographic location at any time.

Clearly, $w = 1$ for any water body, and $0 \leqslant w \leqslant 1$ for land. In general, the water availability on land will be closely related to the precipitation and the water retaining properties of the ground. At the present time, there is little data concerning the values of w on land, but there is some evidence that in vegetated areas w remains fairly high (i.e., close to unity) except under drought conditions when the "field capacity deficit" reaches a critical value, e.g., Thornthwaite & Hare (1965). For dry sand or concrete, w may approach zero.

The water availability will also be close to zero if the surface consists of ice, snow, or frozen soil, except when melting conditions prevail. Under melting conditions, water can rapidly become available, but evaporation will generally be rather small while the melting is in progress since the surface temperature of the ice-water mixture must remain quite cold (i.e., the freezing point, 0°C). In fact, to take into account the melting process represented by $\bar{H}_M^{(4)}$, we can impose the requirement that for surfaces consisting of ice, snow, or frozen soil, we must have $\theta \leqslant 0$°C and for mixtures of these with water (e.g., Spring thaw and Fall freeze conditions), we must have $\theta = 0$°C.

If we substitute the definition (7) into (6), and apply the time average, we obtain

$$\bar{H}^{(4)} = \bar{H}_M^{(4)} + \bar{w}\bar{H}_E^{(4)*} + \overline{w'H_E^{(4)*'}}, \tag{8}$$

where
$$H_E^{(4)*} = -\varrho_w L_e E^*. \tag{9}$$

The departure term $\overline{w'H_E^{(4)*'}}$ measures the effect of the fluctuations of water availability, including, for example, variations of the diurnal frequency and cyclone storm frequencies. We know that in these cases the water content of the ground (though not necessarily w) can increase temporarily with high precipitation due to nocturnal thunderstorms, or traveling cyclones, and subsequently be subject to an above normal potential evaporation with the arrival of clear daytime weather or dryer air masses. Such effects will be present only over land, however, and in all likelihood the magnitudes are small compared with the effects related to the differences in average water availability since w' is probably quite small.

For most of the globe, the largest effect would appear to be represented by $\bar{w}\bar{H}_E^{(4)*}$. To parameterize this term, we now introduce the assumption that the mean potential heating (cooling) due to condensation (evaporation) is proportional to the mean heating due to the convection of sensible heat, $\bar{H}^{(3)}$, which is to express the fact that the same conditions which favor the convection of sensible heat also favor the evaporation of water, cf. Bowen (1926). Thus, we write

$$\bar{H}_E^{(4)*} = e\bar{H}^{(3)} + f$$

or
$$\bar{H}_E^{(4)} = e\bar{w}\bar{H}^{(3)} + f\bar{w}, \tag{10}$$

where e and f are regression coefficients to be determined empirically.

Using the oceanic (i.e., $w = 1$) data along 45° N presented by Jacobs (1950), Clapp (1961), and Zubenok & Strokina (1963), we have obtained the values of the coefficients, e and f, given in Table 2.

5. *Conduction and small-scale convection of heat from below*

We can write for this form of heat transfer the classical formula,

$$H^{(5)} = \varrho S \varkappa \left.\frac{\partial T}{\partial z}\right|_{z=0}, \tag{11 a}$$

or, assuming the departures of $\varkappa$ from monthly average conditions for land and ocean are small,

$$\bar{H}^{(5)} = \varrho S \bar{\varkappa} \left.\frac{\partial \bar{T}}{\partial z}\right|_{z=0}, \tag{11 b}$$

where

z = depth below the surface,
T = temperature,
ϱ = density of the surface medium,
S = specific heat,
$\varkappa$ = molecular or eddy diffusivity.

In order to get a rough estimate of (11) for land, we take for $\bar{T}(z)$ the following solution of the Fickian diffusion equation for the annual evolution of the monthly normal subsurface temperature, e.g., Jeffreys & Jeffreys (1956), p. 572,

$$T(z,t) = A\cos\left(\frac{2\pi}{P}t - \sqrt{\frac{\pi}{P\varkappa}}\,z\right) \times \exp\left(-\sqrt{\frac{\pi}{P\varkappa}}\,z\right) + \boldsymbol{\theta}, \qquad (12)$$

where A is the amplitude of the annual cycle of mean monthly surface temperature about the long-term (e.g., annual) average value, $\boldsymbol{\theta}$ (i.e., $\boldsymbol{\theta} = \tilde{\boldsymbol{\theta}} + A\cos 2\pi/Pt$); P is the annual period (365 days); and $\varkappa$ is a constant thermal diffusivity assumed to be the annual mean value.

With the use of (12) we can write (11) in the following form for land areas (denoted by the subscript L),

$$\bar{H}_L^{(5)} = -\bar{k}_L\left[(\theta - \boldsymbol{\theta}) - A\sin\frac{2\pi}{P}t\right], \qquad (13)$$

where

$$\bar{k}_L = \sigma_L\sqrt{\frac{\pi}{P}},$$

$$\sigma_L = \varrho\bar{S\varkappa}\varkappa^{-\frac{1}{2}}.$$

The quantity σ, called the *conductive capacity* by Priestley (1959), is a spacially variable property of the surface layer.

Since the July and January normal conditions illustrated in Fig. 1 correspond roughly to $t = 0$ and $P/2$ respectively, we have for these two special ensembles the simpler relation,

$$\bar{H}^{(5)} = -\bar{k}(\theta - \boldsymbol{\theta}). \qquad (14)$$

The dynamics governing the profile of temperature in the hydrosphere is much more complicated than for the lithosphere. In this case we shall therefore approximate $\partial T/\partial z_{z=0}$ by a simple linear relation,

$$\frac{\partial T}{\partial z} = -\frac{(\theta - T_D)}{D}, \qquad (15)$$

where D is the depth of the seasonal thermocline and T_D is the temperature at this depth, both of which are to be prescribed as the lower boundary condition. (In a more complete theory, these quantities, which are strongly affected by the horizontal and vertical structure of the ocean currents, should be solved for from the full set of equations governing the ocean and atmosphere dynamics.) Thus, for the hydrosphere (denoted by the subscript H), we have

$$\bar{H}_H^{(5)} = -\bar{k}_H(\theta - T_D), \qquad (16)$$

where

$$\bar{k}_H = \sigma_H\sqrt{\frac{\pi}{P}},$$

$$\sigma_H = \frac{\varrho\bar{S\varkappa}}{D}\sqrt{\frac{P}{\pi}}.$$

6. *Heat addition due to hydrometeors, viscous dissipation, and organic chemical processes (e.g., photosynthesis)*

Systematic effects of these processes are generally small and will be neglected in this study.

If we now substitute the expressions for $\bar{H}^{(n)}$ into the balance equation (1), and solve for θ, we obtain the following formula:

$$\theta = \frac{1}{K}(X + \bar{H}_M^{(4)}), \qquad (17)$$

where

$$\begin{aligned}
X &= [(1-\bar{r})(1-\chi\bar{C})R_0 - (\alpha - \gamma\bar{V})(1-\delta\bar{C}) \\
&\quad + \bar{k}\xi + \varepsilon(a\boldsymbol{T}_7 - bN - e) + f\bar{w}], \\
\xi &= \boldsymbol{\theta} \approx [\theta\ (\text{Jan}) + \theta\ (\text{Jul})]/2\ (\text{land}); \\
&\quad\ T_D\ (\text{ocean}), \\
\varepsilon &= (1 + e\bar{w}), \\
K &= [\bar{k} + (\beta - \lambda\bar{V})(1-\delta\bar{C}) + a\varepsilon], \\
\bar{H}_M^{(4)} &= -X \text{ if } X/K > 0 \text{ for an ice surface.}
\end{aligned}$$

The radiation parameters, α, β, γ, δ, λ, and χ; the clear-sky solar radiation intensity, $R_0(\phi, t)$; and the convection-evaporation parameters a, b, c, e, and f are the fixed constants for the problem.

The surface layer properties $\bar{k}$, $\bar{w}$, and $\bar{r}$, and the atmosphere-ocean variables $\bar{C}$, $\bar{V}$, $\boldsymbol{T}_7$, N, and $\boldsymbol{T}_D$ are all dependent on longitude λ, as well as on ϕ and t. These variables are dynamically related to θ and to each other by the basic hydro-thermodynamical and biophysical laws governing the atmosphere, oceans, and land surface. Here we shall be concerned with determining whether (17) can account for the observed distributions of θ, and hence be a useful interfacial boundary condition for the complete problem, and also with gaining insight

Table 3. *Physio-geographic categories and typical values of surface parameters*

Category	Characteristics	$\bar{k}$ $\frac{\text{ly}}{\text{day deg}}$	$\bar{w}$	$\bar{r}$
L1	Land, 6-month rainfall (in.) <5	1	0.50	0.25
L2	Land, 6-month rainfall (in.) 5–10	1	0.70	0.20
L3	Land, 6-month rainfall (in.) 10–20	1	0.80	0.17
L4	Land, 6-month rainfall (in.) 20–40	1	0.90	0.15
L5	Land, 6-month rainfall (in.) >40	1	1.00	0.10
S	Snow cover	1	0.20	0.40
H1	Ocean, winter (strong mixing)	200	1.00	0.13
H2	Ocean, summer (weak mixing)	20	1.00	0.07

into the relative importance of the various heat transfer processes on the surface temperature. For this purpose, we shall prescribe the values of all of these other dependent variables.

3. Distributions of the prescribed variables

In this section we shall describe our estimates of the surface variables $\bar{k}$, $\bar{w}$, and $\bar{r}$, and the atmosphere-ocean variables $\bar{C}$, $\bar{V}$, $\bar{T}_7$, N, and T_D, for January and July conditions along 45° N.

Recognizing that, for *land*, $\bar{k}$, $\bar{w}$, and $\bar{r}$ are all dependent on the moisture content of the soil and the type of vegetation, which in turn are functions of the seasonal precipitation (k and w generally increase with precipitation and r decreases), and that, for the *oceans*, $\bar{k}$ is dependent on the strength and vertical stability of currents and $\bar{r}$ is dependent on the mean angle of incidence of the solar radiation, both of which are functions of the season, we have itemized the eight physio-geographic categories given in Table 3. Five of the categories represent different land surface conditions corresponding to different six-month precipitation regimes (L1–L5), one represents the condition of snow cover (S), and two represent different hydrospheric conditions (H1–H2).

The values of $\bar{k}$, $\bar{w}$, and $\bar{r}$ assigned to these categories are, at least qualitatively, in agreement with various special studies of these quantities e.g., Sverdrup (1942); Geiger (1950); Budyko (1956); Priestley (1959); Kung, Bryson & Lenschow (1964). It may be hoped that this somewhat arbitrary and crude resolution will be obviated in the future by detailed global mappings of those quantities, such as done by Kung *et al.* (1964) for the albedo of North America, or by an adequate theory of these quantities through which their dependence on the other quantities may be more rigorously expressed.

The distribution of $\bar{k}$, $\bar{w}$, and $\bar{r}$ at every 10° longitude along 45° N, for January and July, were estimated in accordance with the categorizations of Table 3, and are given in Tables 4 and 5, respectively. Since this 10° longitude resolution is inadequate for a correct portrayal of the major inland seas and lakes, (i.e., the Great Lakes, Black Sea, Caspian Sea, and Lake Aral), we have simply assumed a continuous land surface in these regions. The field of snow cover was taken from Lamb (1961), and the precipitation classification categories from the *Life Pictorial Atlas* (1961).

Also given in these tables are the estimates of the observed atmosphere-ocean variables, $\bar{C}$, $\bar{V}$, $\bar{T}_7$, and N, derived from climatological maps presented by Haurwitz & Austin (1944), the U.S. Weather Bureau (Normal Weather Charts) (1952), and Peixoto (1958). The last column gives the observed values of θ which were plotted in Fig. 1.

The temperature at the base of the seasonal thermocline, T_D, is mainly a consequence of the patterns of ocean currents. Generally, T_D is close to the value of the surface temperature in winter, cf. Pickard (1964) and, accordingly, in the absence of an adequate climatology of this quantity we have set $T_D = \theta$ (January). It remains to be verified *a posteriori* whether the solution of (17) will be consistent with this approximation.

Clearly there is much room for improvement in our specification of variables, especially if

Table 4. *Distributions of normal variables at 45° N for January* ($R_0 = 175$ *ly/day*)

λ	Category (see Table 3)	k $\frac{\text{ly}}{\text{day deg}}$	$\bar{w}$ $\times 10^{-1}$	$\bar{r}$ $\times 10^{-2}$	$\bar{C}$ $\times 10^{-2}$	$\bar{V}$ mm$^{\frac{1}{2}}$	$\bar{T}_7$ deg C	N $10^{-6}\frac{\text{deg}}{\text{sec}}$	θ deg C (see Fig. 1)
0	L3	1	8	17	64	2.7	− 6.5	− 16	+ 3.9
E10	L3	1	8	17	50	2.7	− 8.0	+ 2	− 1.1
20	L3	1	8	17	57	2.7	− 9.5	− 11	− 1.1
30	L2	1	7	20	65	2.4	−11.0	− 11	+ 1.1
40	S	1	2	40	75	2.0	−12.5	+ 3	− 1.1
50	S	1	2	40	68	2.3	−13.0	− 5	− 7.2
60	S	1	2	40	62	2.4	−13.0	− 20	− 9.4
70	S	1	2	40	58	2.1	−13.0	− 27	− 7.8
80	S	1	2	40	62	1.9	−13.5	− 13	− 6.7
90	S	1	2	40	54	1.3	−14.5	+ 1	−11.1
100	S	1	2	40	38	1.0	−15.5	+ 1	−11.7
110	S	1	2	40	20	1.0	−16.5	+ 23	−11.7
120	S	1	2	40	18	1.5	−18.0	+ 62	−15.6
130	S	1	2	40	27	1.4	−20.0	+ 98	−18.9
140	H1	200	10	13	40	1.6	−21.0	+113	+ 1.7
150	H1	200	10	13	73	1.6	−20.5	+ 97	− 0.8
160	H1	200	10	13	55	1.6	−18.5	+ 57	+ 1.7
170	H1	200	10	13	60	2.0	−15.5	+ 12	+ 3.6
180	H1	200	10	13	65	2.7	−13.5	− 17	+ 5.0
170	H1	200	10	13	69	2.8	−12.0	− 19	+ 6.4
160	H1	200	10	13	71	2.8	−10.0	− 23	+ 6.7
150	H1	200	10	13	71	2.8	− 8.0	− 17	+ 7.8
140	H1	200	10	13	71	2.7	− 6.5	− 17	+ 9.2
130	H1	200	10	13	75	2.6	− 6.5	− 40	+ 9.7
120	L2	1	7	20	71	2.3	− 8.0	− 35	+ 3.3
110	S	1	2	40	55	1.9	−10.5	− 15	0
100	S	1	2	40	45	1.6	−12.0	+ 7	− 7.8
90	S	1	2	40	60	1.5	−12.5	+ 23	− 7.8
80	S	1	2	40	72	1.7	−13.5	+ 7	− 7.2
70	S	1	2	40	65	2.0	−13.5	+ 22	− 7.2
60	H1	200	10	13	80	2.5	−12.5	+ 45	+ 0.3
50	H1	200	10	13	75	2.8	−10.5	+ 53	+ 3.1
40	H1	200	10	13	70	2.8	− 7.5	+ 29	+13.3
30	H1	200	10	13	72	2.7	− 5.5	+ 12	+13.3
20	H1	200	10	13	68	2.7	− 5.0	− 14	+13.1
10	H1	200	10	13	65	2.9	− 5.5	− 30	+12.2

we wish to account for sub-continental scales of variability. But, let us see how far our approximations may take us in accounting for the largest scale features of the surface temperature distribution which were described in Section 1. The results are given in the next section.

4. Results

The heavy solid curves in Fig. 2 represent the solutions of (17) for θ obtained by specifying the distributions of variables given in Tables 4 and 5 together with the following values of the surface clear-sky radiation, R_0, at 45° N:

$$R_0 \text{ (January)} = 175 \text{ ly/day},$$
$$R_0 \text{ (July)} = 684 \text{ ly/day}.$$

These radiation values are Houghton's (1954) solstice values modified by means of data in the *Smithsonian Meteorological Tables* (1951) to apply to mid-January and mid-July conditions.

The observed values of θ, reproduced from Fig. 1, are shown by the dashed curves. The agreement between the theoretical and observed curves seems quite satisfactory, although there are some systematic differences: e.g., the temperatures are too high for the continents in

Table 5. *Distributions of normal variables at 45° N for July* ($R_0 = 684$ *ly/day*)

λ	Category (see Table 3)	k $\frac{\text{ly}}{\text{day deg}}$	$\bar{w}$ $\times 10^{-1}$	$\bar{r}$ $\times 10^{-2}$	$\bar{C}$ $\times 10^{-2}$	$\bar{V}$ mm$^{\frac{1}{2}}$	$\bar{T}_7$ deg C	N $10^{-6}\frac{\text{deg}}{\text{sec}}$	θ deg C (see Fig. 1)
0	L3	1	8	17	56	3.3	5.0	+14	20.6
E10	L4	1	9	15	35	3.3	5.0	+13	25.6
20	L3	1	8	17	35	3.3	5.5	+ 8	25.6
30	L2	1	7	20	28	3.2	6.5	+ 8	24.4
40	L2	1	7	20	45	3.1	7.5	+ 6	25.0
50	L1	1	5	25	34	3.0	8.5	+ 3	24.4
60	L1	1	5	25	26	3.0	9.5	+ 4	26.7
70	L1	1	5	25	27	2.9	10.5	+ 9	26.1
80	L2	1	7	20	39	2.9	9.5	+ 1	25.6
90	L2	1	7	20	38	3.1	9.0	− 1	26.7
100	L1	1	5	25	43	3.2	9.0	0	26.7
110	L2	1	7	20	48	3.2	9.0	0	27.2
120	L2	1	7	20	57	3.3	8.5	− 3	26.7
130	L3	1	8	17	68	3.3	7.5	−19	21.7
140	H2	20	10	7	62	3.2	6.5	−16	15.6
150	H2	20	10	7	78	3.0	5.5	− 7	11.9
160	H2	20	10	7	69	2.9	4.5	− 8	14.2
170	H2	20	10	7	65	2.8	4.0	− 8	13.3
180	H2	20	10	7	62	2.8	4.5	−13	13.9
170	H2	20	10	7	61	2.9	5.5	−11	15.0
160	H2	20	10	7	60	2.9	6.0	− 6	15.3
150	H2	20	10	7	58	3.0	5.5	− 6	16.1
140	H2	20	10	7	57	3.0	4.0	+10	16.7
130	H2	20	10	7	52	3.0	3.0	+ 1	16.9
120	L2	1	7	20	40	2.8	5.0	− 2	23.3
110	L2	1	7	20	40	2.6	7.5	+ 5	26.7
100	L3	1	8	17	41	2.7	7.0	− 2	24.4
90	L4	1	9	15	51	3.1	5.0	− 7	21.1
80	L3	1	8	17	49	3.1	3.5	− 4	21.1
70	L4	1	9	15	56	3.1	3.0	− 7	20.0
60	H2	20	10	7	63	3.0	3.5	−20	16.1
50	H2	20	10	7	65	3.1	4.0	−10	14.4
40	H2	20	10	7	62	3.3	4.0	− 6	20.0
30	H2	20	10	7	64	3.1	4.0	+ 4	19.7
20	H2	20	10	7	62	3.1	4.5	+10	18.9
10	H2	20	10	7	62	3.2	5.0	+ 9	18.3

summer and a little low for the oceans in both seasons and the continents in winter.

Only a small part of the discrepancy in the summer land surface temperature can be explained by the assumed *linear* dependence on θ of the outgoing radiation, $H^{(2)}$. A larger part of this discrepancy may be due to a bias toward lower daily mean *air* temperatures (used as our estimate of θ) compared to the mean surface *soil* temperatures, e.g., Geiger (1959), p. 74.

A question may be raised concerning the degree to which (17) can account for the observed variations of θ if the longitude-dependence only of the surface characteristics ($\bar{k}$, $\bar{w}$, $\bar{r}$) is prescribed, with *zonal-average* values taken for the remaining atmosphere-ocean variables ($\bar{C}$, $\bar{V}$, $\bar{T}_7$, $\bar{N}$, $\bar{T}_D$). This could represent the first step toward an iterative steady-state solution for the mean atmospheric perturbations forced by mean convective heating, $\bar{H}^{(3)}$, e.g., Döös (1962). The result is given by the thin curve in Fig. 2. This curve is very nearly the same as we obtain if we specify only $\bar{k}(\lambda)$ and use the zonal-average values of $\bar{w}$ and $\bar{r}$ also, thus indicating the prime importance of the variations of the conductive capacity in accounting for the main surface temperature variations. We can see, however, that such features as the cold winter conditions in the eastern portions of the continents and the tendency for a relatively

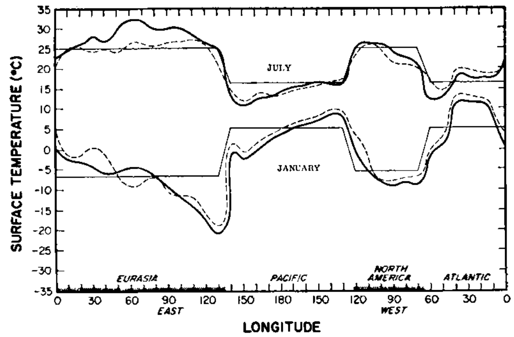

Fig. 2. Theoretical distribution of surface temperature around 45° N latitude for January and July (heavy solid curves). Observed distribution, from Fig. 1, shown by dashed curves. Theoretical values for zonal-average values of atmosphere-ocean variables shown by thin curves.

warmer eastern portion of the oceans remain to be developed as a mutual adjustment with the atmospheric and oceanic current systems as manifested particularly in the variables $\tilde{T}_7$, N, and T_D (cf., Tables 4 and 5).

We note, also, that the ocean surface temperatures are strongly dependent on the prescribed degree of mixing as measured by $\bar{k}$. With the exception of the systematic departures described above, the winter and summer values chosen here, based on the data presented by Sverdrup *et al* (1942), seem to lead to fairly satisfactory values of θ. To avoid the arbitrariness of these choices, however, we shall eventually have to develop the theory to a point where $\bar{k}$ is treated as a dependent variable, presumably of the surface wind conditions and the thermohaline stratification.

In Figs. 3 and 4 we show the five components of the surface heat balance $H^{(n)}$ ($n = 1$–5), and the surface radiation "balance" ($H^{(1)} + H^{(2)}$, indicated by $1+2$), all corresponding to the solution for θ. Some of the more prominent features are, (i) the relatively minor role of conductive heat transfer in the ground as compared to the eddy heat transfer in the oceans, (ii) the marked upward flux of heat from the oceans to the atmosphere in the winter especially in the western portions of the oceans, and the downward flux in the summer, and (iii) the relatively small variation of the radiative fluxes $H^{(1)}$ and $H^{(2)}$ around the latitude circle.

In Table 6 we list the zonal-average values of the heat flux components, denoted by $H_0^{(n)}$. The average Bowen ratio defined by $B_0 = H_0^{(3)}/H_0^{(4)}$ is given in the last column. The values given here, as well as in Figs. 3 and 4, seem to be in good agreement with the estimates of Budyko [1956].

5. Conclusions and recommendations

The general agreement in amplitude and phase between the theoretical and observed distributions of θ indicates the usefulness of a crude representation of the surface heat balance, such as expressed by (17), as a thermal boundary condition at the sea-air-land interface. Through such a condition, it becomes possible to solve for important components of the three-dimensional macroclimatic field (e.g., the stationary planetary waves forced by the mean vertical flux of sensible heat from the surface) by specifying only the radiative input R_0 and the surface state parameters $\bar{k}$, $\bar{w}$, and $\bar{r}$, rather than by specifying the surface temperature *a priori*, e.g., Döös (1962).

However, although the parameters $\bar{k}$, $\bar{w}$, and

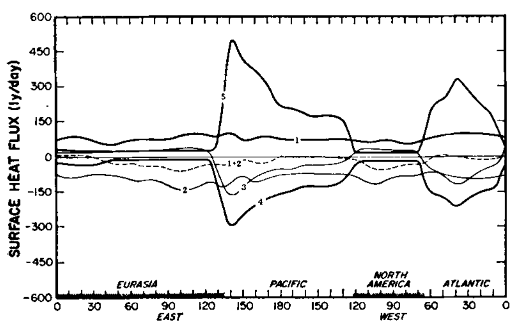

Fig. 3. Theoretical components of the surface heat balance, $H^{(n)}$ ($n = 1$–5, see Table 1), for January.

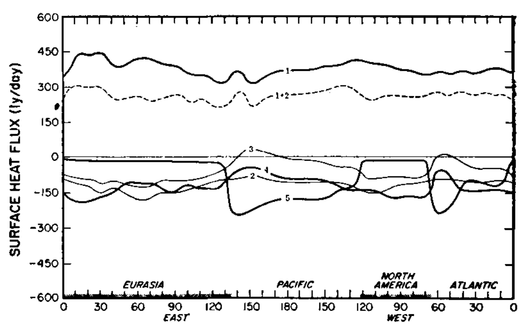

Fig. 4. Theoretical components of the surface heat balance, $H^{(n)}$ ($n = 1$–5, see Table 1), for July.

Table 6. *Zonal average values of the theoretical heat fluxes* $H_0^{(n)}$
Units are ly/day

	$H_0^{(n)}$ (ly/day)						
n ...	1	2	3	4	5	1+2	B_0
January	78	−95	−20	−87	124	−17	0.23
July	385	−117	−60	−123	−84	267	0.49

$\bar{r}$ are more intrinsic properties of the earth than θ, all three are still dependent on other dynamical variables describing the state of the atmosphere and oceans. As a future goal we should seek physical or quasi-physical relationships governing these dependencies not only for these surface variables, but also for the mean cloudiness $\bar{C}$ and mean precipitation. In their ultimate form, the relationships for the surface state parameters should include a description of the interaction between the atmosphere and the land surface characteristics (e.g., vegetation and soil condition) thereby involving physio-biological and physio-geographic variables. With the incorporation of such relationships in a full three-dimensional model of the atmosphere, hydrosphere, and upper lithosphere, we will more nearly approach a detailed deductive theory of "ecoclimate" as is portrayed, for example, by the Köppen or Thornthwaite classifications (see, e.g., Haurwitz & Austin, 1944). A first order improvement of our present study toward this goal would derive from taking the effects of topographic height into account.

Finally, we conclude by stating again the desirability for finding more satisfactory parameterizations of the monthly normal heat transfer processes than we have adopted here. The first aim must be to achieve these from purely physical considerations, but in the absence of success in this approach, better statistical relationships, capable of explaining more of the variance of the heat transfer over the globe, for all months, can probably be obtained by applying such techniques as "multiple discriminant analysis" and "screening regression" (see Miller, 1962). The real challenge, however, will lie in finding firm physical foundations for these statistical relationships.

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О ТЕОРИИ СРЕДНЕЙ ТЕМПЕРАТУРЫ ЗЕМНОЙ ПОВЕРХНОСТИ

Выводится формула для нормальной температуры земной поверхности на основе простого полуэмпирического представления баланса тепла на поверхности Земли. Вычисления, проведенные с помощью этой формулы для января и июля вдоль 45° северной долготы, указывают на полезность такого представления в качестве условия на границе раздела море–воздух–суша для динамических моделей макроклимата.