Eddy vertical transport of latent and sensible heat

By ROY BERGGREN and ALF NYBERG, *Swedish Meteorological and Hydrological Institute, Stockholm*

(Manuscript received September 16, 1965)

ABSTRACT

The result of computations of the vertical transport of latent and sensible heat through the 700 mb surface is discussed. The computations were performed for most of the Northern Hemisphere north of 30° N and for October, 1961. As vertical velocities output data from operational numerical material were used. The total vertical transport through the 700 mb surface turned out to be equivalent to a warming in 24 hours of the layer between 700 mb and 300 (200) mb of 0.4°C at 85° N and 2.6°C at 50° N.

1. Introduction

In connection with studies of energy conversions in the atmosphere a number of investigations have been published regarding horizontal and vertical transport of momentum, heat etc. During the last ten years or so some of these investigations have been based on vertical motions computed in connection with numerical forecasting, either in routine runs or for special studies. Few of these studies have involved computations of the vertical transport of latent heat, while many more have included horizontal transports of momentum and sensible heat. In order to give a contribution to the former type of studies, it was decided that, based on 700 mb data, preliminary computations be performed for one month, representing the conditions at the 700 mb surface. At the same time the vertical transport of sensible heat was included in the study in order to complete the picture. Depending on the result of this effort a more extensive and elaborate investigation would then be performed with the aid of an electronic computer.

2. Material

Through the courtesy of the U.S. Weather Bureau copies of vertical motion charts for October, 1961, were put at the disposal of the Swedish Meteorological and Hydrological Institute. The vertical motion was computed according to the 2-level model described in a paper by Elseasser (1960). The vertical velocities (ω) were given in units of dynes cm^{-2} sec^{-1} at 600 mb. The values were read off in this unit at the grid points given in Fig. 1. Temperature (T) and mixing ratio (s) values were read off at the same grid points. The temperature data were taken from the 700 mb working charts available at the institute, completed at a later stage in areas not covered in the regular analysis. For the humidity values special charts were prepared. All data pertain to 00 GMT. For different reasons, mainly poor data coverage, not all maps could be used. Those not included are the maps for 10–12, 19, 21, and 26 October, 1961. It is reasonable to assume that the result of the computations can be taken as representative for October, 1961, in spite of the six missing days.

In order to get comparable data for the vertical motions, these had to be transformed to represent 700 mb instead of 600 mb. This was done by multiplying the read-off data with a factor 36/49.

3. Vertical transport of latent heat

We introduce deviations from the zonal and time average as follows:

$$\omega = \bar{\omega} + \omega', \quad s = \bar{s} + s',$$

where the bar indicates an average value for the latitude circle and for the month and the

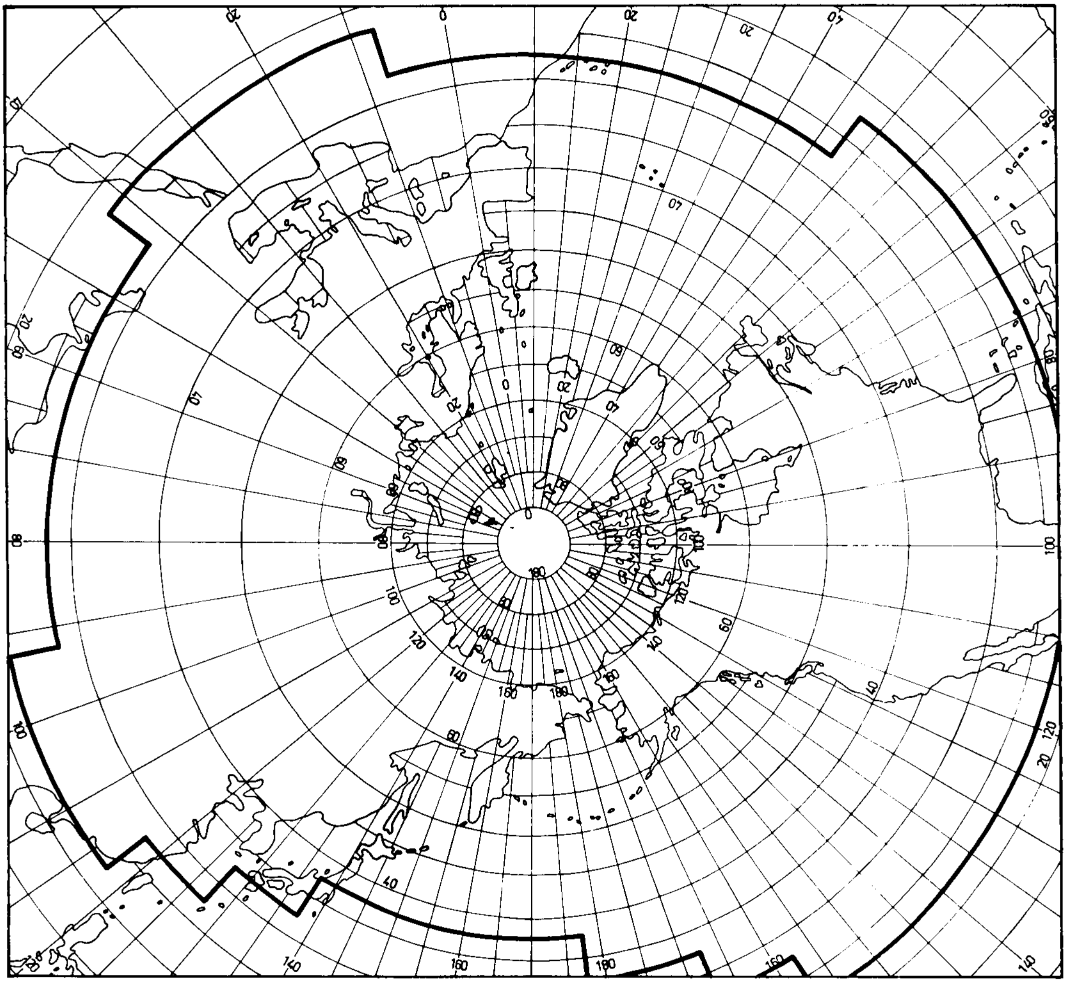

FIG. 1. Area over which calculations were made of vertical transport of latent and sensible heat through the 700 mb surface for October, 1961. Every 5°-intersection between latitudes and longitudes within the irregular frame were used, exept at 80° N and 85° N, where the computations were performed for every 10° longitude.

prime deviation from this mean at each grid point and on each day. The expression

$$\overline{F_M} = -\frac{L}{g}\overline{\omega' s'},$$

where L is the latent heat of evaporation and s the mixing ratio, summed around a latitude circle or part of it (see Fig. 1) and over all available days of the month, represents an eddy vertical transport of latent heat per unit time and unit area. For L the value $2{,}5\cdot 10^6$ MTS units has been used. The computed value of $\bar{\omega}$ and $\bar{s}$ and of the eddy transport term ($\overline{F_M}$) are given in Table 1 and in Figs. 2 and 3.

4. Vertical transport of sensible heat

In this case the following deviations were used:

$$\omega = \bar{\omega} + \omega', \quad T = \bar{T} + T'.$$

The notations are the same as in the above paragraph. The expression

$$\overline{F_T} = \frac{-c_p}{g}\overline{\omega' T'},$$

where c_p is the specific heat at constant pressure, was summed for each available day and for the latitudes indicated in Fig. 1. The computed values of the transport term $\overline{F_T}$, and the mean value of T are given in Table 1 and Figs. 2 and 3.

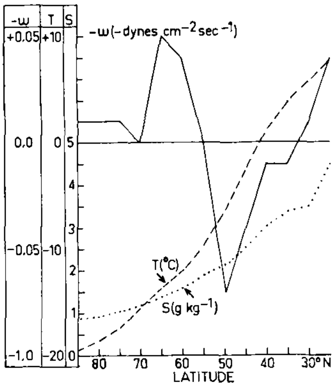

FIG. 2. Latitudinal mean values for October, 1961, for vertical velocity (ω, dynes cm^{-2} sec^{-1}), temperature (degrees Celsius), and specific humidity (g kg^{-1}).

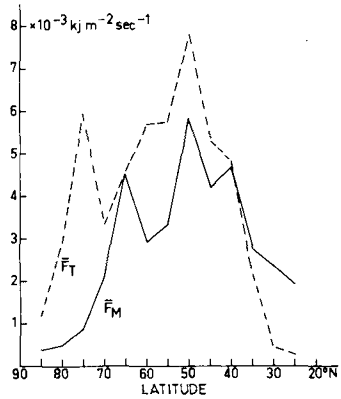

FIG. 3. Latitudinal mean values for

$$-\frac{L}{g}\overline{\omega' s'}\ (\overline{F_M}) \quad \text{and} \quad -\frac{c_p}{g}\overline{\omega' T'}\ (\overline{F_T}),$$

respectively, for October, 1961.

5. The result of the computations

The *mean vertical motion*[1] indicates two well developed meridional cells (see Table 1 and Fig. 2), while the meridional circulation in higher latitudes is weak and undetermined. The maximum values agree reasonably well with, but are smaller than, those published by other authors working with vertical motions derived from numerical forecasting computations (see e.g. WIIN-NIELSEN, 1959). However, in the case reported here the maximum downward motion occurs at 50° N, while in the case reported by WIIN-NIELSEN (January, 1959) the corresponding value is 35° N. The maximum upward motion is found at 65° N and 55–60° N, respectively. The October case is thus characterized by a northward movement of the southern, direct circulation cell (the Hadley cell) or an unusually strong indirect middle latitude cell (the Ferrel circulation) or a combination of both effects.

As can be seen from Table 1 and Fig. 3 the maximum vertical eddy transport of latent heat occurs at 50° N and the minimum at 85° N. For the transport of sensible heat the corresponding values for the maximum is 50° N and for the minimum 25° N. When comparing the values given in Table 1, one should keep in mind that the number of points used for each latitude circle is varying. From Fig. 1 it is clear that especially the values for 25° N might be misleading due to the restricted number of points. Both the latent heat and sensible heat transport terms show more than one maximum. The main maximum occurs in both cases at 50° N while the maxima at higher latitudes are quite far apart at 65° N for $\overline{F_M}$ and at 75° N for $\overline{F_T}$. The numerical values of $\overline{F_T}$ agree quite well with values published by others in connection with studies of energy conversions from available eddy potential energy to eddy kinetic energy. In a paper WHITE & SALTZMAN (1956) give the latitudinal variation of the mean rate of this conversion for January, 1953. The vertical velocities used in this case were vertically-averaged individual pressure changes, computed using the 2-parameter, quasi-geostrophic model developed by THOMPSON (1953). Furthermore, 500 mb temperature were used and the values for both quantities were taken at every 5 degree latitude–longitude intersection from 35° N to 60° N and from 70° W to 120° W. The maximum value, about $12 \cdot 10^{-3}$ kj m^{-2} sec^{-1} occurred at 50° N, i.e. at the same latitude as in October, 1961, studied in this paper, while the numerical value is about 50 % higher. The mean value for the whole area is $6.7 \cdot 10^{-3}$ kj m^{-2} sec^{-1}, as compared to a mean value of $4.2 \cdot 10^{-3}$ kj m^{-2} sec^{-1} from Table 1 valid for a great portion of the Northern Hemisphere down to 25° N. The discrepancy may be explained by the fact that the present paper deals with October data instead of January data and that the White-

[1] The value of $(-\omega)$ at 700 mb is almost numerically equal to the corresponding value of the vertical velocity in cm sec^{-1}.

TABLE 1. *Mean values for the 700 mb surface of vertical velocities, mixing ratios, temperatures, vertical transport of latent and sensible heat for October, 1961, and for the indicated latitudes.*

Lat.	$-\bar{\omega}$ $-$dynes cm^{-2} sec^{-1}	$\bar{S}$ g kg^{-1}	$\bar{T}$ °C	$\overline{F_M}$ kj m^{-2}sec^{-1}	$\overline{F_T}$ kj m^{-2}sec^{-1}
85°N	+0.01	0.90	−19.4	$0.39 \cdot 10^{-3}$	$1.17 \cdot 10^{-3}$
80	+0.01	0.92	−18.6	0.49	2.90
75	+0.01	1.04	−17.4	0.88	5.95
70	0.00	1.16	−15.4	2.10	3.34
65	+0.05	1.37	−13.6	4.50	4.57
60	+0.04	1.59	−12.0	2.92	5.68
55	0.00	1.90	−9.7	3.34	5.74
50	−0.07	2.12	−6.3	5.81	7.79
45	−0.04	2.51	−2.4	4.18	5.28
40	−0.01	3.05	+1.3	4.66	4.83
35	−0.01	3.38	+3.9	2.75	2.17
30	+0.01	3.50	+5.8	2.35	0.44
25	+0.04	4.50	+7.8	1.91	0.26

Saltzman computations were performed for a much smaller area. In a paper WIIN-NIELSEN (1959) gives corresponding values for January and April, 1959, for the entire Northern Hemisphere down to 13° N. The vertical velocities used in this study were taken from the output from the Joint Numerical Weather Prediction Unit in Washington and pertained to the 600 mb surface. The model used was the 2-level thermotropic, quasi-geostrophic one (ELSEASSER, 1960). As temperature parameter the mean temperature between 850 and 500 mb was used. The grid distance was the regular JNWP one, i.a. 381 km at 60° N. For the January case Wiin-Nielsen arrived at a maximum value of $4{,}4 \cdot 10^{-3}$ kj m^{-2} sec^{-1} at 40–45° N, while the mean value for the whole area is $1{,}46 \cdot 10^{-3}$ kj m^{-2} sec^{-1}. For the April case the corresponding values were $2{,}4 \cdot 10^{-3}$ kj m^{-2} sec^{-1} at 45–50° N and $1{,}1 \cdot 10^{-3}$ kj m^{-2} sec^{-1}, respectively. In two papers SALZMAN & FLEISHER (1960, 1961) give values for $\overline{\omega' T'}$, averaged zonally and with time, for February 1959 and January through March and October through December 1959, respectively. In both cases the (main) maximum occurs at 45° N with a secondary maximum at 55° N for February.

It is interesting to note that the values for the correlation between vertical velocity and mean temperature as given by Wiin-Nielsen actually attain negative values around 60° N, indicating that in these latitude bands the warm air in the average is sinking and the cold air rising. The values given by White and Saltzman for the latitudes 35° N through 60° N also have their minimum at 60° N although they are positive. The corresponding values in Fig. 3 however have no such marked minimum in middle latitudes.

It is of interest to compare the values of the frictional dissipation in the atmosphere with the ones given above for the total energy conversions. The present study deals indirectly with the conversion from eddy available potential energy to eddy kinetic energy. However, earlier studies (see e.g. WIIN-NIELSEN, 1959) indicate that the sum of the energy conversion due to a mean vertical motion over the whole area and the energy conversion due to a mean meridional circulation is small compared with the energy conversion in connection with overturning in the zonal planes.

Thus the average figure arrived at in this paper may be taken as a reasonably good estimate of the total energy conversion. Now, if local change of the total kinetic energy is assumed to be approximately zero (for the month studied), we get for the frictional dissipation a value of about $4 \cdot 10^{-3}$ kj m^{-2} sec^{-1}, while WHITE & SALTZMAN (1956) and BRUNO (1941) give the value $5 \cdot 10^{-3}$ kj m^{-2} sec^{-1} and PISHATORY (1954) and WIIN-NIELSEN (1959) give about $2 \cdot 10^{-3}$ kj m^{-2} sec^{-1}.

6. Temperature increase in the middle and upper troposphere due to vertical transport

Using the values $\overline{F_M}$ and $\overline{F_T}$, respectively, given in Table 1 it is possible to compute a temperature increase above the 700 mb surface corresponding to the transport of latent and sensible heat through that surface. We then have to assume an upper level to which the influence extends. The computations have been started at 30° N, as the data for 25° N are somewhat unreliable. As 30° N is rather close to the average latitude of the subtropical jet stream, we have assumed an average pressure of 200 mb at the tropopause from that latitude and northwards. From some middle latitude one would have to lower the tropopause to, say, 300 mb. For the actual month studied in this paper,

TABLE 2. *Temperature increase per 24 hours in middle and upper troposphere due to transport of latent and sensible heat through the 700 mb surface, October, 1961.*

Lati-tudes	Latent heat	Sensible heat	Latent + sensible heat	Layer
85°N	0.08°C	0.25°C	0.3°C	700–300 mb
80	0.10	0.61	0.7	700–300
75	0.19	1.26	1.5	700–300
70	0.44	0.70	1.1	700–300
65	0.95	0.96	1.9	700–300
60	0.49	0.96	1.5	700–200
55	0.56	0.97	1.5	700–200
50	0.98	1.32	2.3	700–200
45	0.71	0.89	1.6	700–200
40	0.79	0.82	1.6	700–200
35	0.46	0.37	0.8	700–200
30	0.40	0.07	0.5	700–200

65° N has been chosen as the average latitude of the shift in tropopause height.

The temperature increase per 24 hours ($\Delta\bar{T}$) in the layer between 700 mb and p mb can be expressed as follows, where the pressure p is given in Table 2, last column:

$$\Delta\bar{T} = \frac{86400 \cdot g \cdot \bar{F}}{c_p\,(70-p)}$$

$\bar{F}$ is equal to $\overline{F_M}$ or $\overline{F_T}$, as the case may be. If the values for these parameters are those given in Table I and the pressure intervals given in Table 2 are used, we arrive at the values for $\Delta\bar{T}$ given in Table 3.

The maximum values for the temperature increase due to the vertical transport of latent heat are about 1°C per 24 hours, observed at 65 and 50° N. The values corresponding to the transport of sensible heat amount to 1.4–1.5°C at 75° and 50° N. This gives a maximum of 2.6°C of the total effect at 50° N.

Now, it is interesting to compare the figures given in Table 2 with (*a*) those given for the temperature increase due to horizontal transport of heat and (*b*) estimates of the radiational cooling observed in the middle and upper part of the atmosphere.

Values of the first type may be found in an earlier paper by one of the authors (NYBERG *et al.*, 1954). In that paper values are given for November, 1947, between 40° N and the pole and from 1000 mb to 300 mb. From these values the figures in Table 3 have been extracted.

TABLE 3. *Temperature increase due to horizontal transport of latent and sensible heat, November 1947* (NYBERG *et al., 1954*).

Latitude band	Latent heat	Sensible heat	Latent + sensible heat	Layer (mb)
40°–50° N	0.37°C	−0.13°C	0.2°C	700–300
50°–60° N	0.23	0.53	0.8	700–300
60°–pole	0.20	0.71	0.9	700–300

A comparison between the values in Table 3 and the mean values computed for the same latitude bands in Table 2 reveals that the vertical transport gives rise to a larger temperature increase than does the horizontal transport. However, great caution should naturally be exercised at this comparison as the data are from entirely different months and also partly for different layers. The conclusion which may be drawn from the indicated figures is that the effect of the vertical transport is at least of the same order of magnitude as that of the horizontal transport and probably one to three times larger.

In his paper DAVIS (1963) gives a Fig. 4 indicating atmospheric infrared cooling rates for average cloudiness for October. In another of his figures Davis gives the average solar heating for October. From these two figures values were read off for every 5° latitude from 700 mb to 200 or 300 mb as required according to Table 2. Mean values for the same layers as indicated in Table 2 were computed and the results are given in Table 4.

We notice from the table that the short wave heating is only a small fraction of the infrared

TABLE 4. *Mean values of the infrared cooling and the short wave heating for October and for latitudes and layers as indicated (based on values published by* DAVIS, *1963).*

	70° N	60	50	40	30° N
Infrared cooling	0.8°C	1.0	1.1	1.4	1.5
Short wawe heating	0.08	0.11	0.17	0.24	0.33
Net	0.7	0.9	1.0	1.1	1.2
Layer	700–300 mb	700–200 mb			

cooling, increasing from 10 % at 70° N to about 22 % at 30° N.

The values given in Table 4 agree well with values given by LONDON (1957) if one in the latter case takes mean values between summer and winter values. The mean values for the infrared cooling and for the layer studied here given by MÖLLER (1951) for middle latitudes and for average cloudiness is slightly above 2°C, i.e. higher than the ones given in Table 4. Values published by KARSTEN, KORB, MANIER & MÖLLER (1959) for the short wave heating indicate also that in most cases the heating is only a small fraction of the cooling. Furthermore, when comparing the net values of Table 4 with the sum of the heating in the middle and upper troposphere corresponding to the computed transports of latent and sensible heat through the 300 mb surface (Table 2) it is obvious that the large scale vertical transport is in itself large enough to compensate for radiational losses in the middle and upper troposphere expect at low latitudes.

This indicates that the vertical transport by small scale processes as cumulus- and cumulunimbus clouds is of little significance compared to the vertical transport by large scale eddies associated with cyclones and anticyclones in middle and high latitudes, at least in layers above the level of a few kilometers.

Finally, it should be pointed out that in the above discussion one term has been neglected and that is the measure of the energy conversion from available potential energy to kinetic energy in the layer in question.

Acknowledgment

Thanks are due to the former Chief of the U.S. Weather Bureau, Dr. F. Reichelderfer, for putting vertical velocity charts for October, 1961, at our disposal. Dr. A. Wiin-Nielsen, University of Michigan, has contributed by very helpful discussions which is greatfully acknowledged.

REFERENCES

BRUNT, D., 1941, *Physical and Dynamical Meteorology*. Cambridge University Press, 428 pp.

DAVIS, P. A., 1963, An analysis of the atmospheric heat budget. *Journal of the Atm. Sciences*, **20**, No. 1, pp. 5–22.

ELLSAESSER, H. W., 1960, JNWP operational models. JNWP Office Note No. 15 (revised), 38 pp.

KASTEN, F., KORB, G., MANIER, G., and MÖLLER, F., 1959, On the heat balance of the troposphere. Final Report, Contract AF 61(052)-18. Met.-Geophysik. Institut, Johannes Gutenberg Universität, Mainz, 89 pp.

LONDON, J., 1957, A study of the atmospheric heat balance. Final Report, Contract AF 19(122)-165, Research Division, College of Engineering, New York University.

MÖLLER, F., 1951, Long-wave radiation. *Compendium of Met.*, Am. Met. Soc. pp. 34–49.

NYBERG, A., DUVEDAL, T., and FRYKLUND, S., 1954, Transport of heat and momentum by horizontal large scale eddies in November 1957, July 1949, and February 1950, north of 40° N. SMHI, Notiser och prel. rapporter, Vol. 1, Nr 4.

PISHAROTY, P. R., 1954, The kinetic energy of the atmosphere. Final Report, Contract AF 19(122)-48. Dept. of Meteorology, University of California at Los Angeles, 154 pp.

SALTZMAN, B., and FLEISHER, A., 1960, The modes of release of available potential energy in the atmosphere. *Journal of Geoph. Res.*, **65**, No. 4, pp. 1215–1222.

SALTZMAN, B., and FLEISHER, A., 1961, Further studies on the modes of release of available potential energy. *Journal of Geoph. Res.*, **66**, No. 7, pp. 2271–2273.

THOMPSON, P. D., 1953, On the theory of large scale disturbances in a two-dimensional baroclinic equivalent of the atmosphere. Q.J.R. M.S., **29**, pp. 51–70.

WHITE, R. M., and SALTZMAN, B., 1956, On conversion between potential and kinetic energy in the atmosphere. *Tellus*, **8**, pp. 357–363.

WIIN-NIELSEN, A., 1959, A study of energy conversion and meridional circulation for the large-scale motion in the atmosphere. *Monthly Weather Rev.*, **87**, No. 9, pp. 319–332.

ТУРБУЛЕНТНЫЙ ВЕРТИКАЛЬНЫЙ ПЕРЕНОС СКРЫТОГО И ОБЫЧНОГО ТЕПЛА

Обсуждаются результаты вычисления вертикального переноса скрытого и обычного тепла через поверхность 700 мбар. Вычисления были выполнены для всей области севернее 30° с. ш. за октябрь 1961 г. Данные о вертикальных скоростях брались из оперционного численного материала. Полный вертикальный поток тепла через эту поверхность оказался эквивалентным теплу, необходимому для нагрева слоя между 700 мбар и 300 (200) мбар на 0,4°C при 85° с. ш., и на 2,6°C при 50° с. ш.