

On atmospheric zonal to eddy kinetic energy exchange for January 1963

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ABSTRACT

An estimate of the exchange of kinetic energy between the large-scale atmospheric eddies and the zonal flow for January 1963 for the troposphere north of 20°N latitude has been made by means of a quasigeostrophic method. This more complete result is compared with the anomalous one obtained earlier by WIIN-NIELSEN, BROWN and DRAKE (1964). The monthly averaged exchange values are in very good agreement. However, the results of the present study indicate that the estimates of the latitudinal and vertical boundary terms must be taken into account when studying the kinetic energy balance over this portion of the atmosphere.

Recent diagnostic studies of the kinetic energy exchange between the zonal flow and the large-scale eddies of the troposphere north of 20°N have indicated the existence of an unusual type of circulation for January 1963 [WIIN-NIELSEN, BROWN & DRAKE (1964) and MURAKAMI & TOMATSU (1965)]. The abnormality of the circulation has also been emphasized particularly by NAMIAS (1963) and O'CONNOR (1963).

The diagnostic calculations indicated an exchange of kinetic energy from the zonal flow to the eddies for this winter period. As has been found, this exchange mechanism is usually in the opposite direction. Since the zonal kinetic energy showed no sign of decreasing in strength during January 1963 [WIIN-NIELSEN (1964)], the energy was evidently supplied through some other mechanism.

The rate of change of zonal kinetic energy, $K_z = \int k_z dM$, is

$$\frac{\partial K_z}{\partial t} = E(K_E, K_z) + C(A_z, K_z) - D(K_z) + B(K_z). \quad (1)$$

Here

$$k_z = \frac{1}{2} \{ [u]^2 + [v]^2 \}$$

$$\begin{aligned} E(K_E, K_z) = & \int [u^* v^*] \cos \varphi \frac{\partial}{\partial \varphi} \frac{[u]}{\cos \varphi} dM \\ & + \int [u^* \omega^*] \frac{\partial [u]}{\partial p} dM + \int [v^* \omega^*] \frac{\partial [v]}{\partial \varphi} dM \\ & + \int [v^* \omega^*] \frac{\partial [v]}{\partial p} dM - \int \frac{[v][u^* \omega^*] \tan \varphi}{a} dM \end{aligned} \quad (2)$$

$$C(A_z, K_z) = - \int [v] \frac{\partial [\Phi]}{a \partial \varphi} dM \quad (3)$$

$$D(K_z) = \int \{ [u][F_\lambda] + [v][F_\varphi] \} dM \quad (4)$$

$$\begin{aligned} B(K_z) = & - \int \frac{1}{a \cos \varphi} \frac{\partial}{\partial \varphi} [u][u^* v^*] dM \\ & - \int \frac{1}{a \cos \varphi} \frac{\partial}{\partial \varphi} [v][v^* \omega^*] dM \\ & - \int \frac{1}{a \cos \varphi} \frac{\partial}{\partial \varphi} [v] k_z dM - \int \frac{\partial}{\partial p} [u][u^* \omega^*] dM \\ & - \int \frac{\partial}{\partial p} [v][v^* \omega^*] dM - \int \frac{\partial}{\partial p} [\omega] k_z dM. \end{aligned} \quad (5)$$

Here

a = mean radius of the earth

Φ = geopotential

φ = latitude

λ = longitude

$u, v, \omega, \lambda, \varphi, p$ components of wind velocity

p = pressure

F = frictional force per unit mass

M = atmospheric mass

Also $[] = 1/(2\pi) \int_0^{2\pi} () d\lambda$ and the asterisk refers to the departure from this zonal average.

Now $E(K_E, K_z)$ represents the exchange of kinetic energy between the eddies and the zonal flow. (Actually this is only strictly true for the case of a closed system.) In the investigation

by Wiin-Nielsen, Brown and Drake only the geostrophic form of the first integral was computed. They used the 850, 700, 500, 300 and 200 mb objective height analyses of the U.S. Weather Bureau's National Meteorological Center. The term $C(A_z, K_z)$ represents the conversion of zonal available potential energy into zonal kinetic energy and has been estimated in the present investigation through use of the omega-equation. It will only be mentioned here that these results using the same 5 levels of data for the period of interest indicated that this conversion acted as a sink of K_z . A similar result was obtained by KRUEGER, WINSTON and HAINES (1965) for January 1963 for the 850–500 mb layer north of 20°N. The term $D(K_z)$, the rate of frictional dissipation of zonal kinetic energy, was not calculated, but it is reasonable to assume that it must have acted as a sink of K_z .

Due to the fact that the calculations are performed over a portion of the atmosphere only, vertical and lateral boundary terms, $B(K_z)$, appear in equation (1). The present investigation is an attempt to estimate these terms together with the complete expression for $E(K_E, K_z)$.

The omega-equation in the following form has been used:

$$\nabla^2 \omega + \frac{f_0^2}{\sigma} \frac{\partial^2 \omega}{\partial p^2} = \frac{f_0}{\sigma} \frac{\partial}{\partial p} (\mathbf{V} \cdot \nabla \eta) - \frac{f_0}{\sigma} \nabla^2 \left(\mathbf{V} \cdot \nabla \frac{\partial \psi}{\partial p} \right) + \frac{gf_0}{\sigma} \frac{\partial^2}{\partial p^2} \left(\frac{\partial \tau_x}{\partial y} - \frac{\partial \tau_y}{\partial x} \right). \quad (6)$$

Here f_0 is the Coriolis parameter at 45°N, $\mathbf{V}$ is the horizontal wind vector, η is the vertical component of the absolute vorticity and σ is a measure of the static stability:

$$\sigma = -\frac{1}{\varrho} \frac{\partial \ln \theta}{\partial p}.$$

Here θ is the potential temperature and ϱ is the air density. σ has been treated as a function of pressure only. Furthermore the stream function was obtained from

$$\nabla^2 \psi = \nabla \cdot \left(\frac{1}{f} \nabla \Phi \right).$$

The surface frictional stress components, τ_x and τ_y , have been approximated by a method de-

scribed by MESINGER (1965). ω was set equal to zero at $p = 0$ and

$$\omega = \mathbf{V} \cdot \nabla p_g$$

at the ground. Here p_g is the pressure at the ground, and this field was taken from the paper by BERKOFKY & BERTONI (1955). The zonally averaged meridional wind was obtained from the resulting ω -fields through use of the continuity equation

$$\frac{\partial[v]}{\partial \varphi} \cos \varphi = -a \cos \varphi \frac{\partial[\omega]}{\partial p} \quad (7)$$

where $[v] = 0$ at $\varphi = 90^\circ\text{N}$.

Table 1 is a summary of the results obtained from the 0000 GMT data for January 1963 using the 850, 700, 500, 300 and 200 mb geopotential height analyses. Terms 1 through 5 represent the five integrals in order of appearance in equation (2), and terms 6 through 11 represent those of equation (5). Columns WBD represent the results obtained by WIIN-NIELSEN, BROWN & DRAKE (1964). Note that here we have furthermore included the results obtained from the objective analyses of the same pressure levels produced by the U.S. Air Force Global Weather Central. All units are in 10^{-4} $\text{kJ m}^{-2} \text{sec}^{-1} \text{cb}^{-1}$. The values given in Table 1 are representative of the atmosphere between 850 and 200 mbs and the area north of 20°N.

From the values presented in Table 1 it is evident that the more complete estimate of $E(K_E, K_z)$ obtained here differs little from that obtained by Wiin-Nielsen, Brown and Drake. Furthermore there is little difference resulting in the monthly averaged results in the two sets of analyses. However, in this respect significant differences, due to analysis differences, can be found on particular days of this month. The differences between the earlier and the present calculated values for term 1 are due to the presence of the velocity potential in the wind components in the present study.

The major result of this study which should be noted is that the total estimated boundary influences obtained from both sets of data are sufficiently large to compensate for the exchange of kinetic energy from the zonal flow to the eddies. One may ask if the boundary terms, $B(K_z)$, act oppositely to give a negative contribution for a period where there existed

TABLE 1. Results of the various components comprising $E(K_E, K_z)$ and $B(K_z)$ of equations (2) and (5) as determined by Wiin-Nielsen, Brown and Drake (WBD) and as determined in the present investigation (B) from the National Meteorological Center and Air Force analyses for 0000 GMT January 1963.

Units are $\times 10^{-6}$ $\text{kJ m}^{-2} \text{sec}^{-1} \text{cb}^{-1}$.

Term	National Meteorological Center Data		U.S. Air Force Data	
	WBD	B	WBD	B
1	-6.51	-3.89	-6.38	-5.52
2	—	-1.87	—	-1.15
3	—	-0.18	—	0.06
4	—	-0.32	—	-0.29
5	—	+0.11	—	0.04
$E(K_E, K_z)$	-6.51	-6.15	-6.38	-6.86
6	5.40	6.40	0.89	1.70
7	—	0.51	—	0.14
8	—	1.17	—	0.58
9	—	3.21	—	1.82
10	—	0.19	—	0.18
11	—	6.36	—	3.22
$B(K_z)$	5.40	17.84	0.89	7.64

a change of K_E into K_z . Calculations were also made on the National Meteorological Center's January 1964 analyses. The resulting monthly averaged value of $E(K_E, K_z)$ was positive ($+3.73 \times 10^{-6} \text{ kJ m}^{-2} \text{sec}^{-1} \text{cb}^{-1}$) but the boundary terms, $B(K_z)$, were also positive ($+7.93 \times 10^{-6} \text{ kJ m}^{-2} \text{sec}^{-1} \text{cb}^{-1}$).

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ОБ ОБМЕНЕ МЕЖДУ ЗОНАЛЬНОЙ И ВИХРЕВОЙ КИНЕТИЧЕСКОЙ
ЭНЕРГИЯМИ ДЛЯ ЯНВАРЯ 1963 г

С помощью квазигеострофического метода произведена оценка обмена кинетической энергией между крупномасштабными атмосферными вихрями и зональным потоком в тропосфере к северу от 20° с. ш. для января 1963 г. Этот более полный результат сравнивается с аналогичным, полученным ранее Вин-Нильсеном, Брауном и Дрейком (1964).

Среднемесячные значения обмена находятся в хорошем согласии. Однако, результаты настоящего исследования указывают, что оценки боковых и вертикальных граничных членов должны приниматься в расчет, когда изучается баланс кинетической энергии в рассматриваемой части атмосферы.