On the mean meridional circulation and the flux of angular momentum over the northern hemisphere[1]

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ABSTRACT

The mean meridional circulation north of the latitude 15° N has been determined by an indirect method, which uses the angular momentum and continuity equations under the assumption that the vertical eddy flux of the zonal momentum is negligible in the free atmosphere. The method requires a knowledge of the (time-space) mean zonal motion and the mean meridional eddy flux of angular momentum. Data for these quantities have been derived for four seasons of the year (December–February, March–May, June–August and September–November from the upper winds statistics charts as published by CRUTCHER (1959). For each season an essentially three-cell circulation pattern is obtained with the strongest cell intensities occurring in winter and the weakest in summer. Except in summer, the direct cell in the lower latitudes is stronger than the indirect cell in the middle latitudes. The direct cell in the high latitudes is found to be very weak in all seasons.

The poleward transport of angular momentum and its different mechanisms are discussed. The role of the mean meridional circulation in the net flux across different latitudinal circles is found to be small except in the low latitudes in winter and spring. The transient eddies are in every season the main agency of the flux, but the standing eddies are also important, particularly in winter. The mean zonal stress of the atmosphere on the earth is derived from the convergence of the angular momentum flux.

1. Introduction

The relatively sparse upper-air wind observations have not yet provided reliable and generally acceptable information about the mean meridional velocities in the atmosphere. For measuring them it is therefore still necessary to resort to indirect methods, which under certain assumptions give approximate values for these velocities from other, more easily measurable quantities.

In the present article, the indirect method introduced by MINTZ & LANG (1955) is used. This method, described in section 2, assumes a knowledge of the mean zonal motion and the mean meridional eddy flux of angular momentum. Data for these basic quantities have been derived from the wind statistics charts in the way described in section 3. The computed mean meridional velocities are discussed in section 4 in comparison with the results from earlier investigations. Finally a study of the poleward flux of angular momentum and the associated zonal stress between the earth and the atmosphere is made in section 5.

The following notation is used:

a = radius of earth
g = acceleration of gravity
$M = a\cos\varphi(a\cos\varphi\Omega + u)$ = absolute angular momentum per unit mass
p = pressure
t = time
u = zonal component of velocity
$u_a = a\cos\varphi\Omega + u$ = absolute zonal velocity
v = meridional component of velocity
λ = longitude
φ = latitude
ϕ = geopotential
τ_λ = zonal component of the frictional stress acting across a horizontal surface

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$\omega = dp/dt$

Ω = angular velocity of the earth

$[x] = \frac{1}{2\pi}\int_0^{2\pi} x\, d\lambda$ = zonal average of x

$x^* = x - [x]$ = deviation from the zonal average

$\bar{x} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} x\, dt$ = time average of x

$x' = x - \bar{x}$ = deviation from the time average

2. An indirect method of determining the mean meridional circulation

The method to be discussed was first used by MINTZ & LANG (1955) and later in a slightly different form by GILMAN (1963). In principle it consists of solving the angular momentum equation and the continuity equation for the mean meridional circulation under the assumption that the vertical eddy stresses are small in the free atmosphere.

If we exclude from our consideration the lower layers of the troposphere where pressure surfaces intersect the earth, we obtain by averaging the equations of angular momentum and mass continuity with respect to longitude and time

$$\frac{\partial\overline{[M]}}{\partial t} + \frac{1}{r}\frac{\partial}{a\partial\varphi}(r[\overline{Mv}]) + \frac{\partial}{\partial p}[\overline{M\omega}] = r[\overline{F}_\lambda] \quad (1)$$

and

$$\frac{1}{r}\frac{\partial}{a\partial\varphi}(r[\bar{v}]) + \frac{\partial p}{\partial[\bar{\omega}]} = 0, \quad (2)$$

where $r = a\cos\varphi$ and F_λ denotes the zonal component of the frictional force. Because $M = ru_a$, we can always write

$$[\overline{Mv}] = r[\bar{u}_a][\bar{v}] + r([\bar{u}^*\bar{v}^*] + [\overline{u'v'}]) \quad (3)$$

and

$$[\overline{M\omega}] = r[\bar{u}_a][\bar{\omega}] + r([\bar{u}^*\bar{\omega}^*] + [\overline{\omega'u'}]). \quad (4)$$

The first term on the right hand side of Eq. (3) represents the contribution to the mean meridional flux of angular momentum by the mean meridional circulation and the terms in the parenthesis the contribution by the large-scale "standing" zonal eddies and the large-scale transient eddies. Eq. (4) is the corresponding representation for the mean vertical flux of angular momentum by the atmosphere's large-scale motion. If we assume that the frictional force is determined primarily by the vertical eddy stress, i.e.

$$F_\lambda = -g\frac{\partial\tau_\lambda}{\partial p},$$

we obtain, by substituting (3) and (4) into (1), the following differential equations for the mean meridional circulation:

$$\frac{\partial}{a\partial\varphi}(r[\bar{u}_a]\cdot r[\bar{v}]) + \frac{\partial}{\partial p}(r[\bar{u}_a]r[\bar{\omega}])$$
$$= -\frac{\partial}{a\partial\varphi}r^2([\bar{u}^*\bar{v}^*] + [\overline{u'v'}]) - r\frac{\partial\overline{[M]}}{\partial t}$$
$$-\frac{\partial}{\partial p}r^2(\bar{u}^*\bar{\omega}^*] + ([\overline{u'\omega'}] + g[\bar{\tau}_\lambda]), \quad (5)$$

$$\frac{\partial}{a\partial\varphi}(r[\bar{v}]) + \frac{\partial}{\partial p}(r[\bar{\omega}]) = 0. \quad (6)$$

From these equations, which so far are quite general, $[\bar{v}]$ and $[\bar{\omega}]$ are to be obtained. $[\bar{u}_a]$, $[\bar{u}^*\bar{v}^*]$ and $[\overline{u'v'}]$ are supposed to be relatively well known from the wind statistics. Also the mean local change of angular momentum, $\partial\overline{[M]}/\partial t$, is a measurable quantity, but we will, due to its smallness in a seasonal picture, neglect it in the present study. The essential physical assumption to be made is that the vertical flux of zonal momentum by large-scale as well as by small-scale turbulence is small in the free atmosphere. Thus, we neglect also the third term on the right-hand side of Eq. (5). Some evidence for the smallness of this term is given in discussion of the results (Section 4).

Equations (5) and (6) are now integrated over a rectangular area in the meridional φ,p-plane. Let the subindices N, S, B and T denote the northern, southern, bottom and top boundaries of the area, respectively. Let further R denote the mean value of $r[\bar{u}_a]$ at the boundary and m the mass flux through the boundary so that, for example,

$$m_N = \frac{2\pi r}{g}\int_{p_T}^{p_B}[\bar{v}]_N\, dp.$$

$$m_T = -\frac{2\pi}{g}\int_{\varphi_S}^{\varphi_N} r[\bar{\omega}]_T\, a\, d\varphi$$

(upward mass flux taken positive).

If the grid size is not taken too large, a good approximate is given by the following expressions:

TABLE 1. *Standing eddy* ($SE = [\bar{u}^*\bar{v}^*]$) *and transient eddy* ($TE = [\overline{u'v'}]$) *part of the poleward flux of linear zonal momentum for winter (December–February), spring (March–May), summer (June–August) and fall (September–November). Unit:* $m^2\ sec^{-2}$.

Presssure	70° N		60° N		50° N		40° N		30° N		20° N		15° N	
level (mb)	SE	TE	SE	TE	SE	TE	SE	TE	SE	TE	SE	TE	SE	TE
Winter														
100	0.2	−11.5	−3.6	−6.4	1.7	8.8	15.8	3.6	26.4	7.6	9.9	6.4	2.4	1.1
200	−0.1	−5.2	2.5	−3.8	10.3	6.4	17.1	34.2	18.9	46.8	7.3	18.4	−7.2	7.9
300	0.4	−3.7	1.6	−8.5	11.9	6.8	18.9	37.8	16.4	42.0	7.5	16.0	−0.3	7.5
500	0.1	4.8	1.4	−1.7	6.3	3.7	4.8	14.3	4.5	16.7	0.4	6.6	0.6	3.0
700	−0.1	0.4	1.0	−2.2	2.3	−2.4	0.0	5.2	−1.1	7.1	−0.1	4.4	−0.4	2.3
850	0.5	0.8	1.4	−3.9	2.2	−4.3	2.4	−4.3	0.5	4.9	−0.7	2.8	−1.6	1.7
Spring														
100	−1.4	1.7	−1.6	0.6	−0.7	1.2	0.6	8.4	5.7	13.8	2.0	3.3	0.8	0.4
200	1.0	−2.4	−1.1	−4.3	1.6	2.5	2.5	25.8	8.8	31.4	−0.6	17.4	1.3	8.9
300	−2.6	−3.6	−2.6	−10.5	2.6	5.8	4.8	27.4	6.0	26.0	−1.3	13.7	−1.8	9.0
500	−1.8	−0.6	−2.2	−1.3	1.6	0.4	3.1	13.6	2.5	13.0	0.5	5.4	−0.1	3.1
700	−0.7	0.1	−1.5	−2.6	1.0	−0.7	1.2	5.6	1.0	5.6	−1.7	3.2	−0.4	2.5
850	0.1	1.1	−0.6	−0.6	0.1	−1.0	0.8	0.6	1.3	2.8	−2.5	2.1	−1.9	1.5
Summer														
100	−0.6	1.3	0.6	0.8	0.0	4.7	1.8	10.1	8.9	6.2	5.9	0.1	4.2	−2.1
200	−0.4	−1.0	0.1	1.5	0.8	19.2	2.9	35.8	4.9	17.3	9.5	6.1	17.0	3.2
300	−0.2	1.3	0.7	−0.2	2.2	11.9	−0.4	23.6	0.9	11.0	2.9	3.5	5.1	1.9
500	−0.7	−1.1	−0.0	0.3	1.6	4.5	−0.1	7.1	−0.2	4.2	0.0	2.3	2.4	1.6
700	0.2	−0.7	−0.1	−0.2	1.7	2.2	1.0	3.3	0.2	2.1	−0.4	0.6	0.9	0.1
850	0.6	−0.8	−0.5	−1.1	0.9	1.2	1.8	0.2	0.5	1.4	−1.3	1.7	−1.2	1.5
Fall														
100	1.3	−0.1	0.6	1.6	2.0	4.9	9.4	8.6	8.6	9.7	−1.1	4.0	0.4	1.1
200	−1.5	−4.5	0.5	1.7	10.8	18.6	7.8	41.1	7.2	34.1	0.2	18.2	0.5	12.4
300	−0.5	−8.3	−2.1	−0.4	12.5	15.0	7.6	34.5	5.3	24.9	1.2	10.0	0.3	6.0
500	−0.6	1.3	−1.2	−0.1	4.1	5.2	2.3	14.9	2.0	7.4	0.1	1.9	0.0	1.4
700	−0.9	0.0	−1.0	−1.0	1.4	0.4	0.7	5.1	1.1	4.9	−0.3	2.3	0.1	1.7
850	1.0	−1.0	−0.8	−2.7	0.4	−1.1	1.2	2.7	0.2	3.7	−0.5	2.2	−1.0	2.1

$$\frac{2\pi}{g}\int_{p_T}^{p_B} r[\bar{u}_a]_N\, r[\bar{v}]_N\, dp = R_N m_N,$$

$$\frac{2\pi}{g}\int_{\varphi_S}^{\varphi_N} r[\bar{u}_a]_T\, r[\bar{\omega}]_T\, a\, d\varphi = -R_T m_T$$

with similar expressions for the southern and bottom boundaries. Thus we obtain from (5) and (6) after integration and multiplication by a constant factor $2\pi/g$,

$$R_N m_N - R_S m_S + R_T m_T - R_B m_B = -(j_N - j_S) \qquad (7)$$

and

$$m_N - m_S + m_T - m_B = 0, \qquad (8)$$

where
$$j = \frac{2\pi r^2}{g}\int_{p_T}^{p_B} ([\bar{u}^*\bar{v}^*] + [\overline{u'v'}])\, dp$$

is the mean poleward eddy flux of angular momentum in the layer considered. Solving (7) and (8) for m_S and m_B in terms of m_N, m_T and the known quantities R and j we obtain

$$m_S = \frac{R_N - R_B}{R_S - R_B} m_N + \frac{R_T - R_B}{R_S - R_B} m_T + \frac{j_N - j_S}{R_S - R_B}, \qquad (9)$$

$$m_B = -\frac{R_N - R_S}{R_S - R_B} m_N - \frac{R_T - R_S}{R_S - R_B} m_T - \frac{j_N - j_S}{R_S - R_B} \qquad (10)$$

At the pole the mean meridional mass flux must be zero. Specifying the mean vertical mass flux at the uppermost pressure level used in the calculations and starting the computations from the northernmost and uppermost block the mean meridional mass circulation and associated velocities can be determined everywhere in the meridional plane except in the lower layers, where the assumption of vanishing stress is not valid.

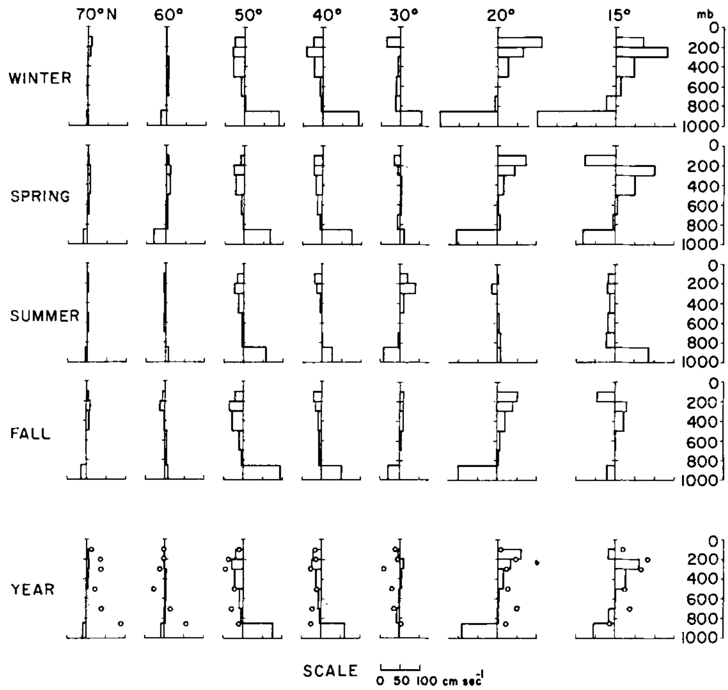

FIG. 1. The computed mean meridional velocities. Circles deonte "observed" velocities.

3. Data

The basic data required by the method just described are the mean zonal velocity $[\bar{u}]$ and the mean meridional eddy flux of angular momentum. These data were derived from the "Upper-wind statistics charts of the Northern Hemisphere" as published by CRUTCHER (1959). The statistics include, among other things, charts of the mean zonal wind component $\bar{u}$, the mean meridional wind component $\bar{v}$, standard deviation, σ_u and σ_v, of the zonal and meridional wind component and the correlation coefficient, $r_{uv'}$, between the two wind components for the 100 mb, 200 mb, 300 mb, 500 mb, 700 mb and 850 mb pressure levels and separately for the four seasons: December–February, March –May, June–August and October–November. The statistics is primarily based upon actual wind observations in the Northern Hemisphere from a period of several years.

The mean zonal wind $[\bar{u}]$, as obtained from this statistics, has been discussed by VUORELA & TUOMINEN (1964) and will not be commented on here. The standing eddy part of the covariance between zonal and meridional wind components was at each latitude computed as

$$[\bar{u}^*\bar{v}^*] = [\overline{uv}] - [\bar{u}][\bar{v}].$$

and the transient eddy part as

$$[\overline{u'v'}] = [r_{uv}\sigma_u\sigma_v].$$

Because at some longitudes the analysis do not cover the very low latitudes, calculations could not be made south of 15° N and already at this latitude some extrapolation had to be made.

The results for the velocity covariance (poleward eddy flux of linear zonal momentum) are given in Table 1. It is seen that both the standing eddy flux and the transient eddy flux of momentum are generally northwards except at the latitudes 60° N and 70° N, where southward fluxes dominate. The standing eddy flux is generally smaller than the transient eddy flux. Both of them have largest magnitude in the upper troposphere. With regard to seasonal variation the values are largest in winter and smallest in summer season.

TABLE 2. *Mean meridional velocity over the Northern Hemisphere in cm sec^{-1}. (a) computed, (b) "observed" (at the upper pressure level).*

Pressure mb	70° N (a)	70° N (b)	60° N (a)	60° N (b)	50° N (a)	50° N (b)	40° N (a)	40° N (b)	30°N (a)	30°N (b)	20° N (a)	20° N (b)	51° N (a)	51° N (b)
Winter														
100–200	8	−20	−1	−15	−26	25	−25	−50	−35	−40	114	70	72	90
200–300	6	0	0	−76	−31	−30	−43	−30	−1	−20	65	170	134	210
300–500	−1	−20	5	−55	−28	−45	−23	−45	−6	−75	27	35	48	150
500–700	−3	−13	4	−40	−10	−35	−7	−40	−9	−50	−1	18	15	20
700–850	−1	0	3	−18	1	−40	−5	−40	−11	−40	−7	60	−22	55
850–1000	−4	64	−16	45	87	−20	91	5	55	20	−147	−25	−199	−103
Spring														
100–200	1	20	4	36	−10	−11	−23	−6	−16	−55	72	−34	−78	4
200–300	4	11	10	29	−27	11	−23	−34	−7	−53	42	19	98	106
300–500	5	76	9	13	−23	−28	−18	−51	3	−140	14	−81	50	−4
500–700	1	21	4	−23	−7	−6	−13	11	3	−39	3	−24	7	−18
700–850	0	11	4	−9	−3	−32	−5	−10	−6	−3	6	86	−6	65
850–1000	−12	69	−30	27	67	−1	76	−16	12	29	−104	1	−82	−43
Summer														
100–200	1	59	−4	−40	−17	−13	−20	−17	20	26	2	6	−17	15
200–300	1	89	−3	6	−24	−70	−14	34	39	65	−14	17	−20	1
300–500	0	84	−2	−11	−14	−51	−4	−14	10	61	0	91	−13	101
500–700	1	29	−3	−7	−5	−22	0	−3	0	17	5	23	−17	30
700–850	0	80	1	31	−5	−31	2	−12	−4	16	8	29	−23	31
850–1000	−4	110	8	79	57	3	26	−22	−42	16	8	120	87	103
Fall														
100–200	3	−7	−8	3	−22	−41	−22	9	10	18	51	−16	−46	−33
200–300	9	41	−12	20	−36	−61	−15	−24	10	−17	41	−23	31	14
300–500	5	−1	−3	3	−28	−61	−8	−9	10	−10	20	42	23	17
500–700	0	46	4	−41	−10	−30	−7	−24	4	−6	7	81	2	63
700–850	0	61	4	49	−5	−11	−7	−35	−2	−37	3	24	−1	5
850–1000	−16	114	7	69	95	−24	51	−75	−30	−54	−100	−9	−21	−5
Year														
100–200	3	13	−2	−4	−19	−10	−22	−16	−5	−13	60	7	−17	19
200–300	5	35	−1	−5	−30	−38	−24	−14	10	−6	34	46	61	83
300–500	2	35	2	−13	−23	−46	−13	−30	4	−41	16	22	27	66
500–700	0	21	3	−28	−8	−23	−7	−14	−1	−20	3	25	1	24
700–850	0	38	3	13	−4	−29	−4	−24	−6	−16	3	50	−13	39
850–1000	−9	89	−8	55	77	−11	61	−27	−1	3	−90	22	−54	−12

The vertically integrated values of momentum flux will be discussed in Section 6 in connection with the poleward flux of angular momentum.

4. Mean meridional circulation

The mean meridional mass circulation and the corresponding meridional and vertical velocities were determined for the layers between 100 mb and 850 mb from Equations (11) and (12), by using the data described in the previous section, under the assumption that the vertical mass flux at the 100 mb level is zero. For the layer from 850 to 1000 mb the mean meridional velocity was then obtained from the requirement of zero net mass flux across a latitude circle. The resulting values of the mean meridional velocities in the layers 100–200 mb, 200–300 mb, 300–500 mb, 500–700 mb, 700–850 mb and 850–1000 mb are given in Table 2 and Fig. 1. For comparison, Table 2 also includes the "observed" values as obtained for the standard pressure levels directly from the wind statistics charts by taking zonal averages of the $\bar{v}$-data.

Considering first the winter season it is seen that a three-cell circulation pattern appears from the calculations with a so-called direct cell in the low and high latitudes and an indirect cell in the middle latitudes. The strongest velocities are found at the latitude 15° N, where the mean southward velocity between 850 and 1000 mb levels gets a value of 2 m sec^{-1} and the maximum northward velocity (between 200 and 300 mb levels) a value of 1.3 m sec^{-1}. At this latitude the total mass flux southward below 700 mb level (and northwards at higher levels) is computed to be 130×10^6 tons sec^{-1}. This value is larger than the intensity of the Hadley-circulation computed by MINTZ & LANG (1955), about 80×10^6 tons sec^{-1}, and by TUCKER (1959, 1960), 110×10^6 tons sec^{-1}, but is smaller than the intensities computed by PALMÉN (1955), PALMÉN, RIEHL & VUORELA (1958), DEFANT & VAN DE BOOGARD (1963) and PALMÉN & VUORELA (1963), which are all between 200 and 260×10^6 tons sec^{-1}. Thus the Hadley-circulation resulting from our calculations does not radically deviate from the results of earlier investigations. There can hardly be any doubt at the present time about the existence of this circulation cell in winter. However, an uncertainty of almost 50 per cent seems still to remain with regard to its mean intensity. It is of interest to notice that the "observed" meridional velocities (Table 2) at 15° N and 20° N are quite similar to the computed velocities. However, due to the fact that geostrophic wind has necessarily to some extent been used in preparation of the wind statistics, too large an emphasis must not be put on the "observed" values.

The indirect Ferrel circulation in winter is, according to our calculations, well defined at the latitudes 40° N and 50° N, where the intensity of the mass circulation is -42×10^6 tons sec^{-1} and -35×10^6 tons sec^{-1}, respectively. These values are somewhat larger than the corresponding values by MINTZ & LANG, but the location of the Ferrel cell is about the same. According to PALMÉN & VUORELA the center of the Ferrel cell is more northward, but their results, based upon the "observed" values, are not very reliable in the middle latitudes. The direct circulation in high latitudes is computed to be very slow, producing velocities of only a few centimeters per second. MINTZ & LANG did not get a direct polar cell from their computations. The reason for this discrepancy between their investigation and the present one is connected with the fact that their values of the mean meridional flux of angular momentum (MINTZ, 1955), differ considerably in the high latitudes from those obtained in the present study (see section 5). For the Southern Hemisphere GILMAN (1963) computed quite large velocities for the direct polar cell. However, due to differences in topography and thermal conditions the circulation patterns for the Northern and Southern polar cap cannot be expected to be similar.

The computed mean meridional velocities are generally strongest in winter and weakest in summer and from winter to summer the whole circulation system shifts northwards. During the summer season we find at the latitude 15° N and to a smaller extent also at 20° N northward motion in the lower troposphere and southward motion higher up. This could be an indication of the Southern Hemisphere Hadley cell extending north to about 20° N. At latitude 30° N the Northern Hemisphere Hadley circulation is found with maximum velocities of 40 cm sec^{-1} occurring in the layers from 200 to 300 mb and from 850 to 1000 mb. The total mass flux southwards in the lower troposphere and northwards higher up at this latitude is 24×10^6 tons sec^{-1}. An indirect meridional circulation is found at 40° N, 50° N and 60° N with maximum intensity, 22×10^6 tons sec^{-1}, occurring at 50° N. A very weak direct circulation is obtained at 70° N. In general, the computed mean meridional circulation in summer fits relatively well together with the scheme presented by VUORELA & TUOMINEN (1964) but differs considerably with respect to the intensity of the circulation from that calculated by MINTZ & LANG (1955).

With regard to the mean meridional velocities in spring and fall one might expect them to be similar to each other and intermediate between the values in winter and summer. However, the results indicate many exceptions to the rule. For example, at latitude 15° N the velocities are larger in spring than in fall and during both seasons the velocity profiles in the upper troposphere differ considerably from those in winter and in summer. In general, the circulation cells in fall seem to be located farther north than in spring. This same feature is also characteristic of angular momentum flux (section 5) and is probably due to the definition of the seasons (Winter: December–February; Spring: March–May; Summer: June–August; fall: September–November): the summer culmination of different atmospheric properties does not occur in the middle of July but later.

The annual mean meridional velocities (average of the seasonal values) show direct circulation at 15° N and 20° N, practically no circulation at 30° N, indirect circulation at 40° N and 50° N and a very weak direct circulation at higher latitudes. It is of interest to compare the computed meridional velocities with "observed" values, which should be most reliable, if at all, in the annual mean picture. It is seen in Fig. 1 that some of the basic features of the computed mean meridional velocities are present also in the "observed" values. Thus, at latitudes 15° N and 20° N the observations give northward velocities, at 40° N and 50° N southward velocities in the free atmosphere, in qualitative agreement with the computed $[\bar{v}]$-values. However, at 30° N, where the computed velocities are very small, the "observed" values indicate a relatively large southward motion and at the latitudes 60° N and 70° N the discrepancy between the two sets of values is very large. Generally, the observed velocities have the same sign in most of the pressure levels and do not result in a vanishing net mass flux across the latitude circle. This may partly be due to the fact that the upper wind statistics do not include any level between 850 and 1000 mb, where according to the computed velocities the mean meridional mass flux is opposite to that in the free atmosphere. Actually, in the trade wind latitudes the mean meridional circulations can probably be best investigated with wind observations at the earth's surface and in the lower troposphere, as has been done, for example, by RIEHL & YEH (1951), PALMÉN (1955), TUCKER (1957) and PALMÉN, RIEHL & VUORELA (1958). However, at higher latitudes, where there are more mountains, it probably is more dangerous to draw conclusions about the mean meridional circulations only on the basis of wind observations close to the earth's surface.

Considering now the reliability of the computed meridional velocities, errors can be expected to arise from the physical assumptions made, uncertainties in data and from the numerical calculation schemes employed.

Physically the most important assumption in derivation of the basic equations (5) and (6) is that the vertical flux of angular momentum by large-scale and small-scale eddies was neglected in comparison with the flux associated with the mean meridional circulations. When looking for evidence to support this assumption it may first be noticed that the meridional eddy flux of angular momentum takes place primarily in the upper troposphere. Accordingly, the magnitude of the vertical momentum flux across some midtropospheric level must be about the same as the flux across the earth's surface. This is measured by the mean zonal component of the surface stress, which is known to be of the order of 1 dyne cm^{-2}. The validity of our assumption depends upon the magnitude of the neglected flux terms in relation to this value.

With regard to the vertical transport of zonal momentum by large-scale disturbances, the first quantitative evaluation is that by WHITE & COOLEY (1952), who obtained values of the same order as the surface stress. However, using a larger body of data and a less crude method of determining vertical velocity, STARR & DICKINSON (1963) obtained flux values which are generally smaller than 0.1 dyne cm^{-2}. Equally small values were found by HOLOPAINEN (1964) for the transient eddy part of the flux over the British Isles. On the other hand, the recent numerical experiments on the atmosphere's general circulation (e.g. SMAGORINSKY, 1963) also indicate that the vertical transfer of zonal momentum in baroclinic waves is much smaller than that associated with the mean meridional circulations.

More uncertain is the effect of the small-scale zonal stress, which SHEPPARD (1953) has argued to be the dominating mechanism of the vertical flux of angular momentum in the middle latitudes. The fact, however, that the frictional forces are known to be important close to the earth's surface, simply means that the internal small-scale stresses in the higher levels are smaller than the surface stress. An exchange coefficient of 10^5 cm^2 sec^{-1} would give typical zonal stress in the middle troposphere of the order of 0.1 dynes cm^{-2}, which is one order of magnitude smaller than the surface stress. Accordingly, the neglect of small-scale internal stress would cause an error of only some ten per cent in the computed mass circulation. If so, the method applied could be considered quite satisfactory, because the error in the observed values of mean meridional velocities is at present much larger.

If the "observed" mean meridional velocities are assumed to be realistic, it is possible to determine from Eqs. (5) and (6) the vertical eddy flux of angular momentum as a residue term. This approach has been used by TUCKER (1960) and GILMAN (1963). But the mere fact that the tentative conclusions of the above authors differ largely from each other, indicates that the method is very inaccurate. In the present study, no attempt has been made to use the "observed" $[\bar{v}]$-values for such a purpose.

In the present calculations the general decrease of mean zonal momentum in spring and increase of it in fall has been neglected. This neglect is justified by a study of the rate of the seasonal change of the mean zonal motion, as evaluated for example by PRIESTLEY (1951), showing that in most latitudes the mean local change of angular momentum during a three-month period is one order of magnitude smaller than the convergence of the meridional momentum flux.

The data used in the calculations for the meridional eddy flux of angular momentum seem to be representative (see the next section). Some errors arise in computations from the use of a rather large grid interval, particularly in the meridional direction. However, an experiment made with a finer grid system indicated that this fact is not of large importance for the gross features of the mean meridional circulation as discussed above.

As a general conclusion from the above discussion we can say that no definite limit can be set for the magnitude of the error in the present calculations. There is, however, some evidence that the method used provides a good approximation to the real mean meridional circulation.

After computing the mean meridional circulation the next natural step would be a study of its role in the budgets of different properties of the atmosphere. The author has in an earlier paper (HOLOPAINEN, 1965) discussed the role of mean meridional circulation in the energy balance of the atmosphere, in light of composite patterns of the mean meridional circulation as obtained from some earlier investigations. The velocities computed in the present study are similar to these composite velocities and would for the same problem give qualitatively similar results. Therefore, the question about the atmospheric energy balance will not be taken up here. However, the role of mean meridional circulation in the angular momentum balance of the atmosphere will be discussed in the next section.

5. Meridional flux of angular momentum

The mean poleward flux of the zonal angular momentum across a latitudinal circle is given by

$$F = \int_0^{2\pi} \int_0^{p_0} \overline{Mv} a \cos\varphi \frac{dp}{g} d\lambda$$
$$= \frac{2\pi r}{g} \int_0^{p_0} (r^2 \Omega [\bar{v}] + r[\overline{uv}])\, dp, \qquad (11)$$

where p_0 denotes the pressure at the earth's surface. On the basis of surface pressure observations we know that the net meridional mass shifts across latitudes are extremely small. Accordingly, the first term on the right-hand side of Eq. (11) can be neglected and we are left with

$$F = \frac{2\pi r^2}{g} \int_0^{p_0} [\overline{uv}]\, dp. \qquad (12)$$

This expression can be written as a sum of three terms:

$$F = \frac{2\pi r^2}{g} \int_0^{p_0} [\bar{u}][\bar{v}]\, dp + \frac{2\pi r^2}{g} \int_0^{p_0} [\bar{u}^* \bar{v}^*]\, dp$$
$$+ \frac{2\pi r^2}{g} \int_0^{p_0} [\overline{u'v'}]\, dp. \qquad (13)$$

The first, second and third term on the right-hand side of Eq. (13) represent, respectively, the contribution made to the poleward flux of angular momentum by the mean meridional circulation, the standing zonal eddies and the transient eddies.

From the wind covariance data given in Table 1 the standing eddy and transient eddy part have been evaluated for the layer between

TABLE 3. *The poleward flux of angular momentum over the Northern Hemisphere between 100 mb and 1000 mb in 10^{25} gm cm^2 sec^{-2}.*

A: flux associated with the mean meridional circulation; *B:* flux associated with the standing zonal eddies; *C:* flux associated with the transient eddies. Total = $A + B + C$.

	70° N	60° N	50° N	40° N	30° N	20° N	15° N
Winter							
A	0.0	0.1	−1.6	−3.7	−3.0	10.7	10.0
B	0.0	0.8	5.2	10.3	12.5	5.3	1.0
C	−0.2	−2.2	1.9	18.8	32.5	16.2	7.8
Total	−0.2	−1.3	5.5	25.4	42.0	32.2	18.8
Spring							
A	0.0	0.2	−1.0	−2.5	−0.9	5.5	2.5
B	−0.2	−1.1	1.1	2.9	5.8	−1.6	−1.2
C	−0.2	−1.8	1.0	16.6	22.9	13.6	8.6
Total	−0.4	−2.7	1.1	17.0	27.8	17.5	9.9
Summer							
A	0.0	−0.1	−0.8	−0.8	0.5	−0.1	1.2
B	0.0	0.0	1.2	1.1	2.2	3.3	7.6
C	−0.1	0.1	5.8	14.7	10.0	4.5	2.6
Total	−0.1	0.0	6.2	15.0	12.7	7.7	11.4
Fall							
A	0.0	0.1	−1.7	−1.6	1.1	1.3	0.4
B	−0.1	−0.6	4.6	4.8	5.0	0.4	−0.5
C	−0.5	−0.3	5.7	21.3	20.5	10.6	7.5
Total	−0.6	−0.8	8.6	24.5	26.6	12.3	7.4
Year							
A	0.0	0.0	−1.3	−2.0	0.4	3.0	1.5
B	−0.1	0.0	3.2	4.0	3.8	−1.0	−0.5
C	−0.2	−1.2	3.5	18.5	23.1	15.4	10.9
Total	−0.3	−1.2	5.4	20.5	27.3	17.4	11.9

100 mb and 1000 mb.[1] At 1000 mb the covariance terms have been assumed to be zero; due to smallness of these terms in the lower troposphere this assumption has no significance for the results. For the evaluation of the role of the mean meridional circulation, the computed mean meridional velocities and the mean zonal motion, as given in the wind statistics, have been used. In this way, the total flux and its different components have been obtained for the atmosphere below 100 mb for each season. With regard to the annual mean conditions, the total flux was computed as an average of the seasonal totals, the fluxes associated with the annual mean meridional velocities and standing eddies were evaluated separately and the transient eddy flux obtained as a residue. The results are given in Table 3.

Considering first the total fluxes in different seasons (Fig. 2) it is seen that the flux generally is northward in the low and middle latitudes and southward at higher latitudes where its magnitude, however, is small. South of about 40° N the flux is largest in winter, smallest in summer and intermediate in spring and in fall, whereas at higher latitudes the seasonal differences are more confused. Between spring and fall there is a systematic difference in that in fall the whole flux pattern is situated further north than in spring. This same fact was already noticed with respect to the patterns of the computed meridional velocities (Fig. 1).

The total flux for winter and summer agrees well in the middle and low latitudes with the geostrophic eddy flux of angular momentum as computed by MINTZ (1955) for the two winter

[1] The contribution to the poleward flux of angular momentum by the layers above the 100 mb level is (e.g. OORT, 1963) one order of magnitude smaller than that by the lower layers.

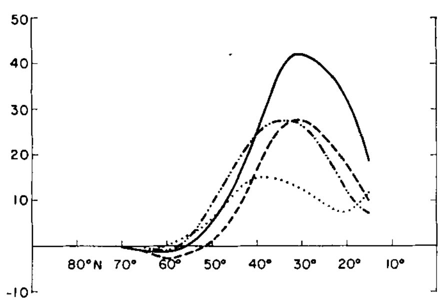

FIG. 2. The total poleward flux of angular momentum over the Northern Hemisphere in winter (heavy line), in spring (dashed line), in summer (dotted line) and in fall (dashed-dotted line). Unit: 10^{25} gm cm^2 sec^{-2}.

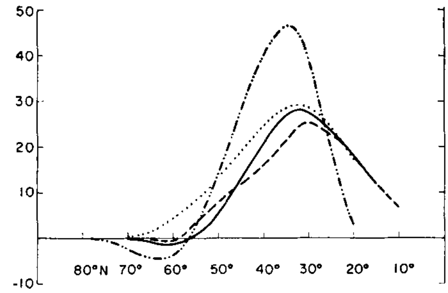

FIG. 4. The annual mean poleward flux of angular momentum over the Northern Hemisphere according to the present study: heavy line. BUCH (1954): dashed line. MINTZ (1955): dotted line. WIIN-NIELSEN *et al.* (1963, 1964): dashed-dotted line. Unit: 10^{25} gm cm^2 sec^{-2}.

months and summer months of 1949. However, MINTZ obtained a northward flux even in the higher latitudes in winter. In this respect his results are probably unrealistic because the southward flux at the latitudes 60° N and 70° N as obtained in the present study is qualitatively supported by the results of STARR & WHITE (1954), BUCH (1954) and WIIN-NIELSEN *et al.* (1963, 1964).

During each season south of about 40° N the main part of the transport is accomplished by transient eddies (see Table 3). At latitudes 50° N and 60° N the standing eddies are, however, very important, particularly in winter. The flux associated with the mean meridional circulation is generally northwards in the low latitudes and southwards in the middle latitudes. Its magnitude is relatively small except in winter and in spring at the latitudes 15° N and 20° N, where it constitutes a large fraction of the total transport. In this connection it must be emphasized that the role of the mean meridional circulation in the poleward flux of angular momentum is small only when the vertically integrated flux values are considered and, accordingly, the term $r^2\Omega[\bar{v}]$ in Eq. (11) integrates to zero. If the poleward flux of angular momentum at different pressure levels is considered, the mean meridional circulation plays by far the most important role.

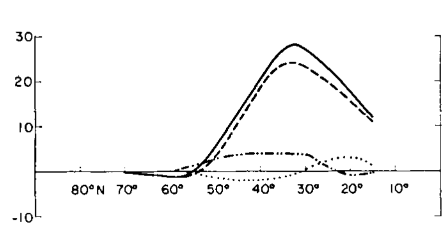

FIG. 3. The annual mean poleward flux of angular momentum. Heavy line: total flux; dashed line: flux associated with mean meridional circulation; dashed-dotted line: flux associated with the standing zonal eddies. Unit: 10^{25} gr cm^2 sec^{-2}.

With regard to the annual mean conditions (Fig. 3) the role of the transient eddies is seen to be even more dominant than during different seasons. Their contribution is almost 90 per cent of the total flux in the region of the flux maximum. The fluxes associated with the annual mean standing eddies and meridional circulations are generally small in magnitude and also have some tendency to balance each other.

Fig. 4 has been constructed in order to compare the results of the present study with earlier calculations of the poleward angular momentum flux over the Northern Hemisphere. The results by BUCH (1954) are for the year 1950 and have been calculated using the actual wind observations. The curve representing MINTZ's (1955) results is an average of the geostrophic angular momentum flux in January, February, July and August of the year 1949 and that representing WIIN-NIELSEN's *et al.* (1963, 1964) results an average of the flux in January, April, July and October of 1962. All the curves refer to the layer between 100 mb and 1000 mb. When constructing the curve representing the results by WIIN-NIELSEN *et al.* it was assumed that the flux at 100 mb is half of that at 200 mb, which was the uppermost level where data were given. It is seen that the results of the present study agree well with the values by

TABLE 4. *Mean zonal stress of the atmosphere on the earth as computed from the divergence of the poleward flux of angular momentum. Unit: dynes cm*$^{-2}$.

	90°–70° N	70°–60°	60°–50°	50°–40°	40°–30°	30°–20°	20°–15°
Winter	< 0	− 0.2	0.7	1.4	0.9	− 0.4	− 1.0
Spring	< 0	− 0.5	0.4	1.1	0.6	− 0.4	− 0.6
Summer	< 0	0.0	0.7	0.6	− 0.1	− 0.2	− 0.3
Fall	< 0	0.0	1.0	1.1	0.1	− 0.6	− 0.4
Year	< 0	− 0.2	0.7	1.1	0.4	− 0.4	− 0.4

BUCH and in the middle and low latitudes also with those by MINTZ. All three curves give for the maximum of the momentum transport, occurring between 30° N and 40° N, a value between 25×10^{25} gr cm^2 sec^{-2} and 30×10^{25} gm cm^2 sec^{-2}. At higher latitudes Mintz's calculations do not show the southward transport found by all other investigators. At this point it is of interest to notice that the poleward flux of angular momentum in the Southern Hemisphere, as calculated by OBASI (1963), shows a remarkable similarity with the flux computed in the present study, except that the equatorward flux at high latitudes in the Southern Hemisphere is stronger than in the Northern Hemisphere.

The most striking feature of Fig. 4 is the deviation of WIIN-NIELSEN's *et al.* (1963, 1964) results from the others in that they give smaller northward flux in the low latitudes and larger magnitude of the flux in the middle and high latitudes. In their calculations the above authors used a stream function obtained from a simplified form of the balance equation and from objectively analyzed geopotential data. The large discrepancy between the angular momentum flux obtained in this way and the other flux evaluations is not only a peculiar characteristic of the year 1962 but appears also in the results computed from later data (WIIN-NIELSEN, to be published) and seems to be qualitatively the same for all seasons. No easy explanation is available for this systematic difference. It has been pointed out by PRIESTLEY & TROUP (1964) that the poleward flux of angular momentum, as obtained from actual wind observations, can be biased owing to a tendency for observations to be missing in cases of strong wind. However, the fact that MINTZ's geostrophic flux calculations give about the same results as those based upon actual winds, suggests that the main source of systematic error should be sought in the scheme of objective analysis and the use of the particular stream function employed by WIIN-NIELSEN *et al.* This question, being of paramount importance for numerical weather prediction, requires thorough investigation.

From the convergence of the poleward flux of angular momentum one can determine the average zonal component of the stress between the earth and the atmosphere. From the angular momentum equation one obtains:

$$\frac{1}{2\pi a^2 \cos\varphi}\frac{\partial F}{\partial \varphi} = -a\cos\varphi\,([\bar{\tau}_{\lambda 0}] + [\bar{\tau}_{\lambda m}]), \qquad (14)$$

where F is the angular momentum flux as defined by Eq. (13), $\tau_{\lambda 0}$ denotes the zonal component of the frictional stress at the earth's surface and $[\tau_{\lambda m}]$ is a measure of the so-called mountain torque, arising from the nonvanishing values in the low troposphere of the zonal pressure gradient, when integrated over the whole latitude. When solved for $[\bar{\tau}_{\lambda 0}] + [\bar{\tau}_{\lambda m}]$, the total zonal stress of the atmosphere on the earth, one obtains

$$[\bar{\tau}_{\lambda 0}] + [\bar{\tau}_{\lambda m}] = -\frac{1}{2\pi a^3 \cos^2\varphi}\frac{\partial F}{\partial \varphi}. \qquad 1(5)$$

The total stress computed from this expression is given in Table 4. It is negative (westward torque exerted by the atmosphere on the earth) in the belts south of 30° N and positive (eastward torque) in the belts between 30° N and 60° N; in summer the mean stress between 30° N and 40° N is a small negative. North of 60° N small negative stress values are again found. All these features are qualitatively well-known characteristics of the angular momentum balance of the atmosphere, empirically established already by WIDGER (1949). Most of the earlier studies consider, however, only periods of a few months or at most one year. As far as the flux of angular momentum, computed in the present study, can be considered representative for the normal conditions over the Northern Hemisphere during different seasons, the stress values in Table 4 should be considered as normals of the dynamic interaction between the earth and the atmosphere in different latitude belts.

The relative contribution to the total stress by the surface friction and mountains is not established with certainty. The frictional stress is negative in the low latitude belts, where the zonal component of the surface wind is predominantly from the east, and positive in the middle latitude westwind belt. These facts are clearly shown in PRIESTLEY'S (1951) results for the mean zonal stress between oceans and the atmosphere. On the other hand it has been shown by WHITE (1949) that the mountain effect is of the same order of magnitude and normally in the same direction as the frictional stress in low and middle latitudes. With regard to the higher latitudes even the sign of the frictional zonal stress is not known with certainty. According to White the mountain effect ($[\tau_{\lambda m}]$) between 60° N and 65° N is negative in most of the months of the year with largest magnitude in spring. This is in agreement with the total stress shown in Table 4. Thus the mountain effect, even if small, is probably an important factor in explaining the negative total stress and, accordingly, the equatorward flux of angular momentum in the high latitudes of the Northern Hemisphere.

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О СРЕДНЕЙ МЕРИДИОНАЛЬНОЙ ЦИРКУЛЯЦИИ И МОМЕНТЕ КОЛИЧЕСТВА ДВИЖЕНИЯ НАД СЕВЕРНЫМ ПОЛУШАРИЕМ

Средняя меридиональная циркуляция была определена методом, использующим уравнения неразрывности и момента количества движения в предположении, что в свободной атмосфере вертикальный турбулентный поток зонального количества движения принебрежимо мал.

Для этого метода необходимо знание среднего зонального движения и среднего меридионального турбулентного потока количества движения как функций пространства и времени. Эти величины для всех сезонов (декабрь–февраль, март–май, июнь–август, сентябрь–ноябрь) были получены из статистических диограмм верхних ветров опубликованных CRUTCHER (1959). Для каждого сезона получена существенно трех-ячеечная модель циркуляции с наибольшей интенсивностью ячеек зимой и наименьшей летом. Для всех сезонов, кроме лета, ячейки с прямой циркуляцией (т. е. против часовой стрелки, если смотреть с востока) выражены в нижних широтах сильнее, чем ячейки с обратной циркуляцией в средних широтах. Ячейки с прямой циркуляцией слабо выраженны в высоких широтах в течение всех сезонов.

Обсуждается перенос момента количества движения к полюсу и его различные механизмы. Влияние средней мериодиональной циркуляции на чистый поток через различные параллели мало за исключением низких широт зимой и весной. Главную часть этого потока составляют во все сезоны осредненые по времени турбулентные члены, хотя и средние по пространству тоже важны, особенно зимой.

Из конвергенции момента количества движения получено напряжение, с которым среднее зональное движение атмосферы действует на землю.