Statistical theory of precipitation process

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ABSTRACT

The present article summarizes the research work on cloud physics in the Institute of Geophysics and Meteorology, Academia Sinica, which began only in 1960. In recent years we set forward the statistical theory of precipitation process on the basis of analysing data of cloud microstructure observed in our country. This theory shows that water content and air current etc. in the cloud all undergo great fluctuations and the process of growth of cloud drops is stochastic in a fluctuating environment, which can only be tackled by the methods of probability. This paper presents the general method of statistical theory of precipitation process, discusses the questions on the precipitation from thin clouds and the formation of hail etc., and gives better results which agree closer with actual conditions than those of earlier theoretical calculations.

The formation of precipitation or the microphysics of the precipitation process is a problem old enough but still not fully solved. During the thirties, Bergeron and Langmuir formulated their theories of precipitation process respectively. And the formation of a large part of the precipitation may be thus explained. But there are also important features which remained unexplained.

1. Frequently warm convective clouds give rain even within half an hour or one after their appearance. This process is too quick to be explained by Langmuir's theory by which several hours are needed for precipitation formation. Clouds with glaciation on the top sometimes also produce rain in a very short time which is also difficult to explain by Bergeron's process, considering the time of falling of the crystals as well as the cloud-drops.

2. Hail forms often rather quickly and it forms even in a cloud not too thick, say 5–6 km or less, while according to the theory, hail could be formed only as thick as 10 km or more, in a period of one or two hours.

3. There are cases in which clouds as shallow as 1–2 km, and even warm shallow clouds give also precipitation. But a thickness of more than 2 km is required for precipitating warm cloud according to the theory accepted.

So it is rather evident that the existing theories of the precipitation process get difficulties when explaining these important facts. It thus appears that the whole essence of the precipitation process is not yet fully revealed.

During the past decades or so, many scientists have tried to improve the theory of the precipitation process by considering different factors involved. But their results are either effective under rather special conditions or improved only very little in one or the other aspect. The basic principles of these theories remain fundamentally the same. In fact, it is not enough to just make some small improvement of the old theories.

If we examine the actual conditions of precipitation in cloud, it is, however, easy to notice that they are very different from the conditions assumed in these theories in the following important aspects:

1. The air current in the cloud is assumed to be uniform and without fluctuation in theories of precipitation process, while the observation shows that the air current is fluctuating in the cloud. The precipitation element is assumed to be brought upward at first and then to fall down after growing, while in fact, it may very likely move up and down with fluctuations until it falls out.

2. The concentration of cloud drops or the water content in the cloud is assumed to be uniform, so is the electric charge of the cloud. In fact, these fields and other meteorological and physical fields should be fluctuating as the field of air motion is fluctuating in the cloud.

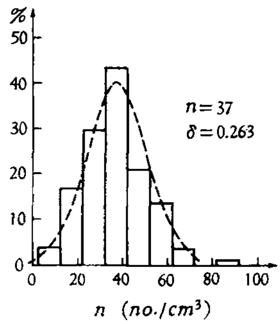


FIG. 1. Comparison of frequency distribution of cloud-drop concentration fluctuation with normal distribution.

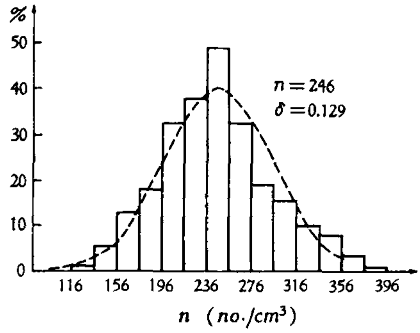


FIG. 2. Comparison of frequency distribution of cloud-drop concentration fluctuation with normal distribution.

In recent years, the cloud physics observation on mountain has been developed in our country. We have obtained some data about the fluctuation of cloud-drop concentration and water content in the stratus cumulus and cug. cumulus.

The two actual examples of the fluctuation spectra of cloud concentration observed in the stratus cumulus are given in Figs. 1 and 2, where n is the mean concentration and δ the concentration deviation (which equals to the ratio of the value of mean square difference to the average value) whose values are 26 % and 13 % respectively, whereas the fluctuation spectra of cloud-drop concentration approximate the normal distribution (see the dashed curves in these figures).

The two actual examples of fluctuation spectra of water content of the same cloud are in Figs. 3 and 4, where q is the mean water content, δ_q the fluctuation of water content whose values are 68 % and 21 % respectively, while the fluctuation spectra of water content are positive skew distributions and the skew coefficients are 1.9 and 1.4 respectively. It should be mentioned that in these cases the clouds had not fully developed and the data were obtained from the lower part of cloud. We believe that in the fully developed nimbocumulus the value of fluctuation must be more larger than that we obtained.

3. It is assumed that each of the cloud-drops (or crystals) of certain size will grow into a precipitation particle. This implies that the concentration of the rain drops and that of large cloud-drops should be the same. Actually, the concentration of the cloud-drops is 10^8–10^9 no./m^3 while that of the rain drop is 10^2–10^3 no./m^3 and even less in showers. So only a very small part of the large cloud-drops is allowed to grow into rain drops.

Now, as the condition in the cloud is fluctuating and the formation of precipitation requires the growth of only a very small part of the large cloud-drops, the problem should be a problem of parbaility. The process of precipitation formation should be some sort of stochastic

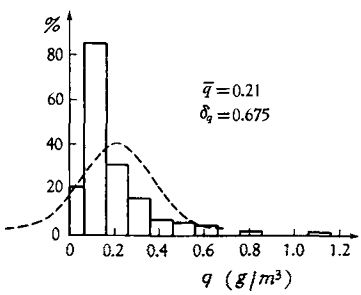


FIG. 3. Comparison of frequency distribution of water content fluctuation with normal distribution.

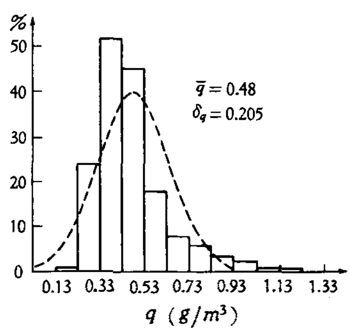


FIG. 4. Comparison of frequency distribution of water content fluctuation with normal distribution.

process. The spectral distribution of the cloud-drops of various size and the fluctuation of the cloud-drop spectra with space and time show clearly that the growth of the cloud-drop should be of statistical nature.

From this point of view, we regard the growth of the cloud-drop in a fluctuating environment as a stochastic process. A stochastic theory of the cloud-drop growth is thus formulated. A large part of our theoretical works on micro-structure of the cloud and the precipitation process are along this line.

Let us turn to the general theory of the stochastic growth of the cloud-drops. Based on the characteristics of the different processes of growth Chou Hsiu-Chi (1963, 1964) has classified the processes into 3 categories:

1. Stochastic process of discontinuous growth of cloud-drops defined over a discontinuous or continuous domain. Let us consider a group of water drops coagulating with a polydispersive cloud, with the coagulation between the water drops themselves neglected. Due to the random space distribution of cloud and the fluctuation of microstructure environment in clouds, the growth of the water drops is fluctuating and the change of drop radius is discontinuous. In this case, as pointed out by Chou Hsiu-chi, after the fluctuating coagulation the radius of the water drop has a probability distribution. The density of distribution satisfies the Kolmogorov-Feller stochastic integro-differential equation:

$$\frac{\partial f(v,t)}{\partial t} = -\int_0^\infty n_0 g(w,t)\,K(v,w,\Delta u)\,f(v,t)\,dw + \int_0^v n_0 g(v-y,t)\,f(y,t)\,K(y,v-y,\Delta u)\,dy, \tag{1}$$

where v is the volume of water drop, ω that of cloud-drop n_0 the concentration of cloud-drops $K(v,\omega,\Delta u) = \pi(\sqrt[3]{3v/4\pi} + \sqrt[3]{3\omega/4\pi})\,E\Delta u$, E is the collision efficiency, and Δu is proportional to the acceleration.

The solution of equation (8) could be given as a series which converges uniformly (Колмогоров, 1938; Feller, 1936, 1946, 1945).

$$f(v,t) = \sum_{n=0}^{\infty} f_n(v,t),$$

$$f_n(v,t) = \exp\left\{-\iint_0^\infty n_0 g(w,t)\,K(v,w,\Delta u)\,dw dt\right\}$$

$$\times\left[\iint_0^\infty n_0 g(v-y,t)\,K(v,v-y)\,f_{n-1}(t,y)\right.$$

$$\left.\times\exp\left(\left(\iint_0^\infty n_0 g(w,t)\,K(v,w,\Delta u)\right)\right)\right\}dt + \mathrm{Cu}\Big], \tag{2}$$

where Cu depends upon initial conditions.

As n_0 and the acceleration are fluctuating, f should be a function defined on the two-dimensional probability space.

Consider now the fluctuating coagulation of a group of water drops in a mono-dispersive cloud. The water drop changes from initial state (volume i) to a state j then the volume of the water drop becomes $i+jw$ after coagulation, where ω is the volume of the cloud drop ($j=1, 2, \ldots$). The probability of such transition satisfies a system of stochastic equations of Markov

$$\frac{dp_{ii}}{dt} = -A_i p_{ii};\quad \frac{dp_{i,i+1}}{dt} = A_i p_{ii} - A_{i+1} p_{i,i+1}, \tag{3}$$

where

$$A_i = \pi\left(\sqrt[3]{\frac{3v}{4\pi}} + \sqrt[3]{\frac{3w}{4\pi}}\right)E_n\Delta u = A_{i+j}.$$

Solution of the equation (3) can be written in series:

$$p_{i+k}(t) = \sum_{r=0}^{k} p_{i+r}(0)\sum_{l=r}^{k-1}\frac{\prod_{p=r}^{k-1} A_{p+i}}{\prod_{k=r\neq l}^{k}(A_{pi+1} - A_{l+k})}. \tag{4}$$

This is one of the special cases considered by Telford's theory (1953).

2. The increase of volume of the large drops by collision with a much smaller cloud drop is very small and thus the volume change could be regarded as continuous. In this case stochastic process of continuous growth of the cloud-drops is defined over a continuous domain. According to the equation of cloud-drop growth, the rate of change of the cloud-drop radius is

$$\frac{dR}{dt} = \frac{Eq\Delta u}{4\varrho} + \frac{D\Delta c}{R}, \tag{5}$$

where q is the water content, Δu the relative velocity of the drops, ϱ the density of the water, D the coefficient of diffusion of the water vapor, E the collision efficiency and Δc the degree of supersaturation of the water vapor.

As Δu, q, Δc are fluctuating, so equation (5) is the stochastic ordinary differential equation for a stochastic variable R in the form:

$$\frac{dX}{dt} = m(X)\,T(t)\,Y(t), \tag{6}$$

where X is a stochastic variable, $m(X)$ and $T(t)$ are given non-stochastic functions, $Y(t)$ is the stochastic variable described the fluctuating environment.

A linear stochastic ordinary differential equation can be regarded as a linear transformation of a certain stochastic function. Its solution is the result of linear transformation of a given stochastic function by a linear integral operator (PYRAYEB, 1960). If now $Y(t)$ is normally distributed, it is easy to find, according to the theory of stochastic function, the normal distribution of the variable $Z = \int_0^x dX/m(X)$, i.e.

$$f(z) = \frac{1}{2\pi\sigma_z} \exp\left(-\frac{(Z-\alpha_z)^2}{2\sigma_z^2}\right), \tag{7}$$

where the mathematical expectation α_z and the square deviation σ_z^2 are respectively

$$\alpha_z = \int_{t_0}^{t} T(t)\,\alpha_r(t)\,dt, \tag{8}$$

$$\sigma_z^2 = \int_{t_0}^{t}\int_{t_0}^{t} H(t'-t'')\,T(t')\,T(t'')\,dt'dt'', \tag{9}$$

where H is the correlative function. The solution is obtained therefrom.

If $Y(t)$ does not satisfy normal distribution, but if the moments from the first up to the Sth degree are known, we can approximately deduce the distributional function density values $f(z_1)$, $f(z_2)$ ... $f(z_s)$ from the following algebraic equation group

$$\left\{\begin{aligned} &\alpha_z = \sum_{n=1}^{S} Z_n f(Z_n)\,\Delta Z, \\ &\sigma_z^2 = \sum_{n=1}^{S} (Z_n-\alpha_z)^2 f(Z_n)\,\Delta t, \\ &\cdots\cdots\cdots\cdots \\ &\sigma_z^S = \sum_{n=1}^{S} (Z_n-\alpha_z)^S f(Z_n)\,\Delta Z; \end{aligned}\right. \tag{10}$$

$$\left\{\begin{aligned} &\alpha_z = \int_0^t T(S)\,\alpha_y\,dS, \\ &\sigma_z^2 = \iint_0^t \sigma_y^2 H(t', t'')\,T(t')\,T(t'')\,dt'dt'', \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ &\sigma_z^S = \iint_0^t \cdots \int \sigma_y^S H(t', \ldots t^S)\,T(t')\,T(t'') \\ &\qquad \ldots T(t^S)\,dt'dt''\ldots d^S. \end{aligned}\right. \tag{11}$$

3. Stochastic process of cloud growth of mixed type. Considering all the factors mentioned above and regarding the cloud as an entirety, then not only the larger drops will collide with the smaller ones but the larger drops will also collide with themselves. The cloud-drop spectrum will thus satisfy the following stochastic equation:

$$\begin{aligned} \frac{\partial n(v,t)}{\partial t} + \operatorname{div}(\mathbf{V}n) + \frac{\partial}{\partial v}\left(\frac{\partial v}{\partial t}\cdot n\right) &= \operatorname{div}(D\nabla n) \\ &+ \int_0^v n(v-v',t)\,n(v',t)\,K(v', v-v', \Delta u)\,dv' \\ &- \int_0^v n(v,t)\,n(v',t)\,K(v,v',\Delta u)\,dv', \end{aligned} \tag{12}$$

where $\mathbf{V}$ is the cloud-drop velocity, D the coefficient turbulent diffusion, $K(v, v', \Delta u)$ the collision efficiency of cloud-drops, and dv/dt the condensational growth rate. All the quantities as $\mathbf{V}$, Δu, Δc are stochastic variables, and $n(v, t, \Delta u, \Delta c, V)$ is a function defined in the 5 dimensional probability space.

1. As mentioned above, our observations show that the water content fluctuates strongly in certain cases. For ordinary cases, the fluctuation is large enough so that $\delta = \sigma/2 = \frac{1}{4}$. This should have a certain effect on the growth of the cloud-drop. To investigate the effect of the water content we may keep the acceleration constant and take q in equation (5) as stochastic with mean values of α_q and the square deviation due to the fluctuation σ_q^2. Applying the general result presented above to the equation of the growth of cloud-drop

$$\frac{dR}{dt} = \frac{E(V_R - V_r)}{4p}\,q$$

and taking $H = 1$, we find the spectral density of the distribution for the stochastic quantity R to be (KOO CHEN-CHAO & TSAI LI-SAN, 1962).

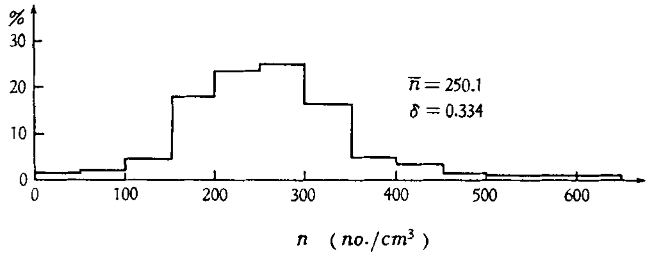


FIG. 5. Fluctuating concentration: distribution of cloud-drop.

$$f(R) = \frac{4}{\sqrt{2R\sigma_q}Et(V_R - V_r)} \times \exp\left(-\frac{(\int_{R_0}^{R} dR/(V_R - V_r) - (\alpha_q/4)Et)^2}{2\sigma_g^2((Et)/4)^2}\right) \quad (13)$$

Assuming that there are cloud-drops of two sizes initially, the larger ones with radius 12.6 μ and centration 10 no./cm^3 and the smaller ones with radius 10 μ, the water content is 1 g/m^3, the collision efficiency $E = \frac{1}{2}$, the mean square deviation of the fluctuation of the water content $\sigma/\alpha = \frac{1}{3}$, then the calculation shows that there are drops of radius 40–60 μ with a concentration 100 no./cm^3, 2000 sec after, and small rain drops of radius 300–400 μ with a concentration 100 no./cm^3, after 3000 sec, and rain drops of radius 400–500 μ with a concentration as large as 70/liter after 4000 sec. But when the fluctuation of the cloud-drop concentration or water content is absent, the cloud-drops of radius 12.6 μ grow only to a radius of 26 μ. Besides, the concentration of large cloud-drops obtained by us is quite near to the observed value and the relation of the concentration of large cloud-drops and the water content is also in broad accordance with the observation.

According to the actual observation data, the fluctuation of water content is generally not necessary of normal distribution; but it is mostly positive skew distribution. What should be the growth rate of cloud-drop in such a fluctuating circumstance? To answer this question we calculate the growth rate taking into account the positive skew distribution of actual fluctuating cloud-drop concentration shown in Fig. 5 in comparison with the normal distribution. As mentioned above, we study only di-dispersional clouds, in which the primitive radius of a great drop is 12.6 μ, the concentration is 10 no./cm^3, the radius of a cloud-drop is 10 μ, the mean concentration is 250 no./cm^3 (in such conditions the water content is 1 g/m^3) and the mean square deviation of fluctuation is 0.334. For simplicity, we divide the spectrum which is formed after the great drops have grown, into 3 intervals: (1) 12.6 μ–21.5 μ, (2) 21.5 μ–62.5 μ, (3) 62.5 μ–600 μ. In this case with numerical calculation of equation (10)–(11) we obtained the values of the density function of cloud-drops concentration distribution after 3000 seconds in each interval. The values of the density function under the same conditions but the concentration of cloud-drops is normally distributed are also obtained. The results are shown in Table 1.

Table 1 shows that the number of great drops with radii larger than 600 μ in actual positive skew distribution is larger than that in normal distribution by one order. Thus we may conclude that the positive skew distribution is favourable to the growth rate in a fluctuating cloud. As to the growth rate in the negative skew distribution, it is still a question to be investigated. Thus the fluctuation of cloud-drop concentration or water content has considerable effect on the formation of large cloud-drops and rain drops. In the same way, the fluctuation in acceleration due to turbulence has also much effect on the precipitation process.

TABLE 1

Interval	12.6μ–21.5μ	21.5μ–62.5μ	62.5μ–600μ
$F(R)$ for positive skew distribution	125	179	2.03
$F(R)$ for normal distribution	724	100	0.10

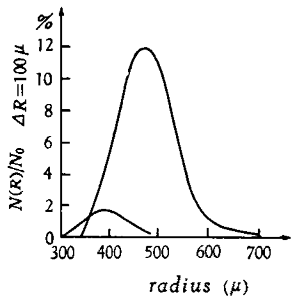


FIG. 6. Distribution of rain-drop in a fluctuating vertical air motion.

2. To explain the precipitation of shallow cloud the fluctuation of the vertical air-current should be taken into consideration. As pointed out above, turbulent motion of almost all scales, which is responsible to the fluctuation in other fields, is present in the clouds. The drops grow up not only in this current by coagulation but also move upward and downward with the current. In this case, the cloud-drops could be regarded as moving with the mean field of the motion and at the same time being diffused by the fluctuation of the air-current. A part of the cloud-drops gets out of the cloud earlier as a result of diffusion, but on the other hand, another part of the cloud-drops linger longer in the cloud and thus have the chance to grow up into larger drops. The spectral density n of the cloud-drops initially with the same size could be described by the equation

$$\frac{\partial n}{\partial t} + \operatorname{div}(\mathbf{V}n) = \operatorname{div}(D\nabla n), \tag{14}$$

where D is the coefficient of turbulent diffusion for the cloud-drop, $\mathbf{V} = \mathbf{W} - \mathbf{w}$ is the velocity of cloud-drops, $\mathbf{W}$, the velocity of the air and $\mathbf{w}$, the terminal velocity of the drops, which is a function of the drop radius supposed to increase under gravitational coagulation. To compute in a somewhat simpler way, a different scheme has been adopted (HSU-HUA-YING & KOO CHEN-CHAO, 1963; HSU HUA-YING, 1963, 1964). Let the air-current possess only vertical component with probability distribution

$$f(w, z, t) = \frac{1}{\sqrt{2\pi}\sigma(z, t)} \exp\left(-\frac{(w-\bar{w})^2}{2\sigma^2(z, t)}\right)$$

We may solve the finite-difference equation equivalent to (14). For the case of $\bar{w} = \text{const.}$, a series of computations is completed. The following example is enough to show the effect of the fluctuation of the vertical current. For water content of cloud to be 1 g/m³, $\bar{w} = 0.5$ m/sec, $\sigma_w = 0.5$ m/sec, initial radius of larger drops to be 25 μ and that of the smaller drops to be 6 μ, precipitation with drop spectrum from 350 to 700 μ can be formed in a cloud 2 km thick only (Fig. 6) while a cloud 1.7 km thick will give drizzle of radius (300–500 μ). In this case a cloud of 3.0 km thick at least is required to form rain drops of radius 700 μ if the air current is uniform with the same w. Besides, the concentration of the rain drops becomes unreasonably large and no rain drop is formed.

The result is even more striking if the vertical velocity of the air current decreases upward (or with a parabolic distribution in the vertical) (HSU-HUA-YING, 1963, 1964), since then the cloud drops are now difficult to get out from the cloud top. Taking $\partial\bar{w}/\partial z = -0.5$ m/5 km the thickness of the cloud 600 m (we divide the cloud into 6 layers), the mean ascending speed of the air 0.2 m/s, the speed in the lower layer 0.35 m/s, the water content 0.2 g/m³, still the initial radius of larger drops 25 μ and smaller ones 6 μ, a computation is carried out, and the result obtained is given in Fig. 7. In this case the largest drop obtainable has a radius of 115 μ, the drops with a radius larger than 100 μ amount to 5 % of the larger drops initially

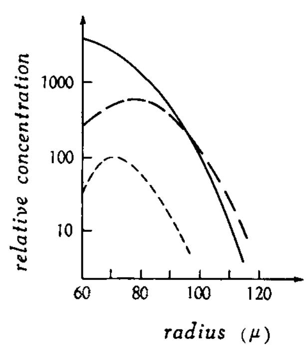


FIG. 7. Distribution of rain-drop: —— cloud thickness is 500 m, the vertical velocity decreased with height; – – cloud thickness is 1 km, the vertical velocity is uniform with height; --- cloud thickness ness is 600 m, the vertical velocity is uniform with height.

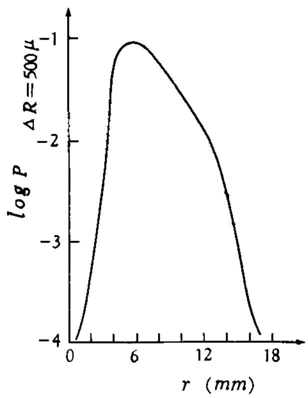
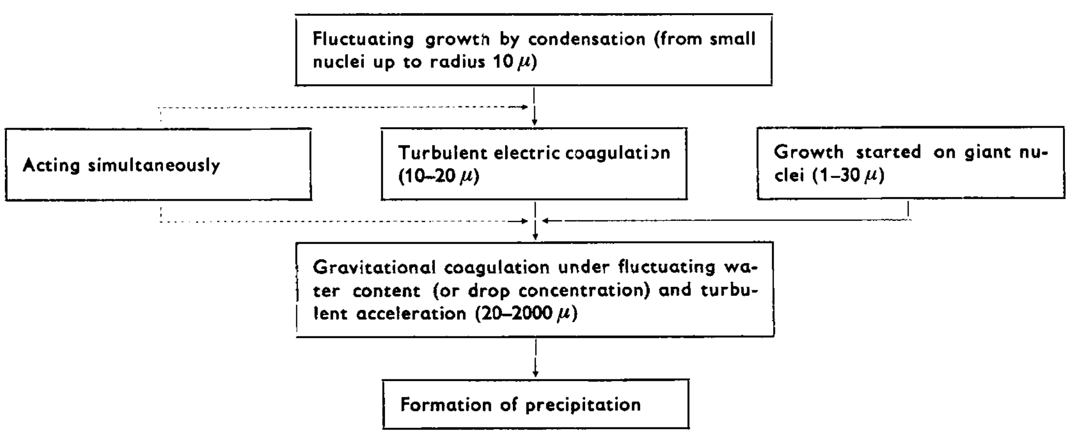


FIG. 8. The formation of hail stone.

present. If the air current is still fluctuating but the mean motion does not decrease upward, with $\sigma_w = \bar{w}$, then the final radius of the large drop will be 95 μ or so, and at the same time the concentration is much less (dotted line in Fig. 7). In order that one still gets drops of radius 115 μ the cloud thickness should increase to 1 km. If the air is not fluctuating and w is kept to be 0.2 m/sec at all height, then no precipitation will be formed as the final drop radius becomes then as small as 57 μ. Through this example, it is clear that the fluctuation of the vertical current, especially when the mean current decreases upward, is very favourable to the formation of precipitation.

3. The formation of hail could be explained in the same way (KUH CHEN-MUO & KOO CHEN-CHAO, 1963, and CHOU HSIU-CHI, 1963). Up to now, there are but a few theories dealing with the rapid growth of the hail in a small cumulonimbus. Considering fluctuation of vertical air current, we have computed the growth of hail with the terminal velocity of hail given by Macklin and Ludlam. We take a case in which the thickness of cloud is 5 km (divide into 5 layers), the water content 1 g/m³, $\varrho_{\text{ice}} = 0.45$ g/cm³, $E = 0.85$, $\sigma_w = \bar{w}/3$, w has a maximum and equals to 10 m/s in the middle of the cloud and decreases to 1.5 m/s at the cloud top and the initial radius of larger-drop is 25 μ. The result shows that quite a part of the drops grows into hail of radius 1.5 cm, the maximum radius amounts to 1.7 cm (Fig. 8). This is rather considerable. Using the model of Biblashvili and Sulakvilitze (without fluctuation in vertical current), hails of such size could only be formed in a cloud of 7 km thickness, and w as large as 15 m/s. And a cloud of 10 km thickness is necessary if the vertical current does not change along the vertical. We may, then, conclude that the fluctuation of vertical current is favourable to the formation of hail. Without the fluctuation of vertical current the mechanism of hail formation cannot be easily understood. Of course, there is still much to be done. For example, though the necessary high concentration of hail given by other theories is much reduced in our computation, a satisfactory result can only be obtained by considering the balance of water in the cloud as a whole.

According to the discussion given above and our other results (CHOU HSIU-CHI, 1964), the mechanism of precipitation process in a warm cloud may be outlined as follows:



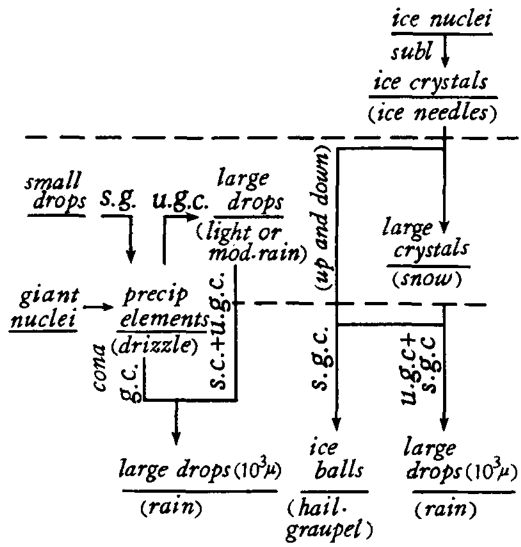


FIG. 9. Scheme of precipitation mechanism.

In a word, the growth of the cloud element will greatly speed up, if all of these factors are operating. The precipitation from warm cloud may then be explained reasonably. In fact, such a mechanism will also operate even in a mixed or ice cloud. Thus the process of precipitation may be revised to a large extent as shown in Fig. 9.

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СТАТИСТИЧЕСКАЯ ТЕОРИЯ ОСАДКОВ

В настоящей статье суммируются результаты исследований по физике облаков, проводимых в Институте геофизики и метеорологии Академии наук КНР только с начала 1960 г. В последние годы на основе анализа данных наблюдений о микроструктуре облаков, проведенных в нашей стране, мы развили статистическую теорию осадков. Согласно этой теории, водность, воздушные токи и другие величины в облаках испытывают большие флуктуации, процесс роста облачных капель в среде с флуктуирующими параметрами является случайным процессом, который может изучаться только вероятностными методами. В данной статье описывается общий метод статистической теории осадков, обсуждаются вопросы осадков из тонких облаков, образование града и т. д. и даются результаты, лучше согласующиеся с реальными условиями, чем результаты более ранних теоретических расчетов.