A simple noctilucent cloud model

By R. J. CHARLSON[1], *Department of Cloud Physics, Imperial College of Science and Technology, London University, London, England*

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ABSTRACT

Particles of dust (of cosmic origin, for example) falling through a supersaturated layer near the mesopause will become coated with ice. The thickness of this coating is an important parameter in discussing the optical properties and mass budget of the system. Calculations of this thickness are presented, including the dependence on the vertical extent of the layer, the humidity, the vertical wind velocity and the density of the dust particles. The results of these calculations indicate the range of conditions that might be sufficient to produce a noctilucent cloud. The effects of sedimentation on the mass budget are considered with the conclusion that an updraft of tens of cm/sec is necessary for a steady-state condition.

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The possibility that water is involved in noctilucent clouds (NLC), particularly that an ice coating effects their visible characteristics, has been a point of speculation for some time. The results of the rocket borne sampling experiments of HEMENWAY, SOBERMAN, WITT and coworkers (1964) showing a residual halo around some of the particles on sampling surfaces has heightened interest in this question. The deposition of ice on particulate matter under assumed mesopause conditions has been discussed in a preliminary fashion in an earlier paper (CHARLSON, 1965). It is the purpose of this paper to continue this line of speculation, to illustrate the amount of ice that might be involved in NLC and to examine the effect of ice deposition on the size spectrum of the cloud particles.

The physical system to be considered consists of a supersaturated layer through which falls a constant flux of particulate matter at ambient temperature. The applicability of this system can be justified briefly by the following considerations:

1. The fall speeds of particles supposed to exist in NLC are of the order of 10 cm sec^{-1}; thus particles can traverse the vertical extent of the clouds (perhaps a km) in times shorter than the observed lifetimes of the clouds as a whole.
2. The H_2O number densities near the mesopause derived by assuming a constant mol fraction or mixing ratio characteristic of the stratosphere (about 2 ppm) is about 10^9/cm^3. At the measured temperature of 130 to 140 deg. K the saturation number densities are between 10^7 and 10^8 providing relative humidities much in excess of 100 % (CHARLSON, 1965).

The mathematical formulation will express the dependence of coating thickness on the distance fallen within the layer. The initial radius of the particle is r, the radius of particle plus ice is R, z is the vertical coordinate and h is the distance fallen by the particle after entering the layer. The fall speed of the particle, dz/dt, can be combined with the rate of its growth, dR/dt, to eliminate time.

The fall speed of a particle may be represented by a kinetic expression or by the Stokes-Cunningham equation since both yield nearly the same result. In either case, the fall speed relative to the air is proportional to the average density of the particle and to its radius:

$$\frac{dz}{dt} = B\bar{\varrho}R - w = B[1 + (\varrho - 1)(r/R)^3]R - w,$$

where $\bar{\varrho}$ is the average density of the particle, ϱ is the density of the nucleus, w is the updraft velocity, and the density of ice is assumed to be

[1] Civil Engineering Department, University of Washington, Seattle, Washington, U.S.A.

1 gram/cc. For the region of the mesopause, B has typical values of about 2×10^6 cm^3 g^{-1} sec^{-1}.

The rate of growth of the particle will be approximated simply, neglecting both the Kelvin or radius effect and re-evaporation. The former is of only minor importance in this temperature regime for the smallest NCL particles. The latter seems reasonable in view of the assumed large supersaturation. Thus the rate of growth can be related to the frequency of collision of water molecules with the surface of the nucleus (assumed to be spherical):

$$\frac{dR}{dt} = \frac{b\eta m_w}{\varrho_{ice}} = bN_w\left(\frac{kT}{2\pi m_w}\right)^{\frac{1}{2}} \cdot m_w = A,$$

where b is the accommodation coefficient, η the collision frequency, m_w the mass of the water molecule, k Boltzmann's constant, T the absolute temperature, the density of ice (ϱ_{ice}) is understood to be one g cm^{-3}, and N_w is the number density of water molecules. Here the quantity A has a value of about $3 \times 10^{-19} \times N_w$ cm sec^{-1} if b is close to unity. Measured values of the accommodation coefficient seem to increase with decreasing T and at 150°, a value of about 0.7 has been cited. This represents a growth rate of about 2 Å min^{-1} if $N_w = 10^9$ cm^{-3}. These two equations may be combined to eliminate the time, t, and integrated to yield a relationship between R, r, h, w, and N_w. If the radius is integrated over a range from r to R and the distance fallen from 0 to h, the following equation results:

$$0 = \frac{B}{2}R^3 - wR^2 + R[B(\varrho - 1)r^2 - Br^2/2 - Ah + wr] - Br^3(\varrho - 1).$$

If the nucleus density is close to one g cm^{-3}, this expression reduces to the quadratic equation which was discussed in the earlier paper (CHARLSON, 1965).

$$0 = \frac{B}{2}R^2 - wR + wr - \frac{Br^2}{2} - Ah.$$

Solutions of the cubic equation have been obtained for a wide range of conditions. The range of assumed quantities, particularly the water number density and the updraft velocity, is hopefully broad enough to include reality. This range includes the following:

1. Water number densities, a factor of ten above and below the value of 10^9 cm^{-3} obtained by extrapolation of stratospheric measurements (using the so-called "dry stratosphere" results of ca. 2 ppm).
2. Vertical velocities between 0 and 100 cm sec^{-1} have been used in logarithmic steps. There are no measurements of this parameter but the calculations of Murgatroyd, Leovy and others suggest at least a mean updraft in the summer polar mesophere of about 1 cm/sec.
3. Nucleus specific gravities from 1 to 8 g cm^{-3} have been used, once again in logarithmic steps.

Figure 1 shows solutions for three different nucleus densities (graphs A, B and C correspond to $\varrho = 1$, 3 and 8 g cm^{-3} respectively) and four vertical velocities: a, b, c, and d curves correspond to 0, 10, 30, and 100 cm/sec. Figure 2 shows similar results for different values of the water vapor concentration: curves a, b, and c correspond to 10^8, 10^9, 10^{10} molecules of water per cc at $w = 30$ cm/sec. In both figures the coating thickness is plotted versus the initial radius, both on log-log scales. These results illustrate that the smaller particles can attain a thicker coating and that an updraft accentuates this effect and increases the coating thickness. Under some conditions, particles which were abundant in the sampling experiments of Hemenway, Soberman, and Witt can more than double their radii. This may be optically important.

One criterion for judging the importance of this sort of radius increase is the effect on the particle size spectrum. Figure 3 (with the 3 graphs again representing nucleus specific gravities 1, 3 and 8) illustrates the effect of this ice deposition on a simple cumulative power law nucleus spectrum. Curves a represent the original inverse r^3 spectra with a cutoff at 0.03 μ and curves b, c, and d correspond to updraft velocities of 3, 10 and 30 cm/sec. The cumulative number of particles per unit column is plotted versus radius, both on logarithmic scales.

Figure 4 shows similar curves for an updraft velocity of 30 cm/sec. Curve a is the nucleus spectrum and curves b and c show the effect of 10^9 and 10^{10} molecules of water per cc respectively. Because of the radius interval used in these calculations, the effect of 10^8 molecules

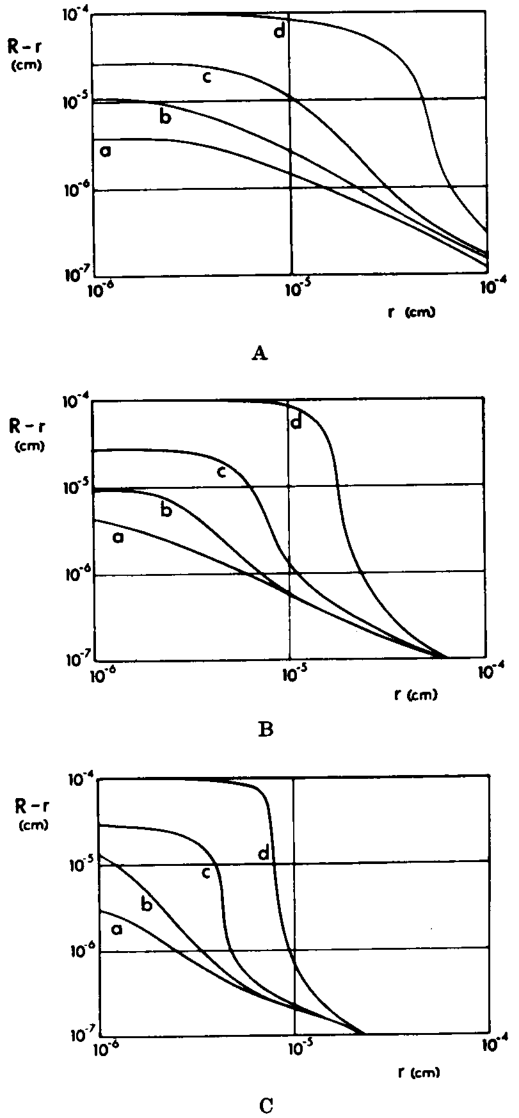

FIG. 1. Coating thickness, R–r, in cm versus nucleus radius, r, in cm for 3 nucleus specific gravities, ϱ. Graphs A, B and C correspond to $\varrho = 1$, 3 and 8 g/cm^3 respectively. The H_2O number density, N_w, is 10^9 cm^{-3}. Curves a, b, c and d correspond to updraft velocities, w, of 0, 10, 30 and 100 cm/sec.

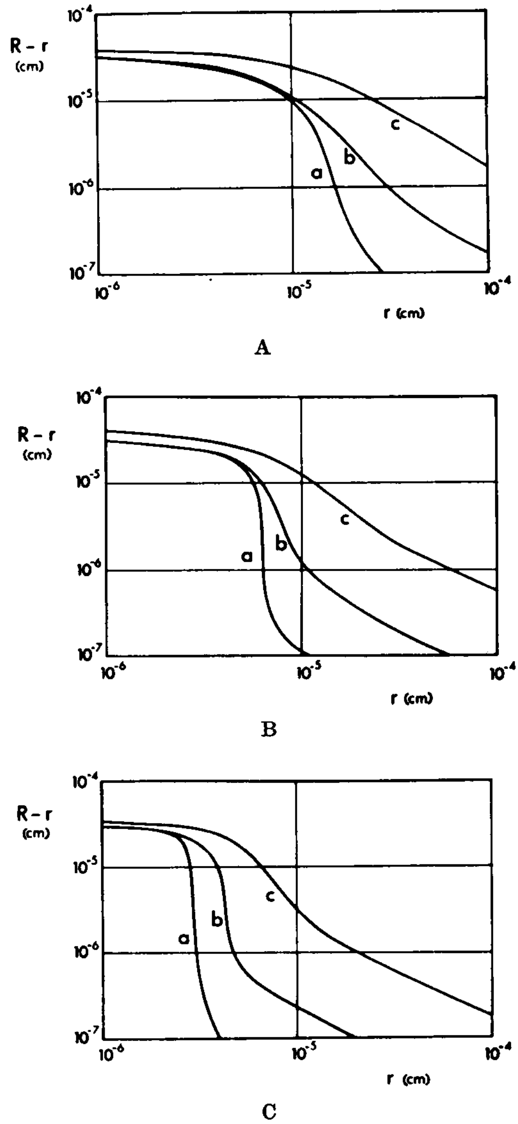

FIG. 2. Coating thickness, R–r, in cm versus nucleus radius, r, for 3 nucleus specific gravities, ϱ. Graphs A, B and C correspond to $\varrho = 1$, 3 and 8 respectively. Updraft velocity, w, is 30 cm sec^{-1}. Curves a, b and c correspond to H_2O number densities, N_w, of 10^8, 10^9 and 10^{10} cm^{-3}.

per cc is not seen to be different from nucleus spectrum.

On the basis of the gross asumptions of this model and the order-of-magnitude nature of the calculations, it is not possible to conclude that ice is definitely involved in noctilucent clouds. However, these results do suggest that ice deposition is a possibility. Even if noctilucent cloud particles are ice coated, the optical importance of such a coating has yet to be defined.

One consequence of such an ice coating would be a depletion of the water vapor within the supersaturated layer. The ratio of mass of ice

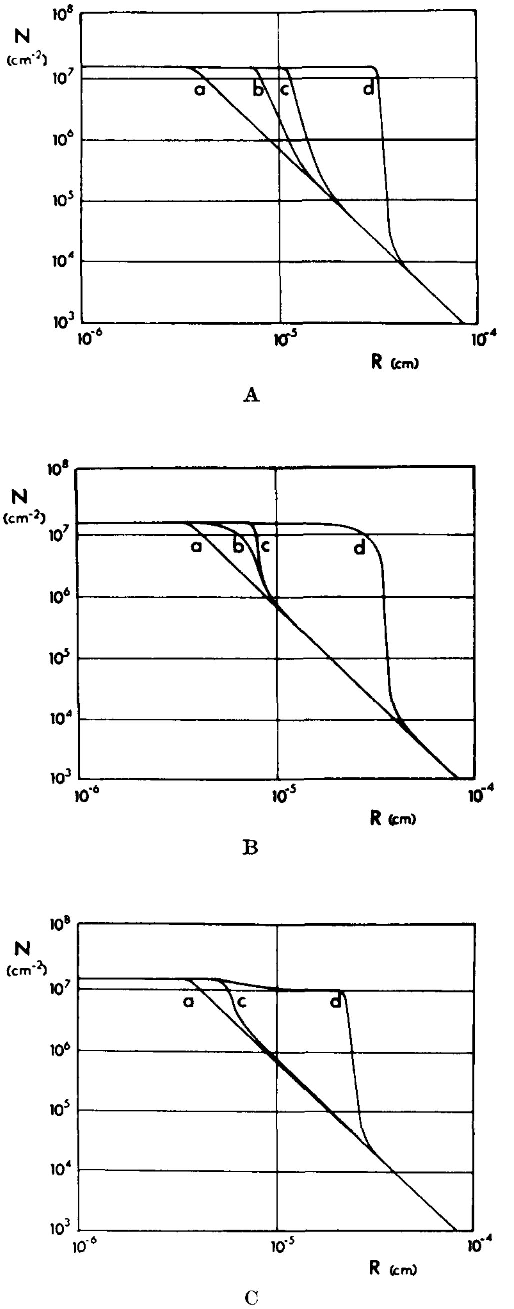

FIG. 3. Cumulative number of particles per unit column, N, versus particle radius, R, for $N_w = 10^9$ cm^{-3} and three nucleus specific gravities. Graphs A, B and C correspond to $\varrho = 1$, 3 and 8 respectively. Curves a represent hypothetical inverse r^3 spectra with an 0.03 μ cutoff; b, c and d correspond to updraft velocities, w, of 3, 10 and 30 cm sec^{-1}.

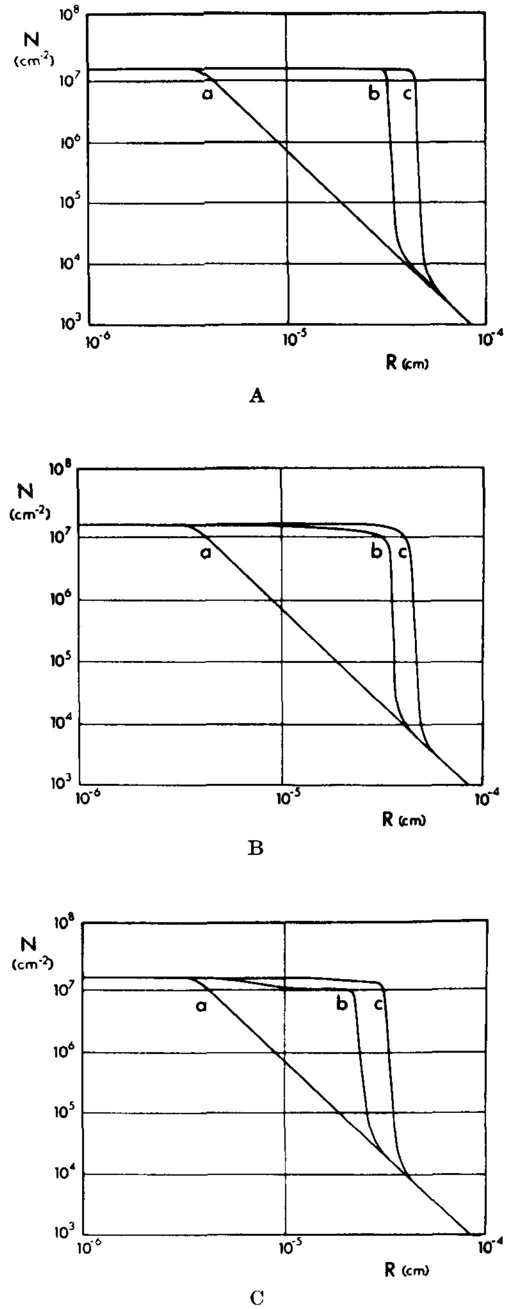

FIG. 4. Cumulative number of particles per unit column, N, versus particle radius, R, for $w = 30$ cm sec^{-1} and three nucleus densities. Graphs correspond to $\varrho = 1$, 3 and 8 respectively. Curves a represent inverse r^3 spectra and b and c correspond to H_2O number densities, N_w, of 10^9 and 10^{10} cm^{-3}.

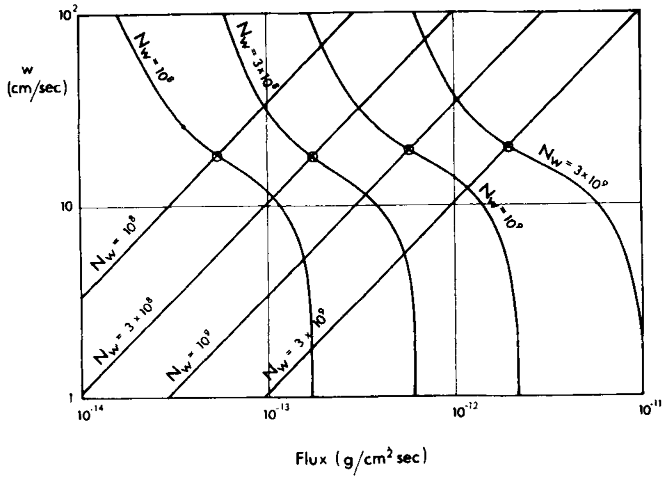

FIG. 5. Mass flux of H_2O versus updraft velocity for the particle number densities found by Hemenway, Soberman and Witt. The curved lines correspond to calculated downward fluxes for four H_2O number densities, N_w. The straight lines correspond to upward fluxes for the same four values of N_w. The points of intersection of the pairs of lines for each value of N_w are circled indicating a neutral flux condition. Note that this condition is nearly independent of updraft veclocity, w.

to mass of vapor per cc is of the order of 1 to 10 for a wide range of conditions as discussed in the earlier paper (CHARLSON, 1965). This ice would be removed by sedimentation and carried into the region below the cloud. Thus, it seems unlikely that water could be involved in such a manner without a source to replace the deposited water molecules. This might be an esoteric source such as meteor-borne water or an *in situ* chemical reaction. However, an updraft might be sufficient to maintain the water vapor concentration.

In this connection it is useful to estimate the downward fluxes corresponding to the calculated radii. A downward flux can be given by the following equation:

$$\text{Flux} \sim \sum_i \frac{\frac{4}{3}\pi(R_i^3 - r_i^3)\varrho_{ice} \cdot N_i \cdot V_i}{h},$$

where N_i = number of particles of radius interval "i" per cm² of layer; V_i = fall velocities of these particles.

Figure 5 is a graph of downward flux as a function of the updraft velocity for several different H_2O number densities. The upward fluxes are also included, showing a flux balance at about 20 cm/sec. It should be noted that the fluxes at zero velocity are large enough to remove all water from the region in times of the order of one hour. Thus, an updraft of tens of cm sec^{-1} seems to be a necessary condition for maintenance of an ice coating.

An updraft of this magnitude is also helpful in explaining the low mesopause temperature during the occurrence of noctilucent clouds, as well as the enhanced number densities of the nuclei reported by HEMENWAY, SOBERMAN & WITT (1964).

One interesting possibility is that ice carried downward by sedimentation will be rapidly evaporated in the warmer air below the cloud. This water vapor, in turn, might be carried into the cold region by the updraft and be redeposited on particulate matter. In this manner the noctilucent cloud becomes a cryogenic trap for water vapor rising from below. Because the saturation number density is very temperature dependent under these conditions (roughly, an order of magnitude per 7 degrees at 130 degrees Kelvin) this sort of trapping might occur over a very small height interval.

It is evident from these computations that the possibility exists for noctilucent clouds to contain ice. Further, an updraft of a few tens of centimeters per second will have a marked effect on the amount of ice deposited and on the mass budget of the cloud. It is difficult to measure this sort of motion, particularly because of the large horizontal velocities and shear in the vicinity. G. Witt has suggested that one result of a strong updraft might be a local lifting of the region of production of excited hydroxyl radicals (WITT, 1965). This, in turn, would perhaps raise the level of the peak OH radiation, providing a possible indicator for vertical motion. Similarly, other air glow constituents might be useful in detecting an updraft.

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ПРОСТАЯ МОДЕЛЬ СЕРЕБРИСТЫХ ОБЛАКОВ

Частицы пыли (например, космического происхождения) падая через перенасыщенный слой вблизи мезопаузы будут накрываться слоем льда. Толщина этого слоя является важным параметром при изучении оптических свойств системы и бюджета ее массы. Представлены расчеты этой толщины, включая ее зависимость от вертикального размера слоя, влажности, вертикальной скорости ветра и плотности частиц пыли. Эти расчеты указывают условия, которые могут быть достаточными для образования серебристых облаков. Из рассмотрения действия осаждения частиц на бюджет массы следует, что для установившегося состояния необходимо движение к верху порядка десятков см/сек.