Methods of measuring solar and terrestrial atmospheric circulations

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ABSTRACT

In order either to support or to cast doubt upon the mode of study of the circulation of the solar photosphere recently employed by Ward, a comparable use of his methods is made in a test study of the terrestrial atmosphere for which extensive investigations by independent procedures are available. In a qualified over-all manner, Ward's procedures for measuring the large-scale horizontal eddy components from sunspot displacements are given vindication.

I. Introduction

According to the recent results of WARD 1964, 1965*a*) the equatorial acceleration of the solar photosphere is maintained against frictional and other retarding effects through the equatorward transportation of angular momentum from higher latitudes in each hemisphere. This momentum convergence is ascribed not to the longitudinally (zonally) averaged motions, but rather to the action of horizontal eddies whose size is of the order of 10^5 km. The longitudinal (east-west) and latitudinal (north-south) components of fluid motion due to these large eddies were found to be correlated so as to bring into existence a Reynolds stress to drive the differential rotation. Since use has already been made of this conception of photospheric motions by STARR & GILMAN (1965*a*, 1965*b*), who have drawn further conclusions, it is time to make such checks of the original premises as are now feasible.

In order to measure the horizontal velocity components, Ward utilized sunspots as tracers of photospheric flow. When this is done there appears to be no doubt as to the significance of the outcome when the data are processed over sufficiently long periods. Nevertheless, questions exist as to the accuracy of the assumption that the daily displacements of spots and small spot groups represent fluid motions associated with large eddies. If they do, there still remains the question whether velocities so measured represent a proper sampling for the computation of the desired Reynolds stresses. Since these questions cannot be immediately answered by securing alternatively measured data from the sun, what will be done here is to study an analogous system for which extensive velocity statistics compiled from direct measurements have been analyzed—namely, the earth's atmosphere. Does an attack similar to Ward's applied to the terrestrial atmosphere yield correct results for the elucidation of the atmospheric differential rotation? If it does, then a somewhat greater credence may be ascribed to the results of the sunspot statistics for the solar photosphere. In pursuance of this thought one may ask the question: If we were to have the data concerning the earth's atmosphere relating only to the daily positions of some easily identifiable feature in the earth's atmosphere, could we deduce more or less accurately the main features of the poleward angular momentum flux solely from this information? The daily positions of the migratory surface cyclones and anticyclones with their attendant weather changes are carefully mapped by meteorologists. It is of course true that the terrestrial disturbances mentioned are not exactly similar to sunspots, but they are perhaps the closest

TABLE 1. *Statistical analysis of displacements.*

N indicates the number of pairs of $\Delta\lambda$ and $\Delta\theta$, the braces denote linear averages over N, Cov. denotes the covariance of N pairs, and r denotes the linear correlation coefficient of N pairs. Displacements in meters per second. Asterisk denotes correlations at or above 95 % significance level by standard tests.

Latitude	10–20	20–30	30–40	40–50	50–60	60–70	70–90
			Displacements of both cyclones and anticyclones				
N	1085	1348	3132	3352	3275	2105	1327
$\{\Delta\lambda\}$	−0.58	1.82	4.91	7.26	5.74	4.46	1.71
$\{\Delta\vartheta\}$	−0.06	0.01	0.12	0.50	0.28	−0.15	0.38
Cov.	−1.32	−0.87	5.01	6.50	1.51	1.52	0.52
r	−0.11*	−0.03	0.14*	0.15*	0.04*	0.05*	0.03
			Displacements of cyclones alone				
N	994	981	1541	1918	2260	1540	828
$\{\Delta\lambda\}$	−0.70	0.96	4.88	7.89	6.20	4.91	2.00
$\{\Delta\vartheta\}$	0.35	0.70	1.10	2.38	2.25	1.93	1.37
Cov.	−1.52	−1.58	8.16	6.81	−0.13	−0.28	0.98
r	−0.12*	−0.06	0.22*	0.17*	0.00	−0.01	0.07*
			Displacements of anticyclones alone				
N	91	367	1591	1434	1015	565	499
$\{\Delta\lambda\}$	0.65	4.13	4.94	6.43	4.72	3.24	1.25
$\{\Delta\vartheta\}$	−0.48	−0.68	−0.86	−1.39	−1.70	−2.22	−0.60
Cov.	1.96	4.23	2.03	2.95	1.14	1.38	−1.16
r	0.21*	0.13*	0.06*	0.07*	0.03	0.04	−0.07

analogoues to them which we have at our disposal for the present purpose. Accordingly, the material which follows presents the results of an experiment of this kind.

II. Procedures and basic numerical results

The daily(24-hour)displacements of all closed cyclonic and anticyclonic centers appearing on the northern hemisphere surface weather maps for 1200 hours Greenwich time were measured for the two-year period from January 1, 1950, to December 31, 1951. To be included, a given center had to be present on at least two consecutive maps. The total number of centers followed was 20,545. Each displacement was resolved into a zonal (eastward) component, $\Delta\lambda$ and a meridional (northward) component $\Delta\theta$, as in Ward's sunspot study. The data were separated by 10° latitude belts except for the region north of 70° N which was treated as a single zone. No data were available for the region between the equator and 10° N.

These data were punched on cards and machine processed in precisely the same way as the sunspot data of Ward were handled by him. In particular, the means, standard deviations, covariances and linear correlations of $\Delta\lambda$ and $\Delta\theta$ were computed by latitude belt for the two-year interval. Table 1 shows the results obtained. For subsequent discussion the calculations were repeated for displacements of cyclones alone and anticyclones alone. Except for anticyclones in the southernmost zone all means were statistically different from zero beyond the 95 % significance level.

III. Comparison with results from direct wind measurements

The mean and eddy components of the earth's atmosphere have been measured by direct observation and reported by STARR *et al.* (1954), and OORT (1964). The mean zonal motions of the atmosphere may be pictured as comprising a large vortex about the polar axis in which the angular velocity is a function of elevation, latitude and time. In Fig. 1, the distribution with latitude of this angular velocity relative to the earth over the northern hemisphere is shown averaged over the period from January 1950 through December 1951 (full curve). The averaging was performed horizontally around the latitude belts and from the surface to 16 km.

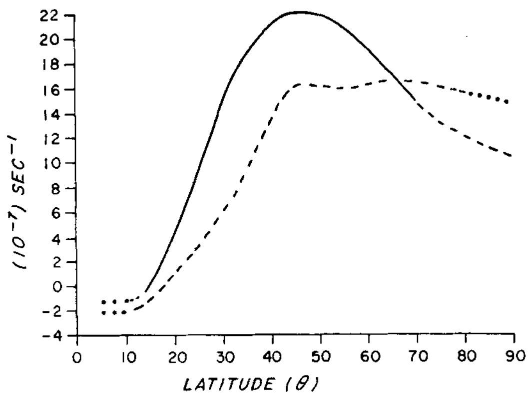

FIG. 1. The meridional profile of the vertically averaged relative angular velocity in the terrestrial atmosphere. The solid curve describes the direct observations of the zonal wind, and the dashed curve shows the zonal wind estimates obtained from the daily displacement, of both cyclones and anticyclones from Table 1. Here $\{u\}$ is the eastward wind velocity averaged as indicated in Table 1. ——, $\{u\}/\cos\theta$; – – –, $\{\Delta\lambda\}/\cos\theta$.

A similar latitudinal profile of the relative angular velocity is also found in the southern hemisphere as recently reported by OBASI (1965).

The effectiveness of the cyclones and anticyclones as tracers of the angular velocity field is shown by the dashed curve. The angular velocity gradient observed directly in the low latitudes is matched very closely by the dashed curve. In the more northerly regions the observed decrease of angular velocity with increasing latitude is not duplicated by the displacements of the cyclones and anticyclones.

The correlation between the latitudinal and longitudinal displacements of the cyclones and anticyclones shown in Table 1 is of the same sign as that found by direct measurements of the wind field. This positive correlation strongly implies that the cyclones and anticyclones are tracing out the large-scale eddy components found in the wind field in the upper troposphere that account for most of the poleward transport of relative angular momentum. The vertically integrated transport of relative angular momentum by these eddies in the northern hemisphere is shown by the full curve on Fig. 2. The eddy flux, calculated from the covariance of the displacements of the surface cyclones and anticyclones is shown by the dashed curve in Fig. 2. In this case it is assumed that the vortices reflect the vertically integrated motion of the total atmospheric mass of 1000 gm. per unit vertical column. The shape of the meridional profile of the transports is quite similar with a maximum flux observed in middle latitudes. The magnitudes are comparable in the high latitudes but differ by a factor of about 3 in the region of the maximum transport and are of opposite sign in the tropical areas. The net equatorward transport of momentum implied from the covariance of the latitudinal and longitudinal displacements of these systems in low latitudes may result from a bias in the annual distribution of these systems. Most of the net, annual flux of momentum toward the poles is observed to take place in the midwinter when these low-latitude cyclone and anticyclone tracers are seldom observed (~ 2 appear each winter month). Conversely, the middle- and high-latitude systems are most prevalent during the wintertime.

It can also be seen from Table 1 that the systematic average poleward motion of cyclonic systems is about balanced by the average equatorward movement of the anticyclones. The net northward component that does appear is small, suggesting that the transport of momentum by the mean meridional motions is negligible. As stated before, direct measure-

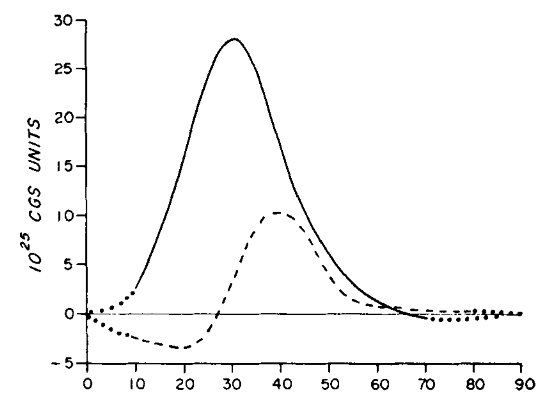

FIG. 2. The meridional distribution of the northward transport of relative angular momentum by the eddy structures in the terrestrial wind field. The solid curve shows the directly measured, vertically integrated transport, while the dashed curve shows the transport estimated from the covariance of the daily zonal and meridional motions of all cyclones and anticyclones. Here v is the northward component of wind velocity. ——, $\{u'v'\}2\pi r^2\cos^2\theta$; r, earth's mean radius; – – –, $\{\Delta\theta',\Delta\lambda'\}2\pi r^2\cos^2\theta$.

TABLE 2. *Correlation between zonal displacement of cyclones and anticyclones and their size.* N indicates the number of pairs of $\Delta\lambda$ and A, respectively, and r is the linear correlation coefficient of N pairs.

Latitude	10–20°	20–30°	30–40°	40–50°	50–60°	60–70°	70–90°
			Cyclones				
N	994	981	1541	1918	2260	1540	828
r	.04	−.06*	−.16*	−.07*	−.11*	−.12*	.02
			Anticyclones				
N	91	367	1591	1434	1015	565	499
r	.01	.06	−.09*	−.16*	−.13*	−.05	−.12*

ments show that the momentum transport by this mode in both hemispheres is not important over time periods of a year or so.

From these data one can conclude that rough estimates of the angular velocity profile of the earth's wind field can be obtained by observing the displacements of the cyclonic and anticyclonic centers. Use of these systems can be made to estimate the eddy flux of angular momentum, but in that case the important middle latitude maximum is underestimated by a factor of two thirds. The general shape of the profile does however resemble the actual one.

IV. Further considerations

In Fig. 3 it can be seen that the small cyclones and anticyclones are more accurate tracers of the earth's angular velocity profile than large systems. The size of each cyclone and anticyclone was determined for each latitude belt by measuring the area contained within the closed isoline of constant pressure located farthest from the center. The average size of these systems is 10^6 km^2 with a range from 5×10^4 to 10^7. The correlation of the zonal displacement of both cyclones and anticyclones with their size can be seen in Table 2 where the asterisks denote the correlations that are significant at or above the 95 % confidence level. The correlations are significantly negative for both types of systems over most middle latitude belts. Thus the small cyclones and anticyclones move more rapidly in their east-to-west direction than the large ones. A similar effect in the proper motion of sunspots has been reported by WARD (1965*b*) who finds that the larger sunspots move more slowly than small spots.

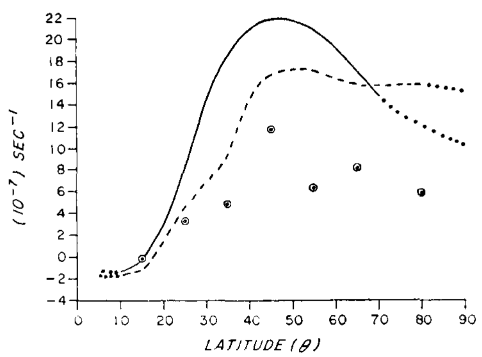

FIG. 3. The meridional profile of the directly measured, vertically averaged angular velocity profile in the terrestrial atmosphere is shown by the solid line. The open circles show the same profile obtained from the average of the daily displacements of all *large* cyclones and anticyclones. The dashed curve is the profile obtained from the motion of all *small* cyclones and anticyclones. ——, $\{u\}/\cos\theta$; ---, $\{\Delta\lambda\}/\cos\theta$ small cyclones and anticyclones ($N = 8197$); ○, $\{\Delta\lambda\}/\cos\theta$ large cyclones and anticyclones ($N = 778$).

A reexamination of Table 1 shows that the average displacement of cyclones is northward toward the pole at all latitudes while anticyclones move consistently toward the equator. It is a well-established observation that in the lower terrestrial atmosphere (northern hemisphere), around regions of relatively low pressure, air tends to spin in a counter-clockwise fashion, that is, in the same sense as the earth's rotation. Conversely, the wind blows clockwise around anticyclones. This difference in the average meridional motion of these systems embedded in the wind field probably is influenced by their sense of rotation.

V. Discussion

Direct measurements of the large-scale motions in the solar atmosphere are not likely to be available in the foreseeable future. As extrasolar observers, we will be forced to infer these motions indirectly using some such assumptions as those made by Ward. An extraterrestrial observer attempting to deduce the nature of the earth's wind field might choose a similar approach by observing the motion of local bright and dark spots in the atmosphere.

The local albedo of the sunlit hemisphere of the earth is ordinarily dependent on the presence or absence of clouds. It is generally found that clouds are observed in cyclones while the atmosphere in anticyclones is more likely to be clear. By measuring the daily displacements of these bright and dark spots (assuming the solid earth was not visible) the observer would probably conclude that part of the earth was fluid since a differential rotation would be observed. In the middle and high latitudes the earth would appear to make 374.3 rotations each sidereal year. Near the equator the rate would be close to that of the true solid-earth rotation of 366.2 rotations/sidereal year.

The extraterrestrial observer would also find that the earth spots, when grouped together, would have an average longitudinal motion of about 27° in their usual lifetime of about five earth rotations. This is much larger than the averaged poleward motion of these systems which average only about 2° of latitude during this time. Yet from a sufficiently detailed analysis and a large enough number of observations the investigator would correctly infer that net meridional motions were negligible. However, he would also find that the instantaneous zonal and meridional motions of these spots are positively correlated indicating the great importance of the eddy mode of angular momentum transport. The estimates of the magnitude of this mode would, nevertheless, be too low by a factor of about 3.

These earth spots are known to be vortices in the terrestrial atmosphere interacting with the wind field. Despite this interaction, the major features and the nature of the terrestrial wind field can be determined, to a first approximation, from the motion of these easily identifiable features. These results can be interpreted as support for Ward's argument that reasonable estimates of the nature of the large-scale motions in the solar atmosphere can be made from the proper motion of sunspots.

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