# Pyrheliometric measurements with special regard to the circumsolar sky radiation

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ABSTRACT

In the present article the influence of aperture and atmospheric turbidity on the readings of pyrheliometers, used within the meteorological network, has been considered on the basis of measurements and observations concerning the radiation intensity within the solar aureole now available. We have emphasized the fact that the constants of secondary pyrheliometers, as these constants generally are determined, are dependent on apertures of involved instruments and on the atmospheric turbidity at the occasion when the standardization is made, and also that secondary instruments generally, if the derived constants are applied to the readings, must show differences from one another which are dependent upon turbidity and aperture conditions. As the effect of these factors is of systematic character, they must be seriously considered if an accuracy surpassing about $\pm 1.5$ per cent shall be reached. Observations made in connection with the International Comparisons of Pyrheliometers at Davos 1964 are found to confirm the ideas expressed here and in previous notes by the present authors. Applications are suggested as regards (1) the difference found to exist between the old pyrheliometric standard scales, as regards (2) the future organization of Intenational Pyrheliometer Comparisons and (3) concerning the desirability of standardizing, to a greater extent than has been made up to now, the aperture conditions of pyrheliometers within the meteorological network.

Sixty years ago, at the time when already comparatively exact pyrheliometers had been constructed, no great attention was yet paid to the problem of comparing the different types with one another.

First after the construction of certain standard types of pyrheliometers—like the Smithsonian silverdisk of Abbot and the compensation instrument of K. Ångström—and after their spreading over the world in comparatively large number, the interest in comparing them in order to establish a "true" and international pyrheliometric scale became imminent.

The first comparisons gave results which were rather startling. Thus C. G. Abbot, in comparing in 1908 the Smithsonian pyrheliometric scale with the one established by K. Ångström, found a difference between both of not less than 9.2 per cent, the Smithsonian scale being the higher one (*Annals*, vol. II, 1908). Later in 1911 the difference between the two scales was given to be 5.5 per cent, and in 1932 it was stated that the Smithsonian scale of 1913, which had been found to be at an average about 3.5 per cent higher than the Ångström scale, ought to be regarded as 2.5 per cent *too high* (ABBOT & ALDRICH, 1932). As there are reasons to believe that the Ångström scale had remained practically constant during the whole time, the conclusion must be that new experiments have successively led to a decrease of the Smithsonian scale, which has been influenced by errors tending to give *too high values* to the readings.

In the meantime experiments carried out on the Ångström compensation pyrheliometer clearly showed that there were sources of error causing the readings to be *too low*. Thus the so-called "edge effect", pointed out and investigated by the present author in 1914 (ÅNGSTRÖM, 1914 and 1958), introduced an error of about $-1.5$ to $-2.5$ per cent and it seems probable that the rather bad heat conduction of the black coating of the receiver works to slight extent in the same direction.

After having subjected the experiments, comparisons and calibrations then available to a thorough examination, the International Radiation Commission at a meeting at Davos, 1956, recommended the adoption of a new pyrheliometric scale to be known as the Inter-

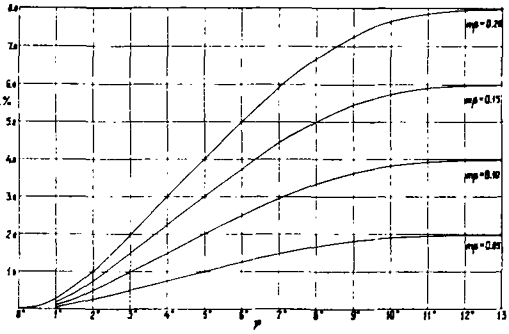


FIG. 1. Radiation from the aureole given in per cent $\varepsilon$ of the direct solar radiation, simultaneously measured. The figure shows $\varepsilon$ in its dependence on turbidity parameter $m\beta$ and on aperture of measuring instrument (pyrheliometer).

national Pyrheliometric Scale 1956. Measurements made according to the original Ångström scale should be increased by 1.5 per cent. As the difference between the Smithsonian scale of radiation as revised in 1913 and the original uncorrected Ångström scale has been found to be 3.5 per cent on the average for measurements of solar radiation, measurements made according to the Smithsonian scale of 1913 should be reduced by 2.0 per cent.

This recommendation was later confirmed by the Commission for Instruments and Methods of the World Meteorological Organization.

At these conclusions one important factor, namely the circumsolar radiation from the sky, was considered in a superficial way, without particular regard to the fact that this factor must cause pyrheliometers with different apertures to present differences from one another, variable with variable turbidity of the sky. Let $Q_0$ be the radiation to the receiver, within the aperture covered by the sun's disk itself, viz. within an aperture of about half a degree. Let the radiation measured with a pyrheliometer within the aperture $\varphi$ be $Q$. We have at the given occasion:

$$Q = Q_0 + D(\varphi), \qquad (1)$$

where $D(\varphi)$ is the circumsolar sky radiation within the aperture angle $\varphi$. As regards this function a considerable amount of measurements and discussions are now found in literature. For a diagram presented in a recent note (ÅNGSTRÖM, 1964*b*), the author used data and deductions presented by ABBOT (*Annals*, nos. 6 and 7), SCHÜEPP (1949), VOLZ (1956), LINKE & ULMITZ (1940) and also a number of results analyzed by van de Hulst and referring to investigations by Rosenberg, Dorno and De Jong (KUIPER, 1949). We have attempted a synthesis of the whole material just mentioned and arrive at the result graphically demonstrated in Fig. 1. For aperture values of 8° and below, the figure agrees closely with the figure in a preliminary report (ÅNGSTRÖM, 1964*b*). For $\varphi > 8°$, where earlier an extrapolation was necessary, the recent Davos comparisons have furnished the possibility to extend the curves up to about 15° aperture on the basis of actual experiments. These experiments suggest for angles above 8° a smaller increase in the circumsolar sky radiation with increasing aperture than was earlier assumed.

In the Fig. 1, $D(\varphi)$ is given in per cent $\varepsilon$ of $Q_0$ and plotted against the aperture angle, $\varphi$, given along the abscissa. On the basis of the investigations by SCHÜEPP and VOLZ (loc. cit.) the intensity of the aureole is assumed to be at all points, proportional to the product $m\beta$, where $m$ is the relative air mass (the vertical air mass being the unit), and $\beta$ is the coefficient of turbidity (A. ÅNGSTRÖM, 1964*a*).

It must be understood in view of what is now known about the solar aureole that the summary presentation here given is to some extent based on an approximation and on a simplification of conditions which occasionally must show considerable deviations from the averages.

## Comparisons of pyrheliometers

Regular observations and, to some extent, registrations of solar radiation are now generally included in the routine work at selected stations within the meteorological network.

As the solar radiation is one of the fundamental factors determining the climate, the importance of the comparability of the results, as well those from different localities as from different epochs at the same place, is evident and now generally realized. As a consequence intercomparisons of pyrheliometers have in recent time been organized on an international basis, and the results have been analyzed and subject to critical discussion.

Let us consider briefly the basic conditions which form the fundaments for these comparisons.

1. The pyrheliometers used at the comparisons have in general been secondary ones, which have been compared at some previous occasion with a standard pyrheliometer. Here generally the standard pyrheliometer alone is an absolute instrument, in the respect that from its known physical properties absolute energy values are derived from the readings. In order to obtain an absolute value for the radiation income from the readings of the secondary instruments these readings must be multiplied by constants obtained through a comparison with a standard instrument.

2. For as well standard as secondary instruments there exists a certain function $F$, in which the readings enter as parameters, and the value of which is pretended to be proportional to the energy falling on the receiver. We are in the following regarding this assumption as valid. The constant with which the function $F$ shall be multiplied in order to give the energy $Q$ of the solar radiation falling on the receiver, is denoted as the constant or the calibration factor $k$ of the instrument. In the case of a *standard pyrheliometer*, the calculation of this constant is based upon the physical qualities of the instrument (ÅNGSTRÖM, 1958). Two standard instruments called ($p$) and ($s$) should give the same result as regards $Q_0$ in (1) but may differ in $D(\varphi)$ if their apertures are not the same,

$$Q'_p = Q_0 + D(\varphi_p, m\beta),$$
$$Q'_s = Q_0 + D(\varphi_s, m\beta). \quad (2)$$

On the other hand, the constant of a *secondary pyrheliometer* is determined by comparing the instrument with a standard pyrheliometer. We call the constant that results in this way $k_{obs}$ in contrary to $k_{calc}$ which is obtained by the way of proceeding in the case of standard instruments. Say, a secondary instrument has the aperture $\varphi_a$ and has been calibrated by means of the standard instrument ($p$) at an occasion when the turbidity parameter $m\beta$ was characterized by the index $c(1)$ and the radiation of the sun's disk was $Q_1$. Consequently, the reading of the secondary instrument is at another time when the turbidity is described by the index $c(3)$ and the radiation of the sun's disk is $Q_3$,

$$Q''_a = \left\{\frac{Q_1 + D[\varphi_p, c(1)]}{Q_1 + D[\varphi_a, c(1)]}\right\}\{Q_3 + D[\varphi_a, c(3)]\} \quad (3)$$

where the ratio in the right-hand side of the equation is identic with $k_{obs}/k_{calc}$. The readings of the standard pyrheliometers are here signed by the mark $'$ and those of the secondary instruments by $''$.

3. Let us now assume that the purpose of the last-mentioned reading under the conditions $c(3)$ and $Q_3$ was to compare the secondary pyrheliometer ($a$) with another secondary one called ($b$) which has previously been compared with the standard pyrheliometer ($s$) when the conditions concerned where $c(2)$ and $Q_2$,

$$Q''_b = \frac{Q_2 + D[\varphi_s, c(2)]}{Q_2 + D[\varphi_b, c(2)]}\{Q_3 + D[\varphi_b, c(3)]\}. \quad (4)$$

Let us assume that $Q_1$, $Q_2$ and $Q_3$ are correct values of the radiation of the sun's disk. We recall the definition of $\varepsilon_c^\varphi$ given in Fig. 1 before we introduced the index notation. Now, the upper index refers to the instrument in question, the lower to the atmospheric turbidity conditions. We easily find that the ratio of the readings (3) and (4) of the secondary pyrheliometers ($a$) and ($b$) can be expressed by

$$Q''_a/Q''_b = R(a/b) = \frac{(1+\varepsilon_1^p)(1+\varepsilon_2^b)(1+\varepsilon_3^a)}{(1+\varepsilon_1^a)(1+\varepsilon_2^s)(1+\varepsilon_3^b)}. \quad (5)$$

Seeing in Fig. 1 that $\varepsilon$ is a small number — practically less than 0.08—we may write (5) in the form

$$R(a/b) = 1 + [(\varepsilon_1^p - \varepsilon_1^a) - (\varepsilon_2^s - \varepsilon_2^b)] + (\varepsilon_3^a - \varepsilon_3^b). \quad (6)$$

The relative difference between the readings of two secondary instruments is

$$\delta(a/b) = R - 1.$$

It is evident from the equations thus derived, that the readings of secondary pyrheliometers relative to one another, are in general dependent on not less than seven variables, namely on their opening angles, and also on those of the respective standard instruments, and on the state of the sky, viz. on the circumsolar sky radiation at three occasions: at the occasions of the calibrations with aid of the standards and at the time of the intercomparison between the secondaries. It is under these circumstances not astonishing that the results of such comparisons show a rather great variety from case to case. Some of the terms in (6) depend upon the conditions which prevailed during the original standardization of the secondary pyrheliometers ($a$) and ($b$), others only depend upon the conditions during the comparison of the two secondary instruments. We call the former group of the terms $M$, the latter $N$,

$$\delta(a/b) = M[\varphi_p, \varphi_s, \varphi_a, \varphi_b, c(1), c(2)] + N[\varphi_a, \varphi_b, c(3)], \quad (7)$$

$$\text{i.e. } M(a/b) = (\varepsilon_1^p - \varepsilon_1^a) - (\varepsilon_2^s - \varepsilon_2^b) \quad (8)$$

$$\text{and } N(a/b) = \varepsilon_3^a - \varepsilon_3^b. \quad (9)$$

The formula (7) is derived under the assumption that no fundamental difference exists between the standards ($p$) and ($s$) on account of technical deficiencies of the instruments. Thus, $M$ is zero if the primary comparisons were made under the most ideal conditions involving $\beta = 0$. However, when there is such a technical difference there might remain a systematic difference $r$, and on that account we instead of (8) write

$$M(a/b) = (\varepsilon_1^p - \varepsilon_1^a) - (\varepsilon_2^s - \varepsilon_2^b) + r. \quad (10)$$

Courvoisier (1957) in his work on comparative pyrheliometry, summarizes the results of comparisons between secondary instruments, representing the Ångström and Smithsonian scales respectively in the following way. At stations at ordinary altitude (0–500 m) the mean difference between the scales is evaluated to be $3.50 \pm 0.5$ per cent, at the high altitude station Davos (1600 m) the mean difference computed from a great number of comparisons is $5.16 \pm 0.3$ per cent (we have here excluded the measurements made earlier than June 1932 from reasons explained in a previous note). In both cases the Smithsonian instrument gives the higher reading. It is evident from a number of investigations that the main part of the difference between the scales must be ascribed to other causes than the circumsolar sky radiation. But on the other hand the fluctuation of the differences—at a given locality within limits of about $\pm 1.0$ per cent—and also those characteristic for high altitude stations relative to low altitude ones, may be largely explained on the basis of the relationships here derived between aureole intensity, aperture and certain atmospheric parameters. Investigations by Schüepp and Volz, upon which we have partly based the drawing of the curves in Fig. 1, show that it is reasonable to assume that the difference between $\varepsilon_3^a$ and $\varepsilon_3^b$ is proportional to the product $m\beta$. Thus we rewrite (7),

$$\delta(a/b) = M + m\beta \cdot f(\varphi_a, \varphi_b) \quad (11)$$

where $f(\varphi_a \cdot \varphi_b)$ is constant when the two instruments concerned are fixed.

Schüepp (1949) has treated a comprehensive series of measurements, carried out at Davos simultaneously with the aid of a recording pyrheliometer of less than 2° aperture and a Michelson actinometer (aperture 6° × 15°). A. Ångström finds with application of a graphical procedure to be described later in this article that the effective aperture of the Michelson instrument in question is 8.5°. The difference between the two instruments reads in form of (11) on the basis of Schüepp's investigation as follows,

$$\delta(8.5°/1.5°) = 0.30\ m\beta. \quad (12)$$

From Fig. 1, which is to some extent based upon the Davos measurements mentioned, we derive

$$\delta(8.5°/1.5°) = 0.32\ m\beta. \quad (13)$$

The difference between the readings of the two instruments is consequently equal to about 3.1 per cent in the case of $m\beta = 0.10$.

The measurements referred to above are now supplemented by recent measurements at the WMO International Comparisons at Davos in August 1964. In connection with this enterprise, the Swedish representative B. Rodhe (1964) made a series of comparisons between the readings of two Ångström pyrheliometers, Å 158 and Å 571, the effective apertures of which with application of a special graphical procedure can be evaluated to be 14.7° and 5.3° respectively. During the comparison, the turbidity of the atmosphere was measured and computed from total and RG2 radiation values (WMO, 1964). The mean value of $m\beta$ was 0.069. Rodhe found that the difference between the readings of the two instruments was correlated with $m\beta$, the correlation coefficient being $0.68 \pm 0.09$. In the form of (11), his measurements give the regression equation

$$\delta(14.7°/5.3°) = -0.020 + 0.115\, m\beta. \quad (14)$$

Thus, he found $f(14.7°, 5.3°) = 0.115$. The diagram in Fig. 1 gives 0.12.

At the same international meeting in August 1964, comparisons were made between the Stockholm Ångström pyrheliometer Å 158 and the Davos one Å 210. The effective apertures of the two instruments can be evaluated to be 14.7° as above for Å 158, and 8.0° for Å 210. The difference between the readings, given in Table III of the report from Davos (WMO, 1964), is found to satisfy the equation

$$\delta(14.7°/8.0°) = -0.003 + 0.074\, m\beta. \quad (15)$$

In this case, the correlation between $\delta$ and $m\beta$ is $0.48 \pm 0.13$, and $f(14.7°, 8.0°) = 0.074$. As regards the latter value, the diagram in Fig. 1 gives 0.068.

A further support for the existence of a combined influence of aperture and turbidity we get from the Tables III and IV of the report on the European regional comparison of national pyrheliometers, held at Davos in September 1964 (DOGNIAUX, 1964). The tables give the mean difference between the readings of each of twelve Ångström pyrheliometers and those of the Davos pyrheliometer Å 210. If we plot the differences against the aperture values we obtain a diagram where the correlation between both is evident. The coefficient of correlation is $0.59 \pm 0.20$. The regression equation has the form of (11) but $(\varphi_a - \varphi_b)$ is the variate and the mean of $m\beta$ during the measurements is involved in the coefficient of the second term,

$$\delta(a/\text{Å } 210) = 0.003 + 0.0032(\varphi_a - 8.0°) \quad (16)$$

The factor 0.0032 is found to agree well with the slope of the curves in Fig. 1, if we assume that the turbidity parameter $m\beta$ was about 0.10 during the measurements and that the apertures of the instruments varied between 5° and 10°, as actually has been the case. The coincidence is rather astonishing as it seems evident that a number of other more accidental factors must have had an influence too. Among those the conditions at the earlier standardizations of the instruments affect $N$ in (7)—or the second term in the right-hand side of (16)—when our previous assumption that $Q_s$ in each of the equations (3) and (4) should be a correct value of the direct radiation of the sun's disk, is not fullfilled. Thus, most of the calibration factors of the instruments which were dealt with at the comparison meeting, seem to fit rather close to the calculated constants [$k_{calc}$, see the discussion of (2) and (3)] of the instruments.

## Aperture angles of pyrheliometers

The modern silver disk pyrheliometer of the Smithsonian Institution and also the latest construction of the Smithsonian Standard (No. 5) are described by ABBOT & ALDRICH (1932).

The aperture is in both cases 5.8° against 16° for previous types of water flow and water stir standards of the Smithsonian. It seems probable that the decrease of the Smithsonian scale after repeated new experiments is, to a large extent, due to a decrease of the measured circumsolar radiation with a decrease in aperture.

The aperture of the K. Ångström compensation pyrheliometers varies highly from case to case. Thus the pyrheliometers represented at the International Comparisons at Davos had apertures which, according to special evaluations at the same occasions, varied between about 2.5° × 13° up to about 12° × 27° (Table 1). The cause of this great spreading is partly to be sought in the fact that a number of different constructors, have, independently of one another, applied different dimensions to the same type, with due regard to the different purposes in view. Thus Dr. Abbot of the Astrophysical Observatory of Smithsonian Institution in pursuing the determination of the solar constant as a main object, constructed and also put on the market a type with a very small aperture (3° × 6°). The original Ångström pyrheliometer, constructed by K. Ångström in 1896, had an aperture of 6° × 24° and the same aperture is characteristic for a number of pyrheliometers delivered from Sweden up to 1920. A fact, which is complicating the matter, must here be considered. The original type of instrument, represented for instance by Å 158, was provided with a filter holder, and with this filter holder in place, the aperture was reduced to the value given above, namely 6° × 24°. In this form it was originally used as a standard. But in practice, the filter holder has in many cases been removed, with the result that the

TABLE 1

| | | Aperture | $\varphi_{eff.}$ |
|---|---|---|---|
| Å 46 | Austria | 5 × 13 | 6.7 |
| 70 | Sweden | 6 × 22 | 8.0 |
| 100B | Kew | 4 × 14 | 5.3 |
| 119 | Congo | 11 × 26 | 14.7 |
| 122 | Hamburg | 7 × 21 | 9.3 |
| 140 | Potsdam | 2.5 × 12.5 | 3.3 |
| 157 | Japan | 8 × 20 | 10.7 |
| 158[b] | Japan | 9 × 21 | 12.0 |
| 158 | Sweden | 11 × 26 | 14.7 |
| 159 | Italy | 5 × 13 | 6.7 |
| 2028 | USA | 4.5 × 10.5 | 5.0 |
| 202 | Japan | 9 × 21 | 12.0 |
| 204 | Norway | 7 × 20 | 9.3 |
| 205 | Tunisia | 8 × 20 | 10.7 |
| 210 | Davos | 6 × 23 | 8.0 |
| 212 | USSR | 6 × 26 | 8.0 |
| 249 | India | 12 × 27 | 16.0 |
| 502 | Australia | 8 × 21 | 10.7 |
| 507 | Norway | 11 × 27 | 14.7 |
| 508 | India | 9 × 21 | 12.0 |
| 511 | Poland | 4.5 × 10 | 6.0 |
| 525 | Davos | 4 × 16 | 5.3 |
| 529 | Hungaria | 4 × 15 | 5.3 |
| 546 | Norway | 8 × 21 | 10.7 |
| 542 | South Africa | 8 × 21 | 10.7 |
| 548 | Valentia | 6 × 24 | 8.0 |
| 568 | Hamburg | 7 × 21 | 9.3 |
| 571 | Sweden | 4 × 15 | 5.3 |
| 576 | Nigeria | 5 × 15 | 6.7 |
| 641 | Davos | 5 × 15 | 6.7 |

aperture has been increased to values of about 11° × 26°. As it is not quite clear, neither as regards the standards nor concerning the standardized secondaries, whether and when in the past they have been used with or without the aperture-limiting filter holder in place, we have here a factor of some uncertainty. A solid point of departure is here given by the report from the last comparisons at Davos. In this report rather complete information is given concerning the apertures of practically all the instruments concerned.

## "Effective" aperture of the compensation pyrheliometers of the K. Ångström type

In a paper now in preparation by one of the present authors (Ångström) and Dr. A. Drummond, we have introduced the term "effective aperture". This aperture is de-

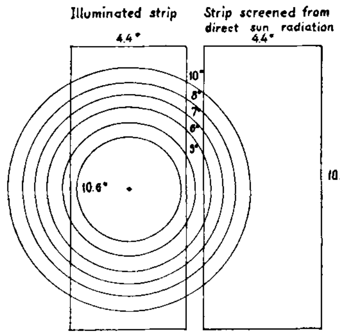


FIG. 2. Graphical procedure for determining effective aperture of compensation pyrheliometers.

fined as the full angle which in the case of a circular opening will give the same radiation to the central part of the receiver as does the actual rectangular aperture. The radiation reaching the centres of the strips was determined through graphical integrations, which here are simply suggested by reference to Fig. 2. The computations made it evident that the effective aperture is, practically, under the turbidity conditions generally occurring, independent of the turbidity. At present, the aperture of pyrheliometers with rectangular openings, is generally defined by the two angles, $\alpha_s$ and $\alpha_l$, corresponding to the short and long sides of the opening respectively. As the intensity of the radiation from the aureole decreases rapidly with the distance from the sun, an increase of the larger angle $\alpha_l$ above a certain limit value, produces no appreciable increase in the effective aperture. In fact, for rings of the sky, with distances above about 5°, (aperture > 10°), an overcompensation of the aureole radiation takes place, in the respect that from such a ring more radiation is falling on the screened than on the exposed strip. What here is said holds for the average type of construction of the compensation pyrheliometer.

As a good thumb rule, valid when $\alpha_l > 2.5\alpha_s$ (Ångström and Drummond, unpubl.), we may apply:

$$\varphi = [\tfrac{4}{3} \pm 0.05]\,\alpha_s. \qquad (17)$$

For an aperture of, for instance 6° × 24°, we thus have $\varphi = 8.0° \pm 0.3$.

The effective aperture, thus computed for the various pyrheliometers, compared at Davos, is given in the third column of Table 1. The frequency of the different apertures of these pyrheliometers on various intervals is graphically demonstrated in Fig. 3. We find that they fall chiefly within 3 main intervals which we design by $A$, $B$ and $C$.

The group $A$, to which belong 14 of the 30 pyrheliometers, viz. 40 per cent of them all, is characterized by apertures between 5° and 8°, the group $B$, to which belong 11 instruments, represents apertures between 9° and 12°, and finally the group $C$, to which belong 4 instruments, embraces apertures between 14° and 16°. The mean apertures within the three main groups are: for $A$ 6.5°, for $B$ 10.7° and for $C$ 15.0°. Let us now imagine a case which is of some interest. Let us assume all the instruments to have been compared at, let us say Davos at turbidity conditions defined by a given mean value for the turbidity parameter $m\beta$ (about 0.075 to 0.10). We apply corrections to their earlier constants in order that they all read equal to a given standard, let us say Å 210, with an effective aperture of 8.0°. Let us now bring all the instruments under conditions characterized by a value of $m\beta$ which is 0.10 higher, viz. about 0.20, a value which often must occur at places like Washington, Kew Observatory and Potsdam at relative air masses between 1.2 and 2.0. The curve in the upper part of Fig. 3, shows how the different instruments may now be expected to deviate from the assumed standard. The instruments of groups $B$ and $C$ will read about 0.5 to 0.7 per cent higher, the instruments $A$ between 0.2 and 1.2 per cent lower. A little group represented by the small aperture instruments of Smithsonian Institution can be expected to read about 2.0 per cent lower than the assumed standard. In the extreme cases (apertures of about 4° on the one hand and about 15° on the other): instruments which were adjusted to give equal readings at the low atmospheric turbidity at Davos, can be expected to differ with about 3 per cent at the turbidity conditions often prevailing at low altitude stations.

## Difference between the Ångström and the Smithsonian scales of radiation

Let us assume that in the formulae (10) and (11) given above, the Ångström Pyrheliometric Scale is correctly represented by the instruments ($s$) and ($b$), the Smithsonian Scale by the instruments ($p$) and ($a$). Then the differences $\varepsilon^p - \varepsilon^a$ and $\varepsilon^s - \varepsilon^b$ are zero under every turbidity condition, and there remains in (10) only the systematic difference $r$ between the two scales. Consequently, equation (11) reads under these conditions,

$$\delta(\mathrm{S/Å}) = r + m\beta \cdot f(\varphi_{\mathrm{S}}, \varphi_{\mathrm{A}}) \tag{18}$$

According to the investigations by Abbot & Aldrich (1932) the official Smithsonian scale of 1913 is 2.5 per cent too high. Recent investigations by F. Lindholm (1958) on the "edge effect" in the case of the Ångström standard No. 70, shows this effect to cause the Ångström scale to be 2.4 per cent too low. If we apply for the apertures the values 6.0 and 8.0 respectively, and as regards the aperture effect the results presented in Fig. 1, we get, as we have $r = 0.025 + 0.024 = +0.049$,

$$\delta(S/Å) = 0.049 - 0.085 m\beta. \tag{19}$$

The result is also presented in Table 2.

As is seen, the result coincides rather well with the findings by laboratory and field measurements. For $m\beta = 0$, the difference is the one caused by technical deficiencies as investigated

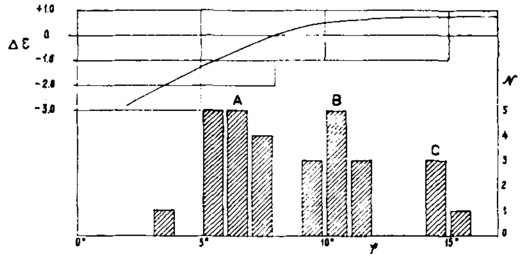


FIG. 3. The figure demonstrates frequency of the apertures of the pyrheliometers represented at the Davos comparisons September 1964 (lower part of figure). If all these pyrheliometers are assumed to be standardized as to give values equal to the Davos standard Å 210 ($\varphi = 8°$) at conditions where the turbidity parameter $m\beta$ has a given value, we get from the upper part of the figure the differences from the standard which are expected to occur, if the comparison is repeated at conditions when the turbidity parameter is 0.10 higher.

TABLE 2

| | | | | | |
|---|---|---|---|---|---|
| $m\beta$ | = 0 | 0.05 | 0.10 | 0.15 | 0.20 |
| $\delta(S/Å)$ | = 4.9 | 4.5 | 4.0 | 3.6 | 3.2 per cent |

by Abbot and Aldrich, A. Ångström and F. Lindholm. For a value of the turbidity parameter of $m\beta = 0.16$, which seems rather typical for low altitude stations, we get the difference 3.5, which is the classical one, on which the agreement on the International Pyrheliometric Scale of 1956 is based.

## Practical consequences as regards construction and comparison of pyrheliometers

It has been shown to full evidence that if we wish to attain an accuracy greater than what corresponds to about 1.5 per cent, in the measurements by pyrheliometers of common construction, attention must be paid to the apertures of the instruments and to the atmospheric turbidity conditions. With an effective aperture of 8° the instrument measures, in addition to the direct radiation within the aperture covered by the sun's disk, a radiation from the aureole which under ordinary conditions amounts to between 1.5 and 7.0 per cent of the direct radiation. With regard to different main purposes of the measurements, however, different attitudes to this additional term may be taken.

1. From climatological point of view the separation of the global radiation in two components, the direct and the diffuse one, is desirable on account of the fact that the direct radiation follows, as also the radiation from the aureole, certain rather simple laws, given by the height and azimuth of the sun. For computations aiming at deriving the energy falling on differently oriented surfaces like slopes, walls or on the ground itself, it seems then practical to include the energy of the aureole in the direct sun radiation component. It is evident from the report here given that the main part of the energy of the aureole falls within an aperture of about 10°. A further increase of the aperture of a compensation pyrheliometer of common construction seems to add only a negligible amount of energy to the measured value.

2. For a number of scientific purposes on the other hand it seems important to get a clear separation between the direct radiation from the sun's disk and the diffuse radiation from the sky. It seems probable, from what is now known, that the radiation from the aureole reaching the receiver of a pyrheliometer is generally reduced below 0.5 per cent for the case that the aperture is less than 1.5°. Such a small aperture has been practicized by Dorno in a recording pyrheliograph and the demand may be easily realized at special observatories with qualified outfit. But for a general use within a meteorological network of radiation stations, pyrheliometers with effective apertures below about 4° are unpractical and the advantage of using them illusory, on account of the difficulties to avoid instability. The recent models of the Ångström pyrheliometer by the Eppley Laboratory Inc., Newport R.I. and by the Meteorological and Hydrological Institute in Stockholm, Sweden have effective apertures of 5° to 6°, and the same aperture is characteristic for the modern silver disk of the Smithsonian Institution.

3. It seems desirable, however, that the apertures of pyrheliometers are standardized and limited to certain preferred dimensions. The use of pyrheliometers with an aperture of 10°–15° seems justified from practical and climatological point of view. An aperture of 5°–6° may be recommended for synoptic studies of the influence of different constituents of the atmosphere upon the radiation exchange within the atmosphere. Finally the use of pyrheliographs of less than 2° aperture at selected qualified observatories ought to furnish records from which the influence of the aureole radiation on the readings of pyrheliometers may be clarified and the conceptions here presented supplemented.

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