The mean meridional circulation of the southern hemisphere inferred from momentum and mass balance

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ABSTRACT

The mean meridional circulation for the southern hemisphere is inferred from momentum and mass balance considerations. Recent calculations by Obasi of the transient eddy transports of momentum and the mean zonal flow are the only data used. The vertical eddy transports are taken into account only in the surface layers, where the stress is assumed to decrease linearly with pressure from its surface value. The surface stresses themselves are calculated from the pressure integral of the momentum equation, and by a short iteration process the effect of horizontal transport of momentum by the mean meridional circulation is taken into account.

The resulting circulation is self consistent, needing only very small corrections to eliminate mass "drifts" across latitude circles, contrary to most direct measurements. The characteristic three cell pattern is very evident. The circulation is compared to that found in a somewhat similar manner by Mintz and Lang for the northern hemisphere. The winter circulations are found to be rather similar, but the polar direct cell is much stronger in the southern hemisphere. In the summer season the southern hemisphere circulation is much stronger.

The kinetic energy balance for the southern hemisphere is briefly considered. Given our assumptions about the vertical eddy fluxes of momentum, about half the kinetic energy fed into the mean flow by the horizontal transient eddies is extracted by the mean meridional motion. The other half is dissipated near the surface. In each season, energy balance is achieved to within 3 %, verifying the consistency of the calculations.

1. Introduction

Direct measurements of the mean meridional circulation in the atmosphere usually suffer from large uncertainties since they are a relatively small residual (of the order 1 m/sec) left from the averaging around latitude circles of time mean meridional winds of the order of 10 m/sec. For this reason, it is desirable to try to infer the mean meridional circulation from more accurately measured quantities. Since the mean meridional circulation is forced (Kuo, 1956; Gilman, 1964*b*), we may infer its strength in the steady state from the requirements that there be a momentum and a heat balance at every point in the meridional cross-section. General theoretical aspects of this problem have been discussed recently by Gilman (1964*b*). In any case, "indirect measurement" of the mean meridional circulation can be made only with sufficient knowledge of the other components (the eddy fluxes and diabatic heating) in the momentum and thermodynamic energy equations. As was pointed out by Gilman, 1964*b*, we have a choice of inferring the mean meridional circulation from the thermal forcing alone, or the mechanical forcing alone. (Ideally, of course, both should give the same result.) The choice would depend on which type of forcing we can measure more accurately.

For the southern hemisphere, Obasi (1963*a*, *b*) has calculated the mean zonal flow and the eddy transports of momentum for the IGY.[1] The eddy heat transports have not yet been computed. Therefore, we must use the mechanical forcing to infer the mean meridional circulation for the southern hemisphere.

Obasi (1963*a*) has also attempted to evaluate the mean meridional circulation directly, but his results suffered greatly from the uncertainty mentioned earlier. This uncertainty manifested itself in the very large meridional "drift" velocities (often as large as 40 cm/sec) which

[1] Certain corrections have been made to Obasi's (1963*a*, *b*) data.

had to be subtracted from the observed values to give zero mass transport across all latitude circles.

Having chosen to infer the mean meridional circulation from the momentum forcing rather than the thermal forcing, we must examine this momentum forcing more carefully. It consists of eddy convergences of momentum, both horizontal and vertical, and "friction". This "friction", for our purposes, is really just vertical eddy stresses on a scale smaller than measurable on a synoptic map. However, Obasi did not attempt to evaluate vertical eddy fluxes of momentum on *any* scale so really we can not separate the scales in our calculations.

Of the various forcing terms, the *transient* eddy convergences of momentum are the most accurately measurable. We may obtain the value of the time correlation of the zonal and meridional components of the wind at each *station*, and rely on map analysis and grid point readings only for obtaining the longitudinal averages of the momentum flux. This is not the case with the horizontal *standing* eddies, since there we must draw maps of the time averaged zonal and meridional components of the wind separately and evaluate the longitudinal correlation by the product of values at grid points. Furthermore, the fields of the meridional component of the wind have a much more frequent reversal in sign along a given latitude circle than will the time correlations of the meridional and zonal wind. For both these reasons, the standing eddy transports are generally less accurate quantities than are the transient eddies. In addition to this difference, OBASI (1963*b*) found that for the southern hemisphere, the standing eddies played a much smaller role in the horizontal transport of momentum than did the transient eddies.

The vertical eddy transports are much harder to evaluate than even the horizontal standing eddies, since the vertical motion is almost impossible to evaluate. Attempts have been made for the northern hemisphere by JENSEN (1961) in which he calculated "adiabatic" vertical motions, but there is some question as to the validity of this method (WIIN-NIELSEN, 1964). Furthermore, as stated earlier, Obasi did not even attempt to calculate the vertical eddy fluxes for the southern hemisphere.

In light of the above, the author decided to attempt to construct a mean meridional circulation for the southern hemisphere using *only* the horizontal transient eddy transports of momentum and the mean zonal wind measured by Obasi, feeling that from these most accurately measurable quantities it was likely that a more self consistent and accurate picture of the mean meridional circulation might be obtained.

2. Notation

The following notation will be adopted (generally the same notation as in GILMAN, 1964 *a*, *b*).

t = time
p = pressure
ϕ = latitude
λ = longitude
u = zonal wind (positive toward east)
v = meridional wind (positive toward north)
ω = dp/dt
Ω = angular velocity of the earth
f = Coriolis parameter = $2\Omega \sin \phi$
a = radius of the earth
g = acceleration of gravity

$$(\overline{}) = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} (\quad)\, dt = \text{time average}$$

$$(\quad)' = (\quad) - (\overline{}) = \text{deviation from time average}$$

$$[(\quad)] = \frac{1}{2\pi}\int_0^{2\pi} (\quad)\, d\lambda = \text{zonal average}$$

$(\quad)^*$ = $(\quad) - [(\quad)]$ = deviation from zonal average
Z = mean absolute vorticity = $f - (a \cos \phi)^{-1}\partial/\partial\phi[\bar{u}] \cos \phi$
τ = stress in the zonal direction
χ = frictional force/unit mass

3. Equations and the method of calculation

The method of calculation of the mean meridional circulation is that given in GILMAN (1963 and 1964 *b*). In brief, the momentum balance equation for the zonal component and the continuity equation (both are time and zonally averaged and are written in the pressure coordinate system) are solved for the vertical motion $[\bar{\omega}]$. The horizontal component $[\bar{v}]$ is then obtained directly from the momentum balance.

Thus we have, for $[\bar{v}]$

$$[\bar{v}] = \frac{[\bar{\omega}]}{Z}\frac{\partial[\bar{u}]}{\partial p} - \frac{G}{Z}, \quad (1)$$

where G is the momentum forcing function, i.e.,

$$G = -(a\cos^2\phi)^{-1}\partial/\partial\phi[\overline{u'v'}]\cos^2\phi + [\bar{\chi}]. \quad (2)$$

Here we have omitted the standing eddy convergences ($[\bar{u}^*\bar{v}^*]$ and $[\bar{u}^*\bar{\omega}^*]$) and grouped the vertical eddy fluxes of all scales in the friction term $[\bar{\chi}]$, in accordance with the remarks in section 1.

For $[\bar{\omega}]$ we have

$$[\bar{\omega}] = \left(\exp - \int_0^p J dp'\right)\int_0^p \left(\exp\int_0^{p'} J dp''\right) K dp', \quad (3)$$

where the integration is along the absolute angular momentum isolines, which are the characteristics of the problem. Here

$$K = -(\cos\phi)^{-1}\partial/\partial\phi\, G\cos\phi, \quad (4)$$

and

$$J = (\cos\phi)^{-1}\partial/\partial\phi\, s\cos\phi, \quad (5)$$

where s is the slope of the angular momentum characteristics, given by

$$s = \frac{1}{aZ}\frac{\partial[\bar{u}]}{\partial p}. \quad (6)$$

We have no direct measurements of the vertical eddy fluxes of momentum for the southern hemisphere. However, their effect cannot be neglected at least near the surface because the horizontal transient eddy forcing alone will not give the return flow required to balance the mass transport across latitude circles at higher levels. Therefore we must make some assumption about the vertical eddy transports in the lowest layers. The simplest assumption to make would be that the stress due to these fluxes decreases from its surface value to a negligible value a certain pressure height above the surface. We chose a decrease linear in pressure, reaching zero at 850 mb (700 mb over the Antarctic continent). That is, for

$$[\bar{\chi}] = -g\frac{\partial[\bar{\tau}]}{\partial p} \quad (7)$$

TABLE 1. *Surface stress and drift velocity for the* WINTER *season.*

Corrected surface stress includes the effect of horizontal convergence of momentum due to the mean meridional motions.

Latitude (°S)	Surface stress (dyne/cm²) Uncorrected	Corrected	Drift (cm/sec)
77.5	−2.00	−1.89	0
72.5	−1.91	−1.88	4
67.5	−0.75	−0.83	2
62.5	0.48	0.36	0
57.5	0.83	0.70	0
52.5	1.14	1.07	0
47.5	1.12	0.99	1
42.5	1.07	1.03	0
37.5	0.70	0.82	−1
32.5	0.12	0.27	0
27.5	−0.09	0.14	−3
22.5	−0.67	−0.52	3
17.5	−0.46	−0.49	1
12.5	−0.43	−0.63	2
7.5	−0.36	−0.65	4
2.5	−0.28		

we approximate

$$[\bar{\chi}] = -g\frac{[\bar{\tau}]}{\Delta p}, \quad (8)$$

where Δp is the pressure depth of the surface layer. Now the surface stress can be inferred from the horizontal transient eddy transports of momentum themselves, simply by integrating the momentum equation in the vertical to the "top" of the atmosphere. (This method was used by Obasi in his estimation of the surface stress.) Furthermore, to include the horizontal transports of momentum by the mean meridional motion (the term $[\bar{u}][\bar{v}]$) in the calculation, a short iteration procedure was used. A first estimate of the surface stress was obtained from the vertical integral of the horizontal transient eddy convergences of momentum alone. These stress values were then used, together with the assumption about the rate of decrease of stress with height and the unintegrated distribution of horizontal transient eddy convergences of momentum, to calculate a first mean meridional circulation. The mean meridional circulation was then used to correct the surface stress values from which a second mean meridional circulation was then obtained. That this correction (see Tables 1 and 2) was not

TABLE 2. *Surface stress and drift velocity for the* SUMMER *season.*

Corrected surface stress includes the effect of horizontal convergence of momentum due to the mean meridional motion.

Latitude (°S)	Surface stress (dyne/cm²) Uncorrected	Corrected	Drift (cm/sec)
77.5	−1.39	−1.35	2
72.5	−0.93	−0.97	4
67.5	0.23	0.10	1
62.5	1.13	1.00	0
57.5	1.45	1.36	1
52.5	1.62	1.57	−1
47.5	1.16	1.17	0
42.5	0.46	0.50	0
37.5	0.10	0.17	0
32.5	−0.12	−0.02	0
27.5	−0.30	−0.20	0
22.5	−0.42	−0.38	0
17.5	−0.43	−0.46	0
12.5	−0.37	−0.45	−1
7.5	−0.25	−0.35	1
2.5	−0.22		

too large indicates the relatively small contribution of the mean meridional circulation in determining the *horizontal* transport of momentum. By the nature of our assumptions, however, we have forced it to be the mode of *vertical* transport between the upper levels and the surface layer; but this does not seem too unreasonable, as discussed by GILMAN (1964*a*, PHILLIPS (1954)) and others.

To actually carry out the integration for $[\bar{\omega}]$ indicated in (3), we must interpolate the integrand onto the characteristic curves, whose profiles must first themselves be calculated. The equation defining the characteristic in p, ϕ space is just the definition for absolute angular momentum/unit mass m, i.e.,

$$m = \Omega a^2 \cos^2 \phi + a[\bar{u}] \cos \phi. \tag{9}$$

To find ϕ as a function of p for $m = m_0$, the angular momentum at evenly spaced latitudes, we solve for $\cos \phi$, using the fact that

$[\bar{u}]/4\sqrt{\Omega m_0} < 1$, to obtain

$$\cos \phi = \frac{2\sqrt{\Omega m_0} - [\bar{u}]}{2\Omega a}.$$

Strictly speaking, we must use $[\bar{u}]$ at ϕ to determine $\cos \phi$, but we may to first order use $[\bar{u}]$ at ϕ_0, since for a given characteristic $|\phi - \phi_0| < 5°$ generally. Once the position of the characteristics was known, we interpolated the integrand values linearly onto them. The m_0 values (and therefore the ϕ_0 values) were chosen to coincide with the original latitude grid for the integrand, making the interpolation particularly simple. The results of the integration were then interpolated back to the ϕ_0 latitudes, from which the figures were made.

On a meridional cross-section, the characteristics appear as nearly vertical lines, each one not deviating from its reference latitude ϕ_0 by more than 5°. Since we may say that at the top of the atmosphere $[\bar{\omega}] = 0$, the solution (3) was chosen so that the integration along the characteristic begins at $p = 0$, and proceeds to the surface. Had we found the mean meridional circulation in the manner of KUO (1956) solving a second order equation for the meridional stream function, we would have imposed boundary conditions such that $[\bar{\omega}] = 0$ would hold both at $p = 0$, and at the surface. But here we are allowed to specify $[\bar{\omega}]$ at only one point on each characteristic. Nevertheless, $[\bar{\omega}]$ will still approach zero near the surface, as can be seen by a closer examination of (3). If the factors $\exp \pm \int J\,dp$ were deleted, and the characteristics straightened to become lines of constant latitude, (3) would just be the integrated continuity equation if we assumed that $[\bar{v}] = -G/Z$. In this simpler calculation we would expect $[\bar{\omega}] \to 0$ at the surface. Since, (*a*) $0.5 \leq \exp \pm \int J\,dp \leq 2.0$ generally, (*b*) the factors inside and outside the integral in (3) tend to cancel, and (*c*) the characteristics are nearly vertical, we should therefore expect $[\bar{\omega}]$ to be not radically different from the less refined calculation (10–30% was the result found in GILMAN 1963).

It is evident then that the above method of calculation of the mean meridional circulation will give a set of values to which no large "drift" correction will have to be applied to ensure vanishing meridional mass transport. We may therefore say that we have at least arrived at a self consistent mean meridional circulation. This does not say we have arrived at the correct one, but since the assumptions made in our calculation do not seem to be too unreasonable, it is probable that our results are more accurate than directly measured values would be.

4. Results and comparisons

The calculated mean meridional cross-sections for the winter (April–September 1958) and summer (January–March; October–December 1958) are presented in Figs. 1 through 4. The surface stress values used, both before and after correction for the horizontal convergence of momentum due to the mean meridional circulation, are presented in Tables 1 and 2, along with the meridional drift velocities which resulted from our method of calculation.

From examination of the results, the following facts are of note:

(*a*) Both seasons show very distinctly the characteristic three cell pattern, though the equatorward boundary of the direct cell in low latitudes is not in evidence. There is also evident a shift of the cell pattern equatorward in the winter season, as would be expected from the shifting of the heat equator. The amount of this shift is generally 10–20° latitude, and is more strongly evident in the vertical motion sections (Figs. 2 and 4), indicating a change in the shape of the cells with season.

(*b*) A calculation somewhat similar to ours was performed for the northern hemisphere by MINTZ & LANG (1955). For the *winter* season in each hemisphere the boundaries between cells are found to be between 30° and 35°, and between 60 and 65°. The indirect cells in the two hemispheres have about the same mass circulation, as do the equatorial direct cells. In both cases, the equatorial direct cell has about three times the mass circulation of the middle latitude indirect cell. However, the southern hemisphere results show a very much stronger polar direct cell than has the northern hemisphere. Furthermore, the winter circulation in the southern hemisphere appears to overlap more across the equator than does the northern hemisphere circulation in its winter. This last result seems to be in agreement with the general conclusion of OBASI (1963*a*, *b*) that the southern hemisphere circulation is generally stronger. This conclusion is reinforced by the fact that Mintz and Lang's values are averages only over the most winterlike months, January and February.

In the *summer* season, the southern hemisphere mean meridional circulation is seen to be very much stronger. The polar cell is still very strong, while in the northern hemisphere it is practically non-existent. Undoubtedly the weaker northern hemisphere circulation is due partly to the choice by Mintz and Lang of two month averages over July and August, but it would appear to be so weak that even a 6 month average would be weaker than the southern hemisphere result.

The much stronger polar direct cell in the southern hemisphere during both seasons may be a manifestation of the katabatic wind descending off the antarctic continent.

(*c*) The surface stress values presented in Tables 1 and 2 indicate that the inclusion of the horizontal momentum convergences due to the mean meridional circulation does make some difference, but not a large amount, being generally on the order of 10–20%. The uncorrected values are slightly different from those calculated by OBASI (1963*b*), as the numerical integration procedures were slightly different.

(*d*) The drift velocities resulting from our method of calculation (which were subtracted out before the sections were drawn) are never larger than 4 cm/sec and average about 1 cm/sec. These drifts are probably a result of the finite difference computation methods. It should be noted that had the stress been assumed to decay more slowly with the decrease in pressure, the drifts would have been about the same, since the total meridional mass flow in the lower layers from the vertical convergence part of the forcing depends only on the size of the surface stress.

5. Energy computations

As a further check on the self consistency of our computations, the kinetic energy balance for the southern hemisphere is briefly considered. OBASI (1965) calculated the conversion of transient eddy kinetic energy to mean zonal kinetic energy for the hemisphere. The calculation of the conversion of mean meridional kinetic to zonal kinetic energy via the Coriolis parameter, however, was considered to be unreliable due to the uncertainty in the directly measured $[\bar{v}]$ values. Using our indirectly measured mean meridional circulation and mean zonal winds of OBASI (1963*a*) (extrapolating for the layer 850–1000 mb), we may make a first estimate of this conversion integral. Furthermore, we may estimate the dissipation of mean zonal kinetic energy near the ground in our model by using the calculated surface stress values, and the extrapolated zonal winds.

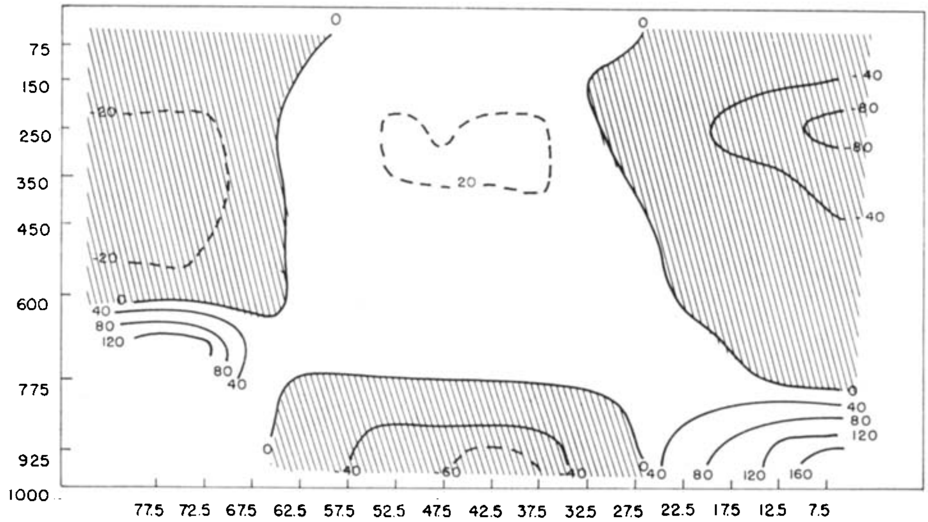

FIG. 1. Mean meridional motion $[\bar{v}]$ (cm/sec) for the southern hemisphere *winter* season, calculated from momentum and mass balance. Shaded areas indicate motion toward the south pole.

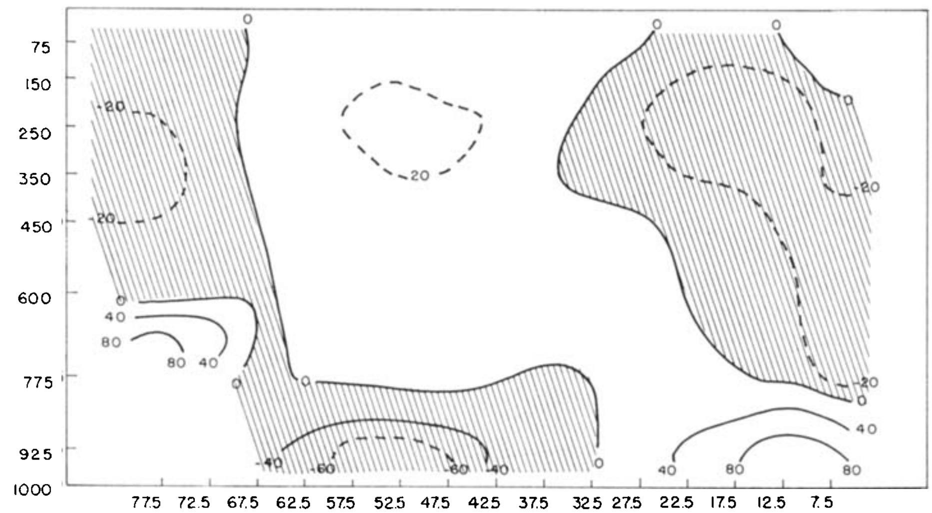

FIG. 3. Mean meridional motion $[\bar{v}]$ (cm/sec) for the southern hemisphere *summer* season, calculated from momentum and mass balance. Shaded areas indicate motion toward the south pole.

Mathematically, the conversion via the Coriolis parameter is given by the integral

$$\frac{2\pi a^2}{g}\int_0^{p_g}\int_0^{\pi/2} f[\bar{u}][\bar{v}]\cos\phi\, d\phi\, dp$$

while the dissipation in the surface layers is obtained from the integral

$$\frac{2\pi a^2}{g}\int_0^{p_g}\int_0^{\pi/2} [\bar{u}][\bar{\chi}]\cos\phi\, d\phi\, dp$$

which becomes, under our assumptions,

$$2\pi a^2\int_0^{\pi/2} [\bar{u}]/\,[\bar{\tau}]_{p_g-\Delta p/2}\cos\phi\, d\phi.$$

Here the subscript g refers to the ground level. The results of these integrations are presented in Table 3. As can be seen from the last column, which gives the amount of imbalance, our method of calculation arrives at energy conversion rates which balance to within 3%. In

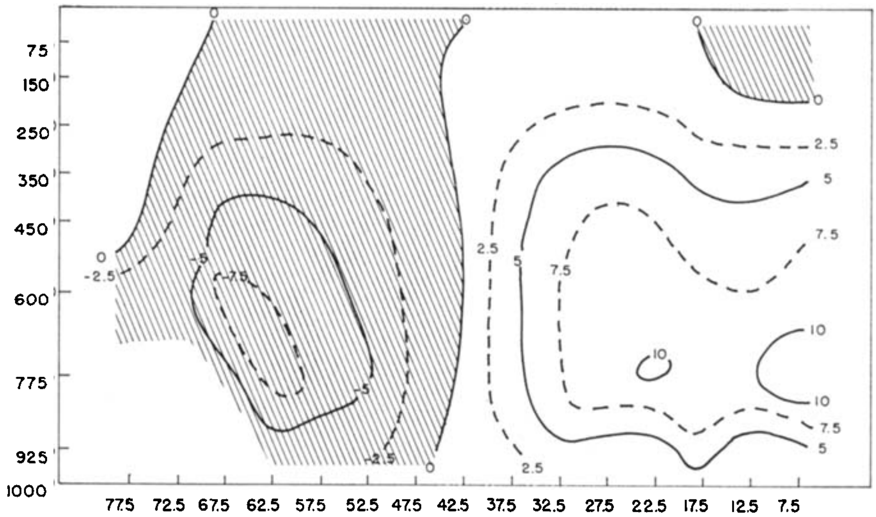

FIG. 2. Mean vertical motion $[\bar{\omega}]$ (10^{-5} mb/sec) for the southern hemisphere *winter* season, calculated from momentum and mass balance. Shaded areas indicate "upward" motion.

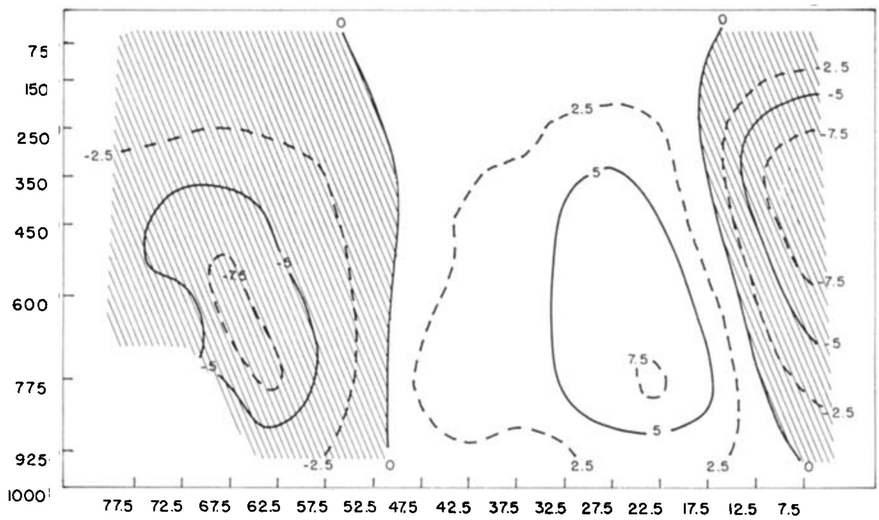

FIG. 4. Mean vertical motion $[\bar{\omega}]$ (10^{-5} mb/sec) for the southern hemisphere *summer* season, calculated from momentum and mass balance. Shaded areas indicate "upward" motion.

this balance the mean meridional circulation acts to remove about half the energy fed into the mean flow, the other half being dissipated. The degree of consistency will not be affected appreciably by a change in the extrapolated zonal wind, since a decrease in dissipation due to such a change will, by the nature of our method of calculation, be accompanied automatically by an increase in the conversion from mean zonal to mean meridional kinetic energy through the Coriolis parameter.

TABLE 3. *Comparison of the various energy conversions in the mean zonal kinetic energy balance.* A positive value indicates a conversion to mean zonal kinetic energy.

	Energy conversions (10^{20} ergs/sec)			
	Eddy to zonal	Meridional to zonal	Dissipation	Residual
Winter	+ 9.63	− 3.85	− 6.17	− 0.39
Summer	+ 9.72	− 5.40	− 4.02	+ 0.30

The values of the conversion from mean zonal to mean meridional kinetic energy via the Coriolis parameter are on the average about 75% greater than those obtained by direct measurement by STARR, 1959 for the northern hemisphere. This is a reasonable result since Obasi showed the rate of conversion of eddy kinetic to zonal kinetic energy to be almost twice as large in the southern hemisphere as in the northern.

6. Concluding remarks

Recapitulating, we have calculated a reasonable mean meridional circulation, surface stress distribution, and kinetic energy balance entirely from the observed horizontal transient eddy convergences of momentum and mean zonal wind. Since all the balance requirements are met, these statistics form a self consistent set. They are undoubtedly not the correct set. However, since the approximations made seem physically reasonable, this mean meridional circulation is probably more accurate than can be obtained at present by direct measurements. Even so, improvements in this method could be made. For example, more attention could be paid to the surface zonal wind values (though this would require a great deal more work). Even with better surface wind values, however, it will probably still be profitable to estimate the surface stress from the momentum convergences, rather than using drag coefficient assumptions, if, again, only for self consistency.

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