On the structure and energetics of large scale hydromagnetic disturbances in the solar photosphere

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ABSTRACT

It is suggested that the solar differential rotation is maintained against frictional dissipation by large scale quasi-horizontal hydromagnetic disturbances, which possess tilted structures similar to upper level troughs in the westerlies of the earth's atmosphere. On the sun, these tilts would be directed from the equator toward the east limb in each hemisphere. Horizontal Reynolds stresses in these disturbances thus convert eddy kinetic energy into kinetic energy of the zonal flow, while horizontal Maxwell stresses convert the mean zonal kinetic energy into "eddy" magnetic energy. In order for the disturbances as a whole to feed kinetic energy into the differential rotation, the horizontal eddy magnetic fields must be somewhat less than 7 gauss at the level considered.

Measurements of the large scale patterns of the line of sight component of the magnetic field, made by Howard, show clearly the required tilt in the patterns toward the east limb in each hemisphere. These measurements also show that the patterns do have a very large horizontal scale, possessing a longitudinal wave number of about 6, and a latitudinal extent of about 60°. The magnitudes of these fields appear to be compatible with horizontal eddy magnetic fields less than 7 gauss. The braking action by the symmetric steady component of the horizontal Maxwell stress appears from the observations to be small compared with that due to the eddy component.

1. Introduction

The problem of the maintenance of the solar differential rotation was recently given new life with the discovery by our colleague, Dr. Fred Ward (WARD 1964, 1965), that the longitudinal and latitudinal movements of sunspots are systematically correlated. If the sunspot movements are assumed to trace large scale mass motions, then these correlations imply an "eddy" angular momentum flux into the equatorial regions, where the angular velocity is greatest. Ward estimated that if these fluxes were totally suppressed, allowing all other momentum exchange processes to continue, the differential rotation would be virtually destroyed within the time of a few solar rotations.

The writers subsequently showed (STARR & GILMAN, 1965) that these equatorward momentum fluxes imply that the differential rotation is gaining kinetic energy at the expense of the kinetic energy of the eddies. The writers emphasized that this conversion process acted in a manner contrary to molecular and eddy viscosity, and indeed could comprise the primary mechanism by which the differential rotation is maintained against the sum total of dissipative effects.

However, though the eddy momentum fluxes as inferred by Ward were shown by the writers to feed eddy kinetic energy into the differential rotation, the eddy structure required to give the flux in the correct sense was not discussed. One purpose of the present paper is therefore to suggest a simple flow disturbance capable of effecting the proper flux. This disturbance, commonly called the "tilted trough" in meteorology, was first suggested by STARR (1948) to be responsible for the bulk of the horizontal eddy flux of momentum in the earth's atmosphere.

In the earlier paper, the writers also pointed out that the magnetic energy associated with bi-polar magnetic regions seemed to be of the same order of magnitude as the kinetic energy linked to the spot motions. Furthermore, the horizontal scale of these bi-polar magnetic regions was noted to be the same as the scale of eddies in the flow field needed to carry the sunspots. Though these facts indicate the probable importance of the eddy magnetic fields in the energy exchange processes, the precise role which they might play was not further investigated. A second purpose of the present paper is then to show how these eddy magnetic fields, in the form of Maxwell stresses, could act as a significant brake on the differential rotation, opposing the gain derived from eddy kinetic energy. Observations of the large scale magnetic fields on the sun are used as evidence to support the characteristics and importance of this braking action.

Finally, it is well known from the equations of magnetohydrodynamics that for fluids of sufficiently high electrical conductivity, the magnetic fields tend to be "frozen" to the fluid particles, i.e., the field lines move with the flow. Since solar physicists generally agree that the conductivity in the photosphere and adjacent layers below is very large, it is reasonable to expect an intimate relation between the large scale eddy magnetic fields and the eddy flow. The streamline and magnetic field patterns suggested here can easily be seen to possess such a close connection. In fact, they may well be distinct manifestations of the same hydromagnetic disturbance.

2. The tilted streamline pattern

It is well known to any synoptic meteorologist that the upper level troughs in the westerlies usually show a characteristic tilt in the flow pattern, from northeast to southwest in the subtropics and middle latitudes. An idealized trough is shown in Fig. 1. As was first pointed out by STARR (1948), this tilt is such as to imply a zonal momentum flux poleward across the latitude circles which intersect the trough. The sum total effect of many such troughs occurring over a long time and at most longitudes in the subtropics and temperate zones should be to give a mean eddy momentum flux poleward, and indeed this has been shown to be the case by extensive observational studies of the General Circulation Project at MIT and others. Furthermore, much theoretical justification has been found for the importance

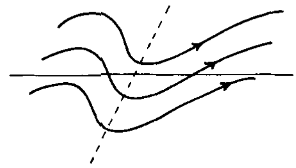

FIG. 1. Idealized tilted trough streamline pattern, taken from STARR (1948). On the sun, this pattern would indicate momentum flux toward the equator in the southern hemisphere. The east limb is toward the left, and north is toward the top. The corresponding pattern for the northern hemisphere of the sun is obtained by reflecting the figure about the east-west line.

of the tilted trough mechanism, especially by KUO (1951*b*, 1953). By considering wave disturbances on a zonal flow with horizontal shear, Kuo showed that when the basic flow was barotropically stable, the perturbations were damped in time, losing their kinetic energy to the zonal flow. An examination of the shape of these disturbances showed they possessed troughs titled in the same sense as in the atmosphere.

The zonal flow in the photospheric layers of the sun is certainly barotropically stable in the long term average, having an approximately parabolic shape. Kuo's theoretical work would seem to be applicable, and would imply the existence of large scale quasi-horizontal disturbances in low and middle latitudes on either side of the equator, possessing tilted troughs extending away from the solar equator toward the east limb. (The reader must bear in mind that in the solar direction convention "east" and "west" are reversed from their meanings for the earth.) Thus Fig. 1 could apply to a disturbance in the *southern* hemisphere of the sun. The corresponding pattern for the northern hemisphere would be obtained by reflecting the streamlines in Fig. 1 about the equator.

However, solar astronomers have not yet been able to draw synoptic maps of the large scale flow, and are therefore in this respect at a distinct disadvantage to the meteorologists. This is because there are not enough spots on the solar disk at any one time to trace the actual streamlines, and because Doppler shift measurements of velocity are generally taken for too short a time period, and are usually dominated by the larger velocities of smaller scale turbulence. Therefore, direct observational evidence for the tilted streamlines would seem to be lacking. However, as will be discussed later, larger scale magnetic field observations on the sun do seem to support their existence.

3. Maxwell stresses and their possible braking action

In magnetohydrodynamic flow the fluid experiences a volume force due to the flow of electric currents in the fluid across magnetic field lines. The mathematical representation of this force may be rearranged (see, for example Cowling (1953)) so that it appears in the equations of motion as derivatives of a stress tensor. Part of the force thus becomes the gradient of a magnetic pressure, while the other part is analogous to derivatives of the Reynolds stresses. To be more precise, let us take u as the zonal velocity, positive in the direction of rotation (toward the west limb); v the meridional velocity, positive toward the north; H_x and H_y as the corresponding zonal and meridional components of the magnetic field, respectively; ϱ the density and μ the magnetic permeability. The Maxwell stress, in the cgs system, analogous to the horizontal Reynolds stress $-\varrho uv$, is then written as

$$+\frac{\mu}{4\pi} H_x H_y. \tag{1}$$

To represent the average value of this stress over long time periods and at all longitudes, it is convenient to use a bar to represent a time average, a bracket to represent a zonal average, and primes to denote deviations from the space-time average. The space-time mean of the Maxwell stress then gives two terms, one representing a contribution from the axially symmetric steady part of the field, the other giving the effects due to all deviations from symmetric and steady fields. Thus, applying the space-time averaging operator to (1), we obtain

$$+\frac{\mu}{4\pi}\overline{[H_x' H_y']} + \frac{\mu}{4\pi}[\overline{H}_x][\overline{H}_y]. \tag{2}$$

Let us for the moment consider only the first, or eddy, term. The net force on an annular volume of material in the sun of unit depth and of latitude width $d\phi$ due to this stress is given by

$$+\frac{\mu}{2}\frac{\partial}{\partial\phi}\{\overline{[H_x' H_y']} R\cos\phi\}\, d\phi, \tag{3}$$

where R is the solar radius and ϕ the latitude. The sun is assumed spherical for our purposes. This force does work on the fluid, by virtue of the mean zonal velocity $[\bar{u}]$ present, of an amount

$$[\bar{u}]\frac{\partial}{\partial\phi}\left\{\frac{\mu}{2}\overline{[H_x' H_y']} R\cos\phi\right\} d\phi \tag{4}$$

which, when integrated from pole to pole, gives

$$\int_{-\pi/2}^{\pi/2}[\bar{u}]\frac{\partial}{\partial\phi}\left\{\frac{\mu}{2}\overline{[H_x' H_y']} R\cos\phi\right\} d\phi. \tag{5}$$

After some integration by parts, (5) may be rewritten as

$$-\int_{-\pi/2}^{\pi/2}\frac{\mu}{2}\overline{[H_x' H_y']} R^2\cos^2\phi\,\frac{\partial}{\partial\phi}\frac{[\bar{u}]}{R\cos\phi}\,d\phi. \tag{6}$$

A negative value for this expression (positive value for the integral) gives the rate of conversion of kinetic energy of the differential rotation to "eddy" magnetic energy. By comparing (6) with the analogous integral giving the conversion of eddy kinetic to zonal kinetic energy by the action of Reynolds stresses (Kuo 1951*a*; Starr & Gilman, 1965), we may interpret the quantity $\mu/2\overline{[H_x' H_y']}R^2\cos^2\phi$ as the total angular momentum flux due to eddy Maxwell stresses across a latitude wall of unit height.

In order for these Maxwell stresses to act as a brake on the differential rotation, as stated before, expression (6) must have a negative value. Since the angular velocity everywhere increases toward the equator, the horizontal magnetic fields would have to be arranged in such a way as to give Maxwell stresses transporting, in the net, momentum toward the poles. This result could be obtained very simply if the horizontal magnetic flux lines were in the form of tilted wave patterns—tilted away from the equator toward the east limb in each hemisphere. *The required tilt would therefore be in the same sense as we showed was needed in the streamline patterns to maintain the differential* *rotation*. It is to be noted that the braking effect is independent of the direction of the magnetic flux lines in disturbances tilted this way, so that the change of field direction with succeeding sunspot cycles does not modify the process.

The suggestion of tilted magnetic flux patterns can be supported, and indeed was motivated, by observational evidence. Dr. Robert Howard of the Mount Wilson and Palomar Observatories has kindly consented to our use of his unpublished schematic drawings of the large scale magnetic fields on the sun (Fig. 2).[1] These drawings were made from solar magnetograms, which are photographic representations of the magnetic field distribution as measured by the Zeeman splitting of selected spectral lines. The abscissa of the figures represents time, measured positive toward the left, but due to the rotation of the surface past the central meridian, on which the optical instruments were most often fixed, and due to the persistance of the patterns, it may be interpreted as solar longitude, measured positive toward the right. The ordinate is the sine of the latitude. The magnetic field these magnetograms actually measure is the component in the line of sight of the observer. The lightly shaded areas are regions of magnetic flux of greater than 2 gauss directed toward the observer, and the darkly shaded areas are regions of magnetic flux of greater than 2 gauss directed away from the observer. Since most of the measurements are made along a polar diameter of the disk, the line of sight component would be in meridional planes. That is, H_x would not be measured by this technique. It would seem reasonable, however, to assume that gross properties of the line of sight component of the field would be reflected in the transverse field. Certainly in the earth's atmosphere, similarities (specifically relating to tilt, scale, etc.) are seen among the large scale horizontal flow, temperature, and vertical motion patterns. The following characteristics of Fig. 2 are then to be noted.

1. The patterns of the sign of the magnetic flux show a marked tilt away from the equator toward the east limb in each hemisphere.

[1] Synoptic charts of magnetic features in the chromosphere, published by the Meudon Observatory, examples of which may be found in Abetti's *The Sun*, also show many of the features to be discussed.

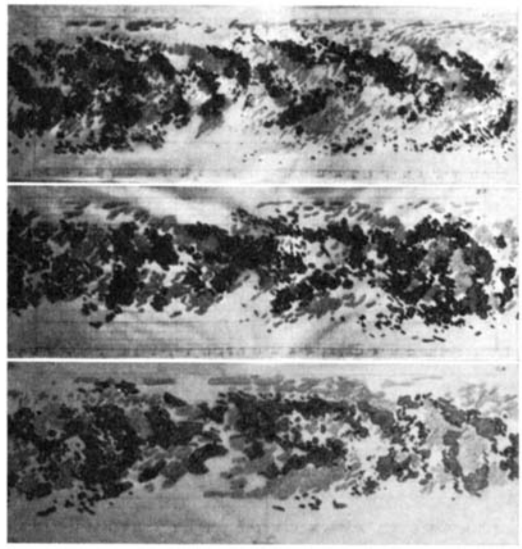

Fig. 2. Schematic drawings of large scale patterns of the line of sight component of the magnetic field on the sun, for 3 successive solar rotations, made by Dr. Robert Howard of the Mount Wilson and Palomar Observatories. The drawings are based on solar magnetograph records, which measure the line of sight magnetic field by the Zeeman splitting of selected spectral lines.

The ordinate is the sine of the solar latitude, the equator being at the middle of each drawing. The abscissa is time measured positive toward the left, but due to the rotation of the sun past the central meridian, where most of the measurements were made, and due to the persistence of the patterns, the abscissa may be interpeted as solar longitude, measured positive toward the right.

The lightly shaded areas represent magnetic fields greater than 2 gauss directed toward the observer, while the darkly shaded areas are magnetic fields greater than 2 gauss directed away from the observer.

2. The patterns are of quite large scale, appearing to give a typical zonal wave number of about 6, and a latitudinal extent of approximately 60°.
3. The patterns at very high latitudes generally represent magnetic fields of a single sign, the sign being opposite in the two hemispheres. The periodic character of the patterns is almost entirely wiped out.

From these facts, the following implications are suggested.

1. The tilt toward the east limb indicates that the large scale horizontal magnetic fields are indeed acting as a brake on the differential rotation.

2. The zonal wave number of about 6 and the latitudinal extent of 60° imply the existence of hydromagnetic disturbances on the sun with a scale much larger than the sunspots.
3. The change of pattern at very high latitudes suggests that a different kind of large scale disturbance predominates at these latitudes.

4. Further discussion of the results

The magnetic flux patterns have been seen to imply a braking action by the Maxwell stresses on the differential rotation. An idea of the strength of this braking action may be obtained by calculating the approximate eddy magnetic field strengths needed to make the Maxwell stresses the same order of magnitude as the Reynolds stresses taken from Ward's data.

Letting σ denote the standard deviation and r the correlation coefficient, the stresses will be comparable if

$$\varrho r(u, v)\,\sigma(u)\,\sigma(v) \sim \frac{\mu}{4\pi} r(H_x, H_y)\,\sigma(H_x)\,\sigma(H_y). \quad (7)$$

For the solar gas, $\mu = 1$, and a typical density for the photospheric layers is 3×10^{-7} gm cm^{-3}. If we assume for the present that

$$r(u, v) \sim r(H_x, H_y)$$

and take values of $\sigma(u) = 50$ m/sec, $\sigma(v) = 30$ m/sec from Ward's data, we conclude that $\sigma(H_x)$ and $\sigma(H_y)$ must have values of approximately 7 gauss. Howard's patterns show very large areas in which the line of sight magnetic fields are greater than 2 gauss. Since the vertical scale of these fields in the photosphere is most likely less than their horizontal scale, the non-divergent nature of magnetic fields then suggests that $\sigma(H_x)$ and $\sigma(H_y)$ could well approach 7 gauss. We therefore conclude that the braking effect due to horizontal Maxwell stresses could easily approach in magnitude the accelerating effect of the horizontal Reynolds stresses.

Any energy drained from the differential rotation in this way would be converted by the Maxwell stresses into eddy magnetic energy. It is therefore possible that this conversion is largely responsible for the maintenance of the eddy magnetic energy against dissipation by joule heating.

We may form an energy conversion integral similar to (6) for the symmetric components of the mean stress $\mu/4\pi[\bar{H}_x][\bar{H}_y]$, namely

$$-\int_{-\pi/2}^{\pi/2} \frac{\mu}{2}[\bar{H}_x][\bar{H}_y]\,R^2\cos^2\phi\,\frac{\partial}{\partial\phi}\frac{[\bar{u}]}{R\cos\phi}\,d\phi. \quad (8)$$

The symmetric component could also therefore act as a brake on the differential rotation if $[\bar{H}_x]$ and $[\bar{H}_y]$ have the same sign within each hemisphere. The possible effects of this component have been discussed by BABCOCK (1961) and others. In order for this part of the stress to be a significant brake, however, it must also be comparable in size to the horizontal Reynolds stress. That is, we would have to have

$$\varrho r(u, v)\,\sigma(u)\,\sigma(v) \sim \frac{\mu}{4\pi}[\bar{H}_x][\bar{H}_y]. \quad (9)$$

From Fig. 1 in high latitudes, and from many other measurements, an upper limit of about 1 gauss may be placed on $[\bar{H}_y]$. Using the same values of the other quantities that were used to estimate $\sigma(H_x)$ and $\sigma(H_y)$ together with the value $r(u,v) = 0.25$ from Ward's calculations, we conclude that $[\bar{H}_x]$ must be of the order of 15 gauss. There does not appear to be any observational evidence that *symmetric* fields of this magnitude exist in the sun at these levels. Therefore it would appear that this symmetric component of the Maxwell stress is a less important braking mechanism than the eddy component.

As stated earlier, the electrical conductivity in the photosphere and adjacent layers below is considered by solar physicists to be high enough so that the magnetic fields should be closely tied to the motion fields. This certainly suggests that the tilted magnetic field patterns observed by Howard should be associated with tilted troughs in the large scale flow patterns. We should therefore consider the tilted magnetic patterns as further evidence for the existence of tilted troughs in the streamlines. The assumption made earlier, that $r(u,v) \sim r(H_x,H_y)$ seems quite reasonable in the light of these considerations. Indeed, it would appear that *we should consider the tilted magnetic field and flow patterns as different manifestations of the same hydromagnetic disturbance, which as a whole still feeds energy into the differential rotation but at a slower rate than the writers found the hori-*

zontal Reynolds stresses to do alone. This is not to say that the horizontal magnetic field lines and streamlines coincide, however. A phase shift between the two patterns would seem more likely, just as in the earth's atmosphere there is a phase shift between the upper level streamlines and the isotherms. Furthermore, just as this phase shift in the terrestrial atmosphere implies a flux of heat and an exchange between potential and kinetic energy, so will a phase shift between the magnetic field lines and the streamlines in the sun imply an exchange between components of kinetic and magnetic energies, the exact nature of which should be considered further.

Up to now, we have considered Maxwell stresses only in a horizontal plane. Analogous to the "vertical" Reynolds stress $-\varrho uw$, where w is the velocity in the vertical direction, there is a corresponding Maxwell stress, given by

$$+\frac{\mu}{4\pi} H_x H_z, \tag{10}$$

where H_z is the component of the magnetic field in the vertical direction. Furthermore, the energy conversion integrals corresponding to (6) and (8), but for a spherical shell of thickness $z_2 - z_1$, may be obtained from the expressions

$$\int_{-\pi/2}^{\pi/2} \int_{z_1}^{z_2} [\bar{u}] \frac{\partial}{\partial z} \left\{ \frac{\mu}{2} [\overline{H_x' H_z'}] R \cos \phi \right\} dz R d\phi \tag{11}$$

and

$$\int_{-\pi/2}^{\pi/2} \int_{z_1}^{z_2} [\bar{u}] \frac{\partial}{\partial z} \left\{ \frac{\mu}{2} [\overline{H}_x][\overline{H}_z] R \cos \phi \right\} dz R d\phi \tag{12}$$

which are derived from similar considerations as previously. However, the integrable parts of these expressions will give quantities interpretable as work done against the vertical Maxwell stresses at the upper and lower boundaries of the spherical stratum. The amount of this work will in general be different from zero. For the earth's atmosphere scale analysis considerations and calculations of STARR and DICKINSON (1963) and others indicate that the vertical Reynolds stresses due to cyclone scale disturbances are small contributors to the kinetic energy balance of the zonal flow as compared with their horizontal counterparts. It is not yet possible to say whether the vertical Maxwell stresses in the sun similarly play a smaller role than do the horizontal Maxwell stresses.

5. Supplementary note

Since the hydromagnetic disturbances are of much larger scale than the sunspots themselves, it seems reasonable to expect that the spots trace the mass motions and momentum fluxes associated with these disturbances. Supporting evidence for this expectation has recently been found by MACDONALD (to be published). MacDonald calculated the longitudinal and latitudinal movements of high and low pressure centers in the earth's atmosphere, as well as the correlation of these motions. He found the eastward and northward movements positively correlated in temperate and subtropical latitudes. The correlation coefficient was generally about 0.2, and the standard deviations of the longitudinal and latitudinal movements were such that momentum fluxes calculated from these statistics compare very well with those derived from station data. The implication is that if movements of highs and lows in the atmosphere reflect momentum fluxes well, sunspot movements likewise probably indicate momentum fluxes on the sun.

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