Quantitative forecasting precipitation results obtained by means of three simplified computation methods for vertical velocities from amounts recorded at Bucharest and Cluj

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(Manuscript received March 17, revised version August 15, 1964)

ABSTRACT

In this paper are computed the recorded precipitations in 24 hrs from the Bucharest and Cluj stations, making use of three simplified methods for the estimation of the vertical velocities and of their variations with height.

The methods differ from one another in that the first assumes the atmosphere as adiabatic and saturated, whereas the second and the third take into account the individual variations of temperature and specific humidity, using thus in the calculus only "useful" vertical velocities.

There have been analysed 29 rainy periods in the cold season of the years 1957, 1958 and 1962. The results have shown that the computation methods for the vertical velocities using the Laplace operator and its variation with time, and those using a diabatic atmosphere divided into layers, have yielded the best results.

The estimation of the precipitation amounts falling during a certain interval of time and within a limited area, is of great interest to the forecasting routine.

That is why three computation methods of precipitations have been simultaneously tested at the Bucharest and Cluj meteorological stations.

The results of these experimentations are presented in the paper, and they refer to the cold season of the year, since it is during this period of the year that the turbulence effects can be neglected when computing vertical velocities. The influences due to orography have been neglected as well.

One of the methods uses a computation of the large scale vertical velocities. The atmosphere is considered to be barocline, quasi-geostrophic, adiabatic and saturated all through the forecast period. In the quasi-geostrophic vortex equation, the vortex advection at the ground level is left out, and the gravitational acceleration and the Coriolis parameter are assumed as being constant (ESTOQUE, 1957).

In computing the vertical velocity, a sinusoidal parametral distribution of it is assumed in the vertical. The constants are chosen in such a manner that the velocity be zero at the ground level, and reach its maximum at the nondivergence level (500 mb).

The determination of the precipitation amounts is achieved by means of a similar relation to that advanced by DIUBUCK (1947), ESTOQUE (1957) and FULKS (1935).

For the computation of the precipitation amounts fallen during the forecasting interval, the following relation was used as starting point:

$$P = \int_0^{\Delta t} \int_{p_0}^{0} \frac{\omega}{g} \frac{\partial s}{\partial p} \, dp \, dt \approx \frac{\Delta t}{g} \int_{p_0}^{0} \omega_t \left(\frac{\partial s}{\partial t}\right)_t dp,$$

where

- s is the maximum specific humidity expressed in gr/kg,
- ω is the generalized vertical component of velocity, and the index t shows the mean over the chosen time period.

In ascertaining the relation above, the following assumptions have been made:

The amount of water fallen to the ground is equal to that of condensed water.

(*b*) During the forecast interval the air keeps saturated from the surface boundary layer to the top limit of the atmosphere.

(*c*) In the expression $ds/dt = -\partial s/\partial t + \mathbf{v} \cdot \nabla s + \omega(\partial s/\partial p)$ the first two terms in the right-hand member have been neglected.

Taking into account the vertical velocity expression (ESTOQUE, 1957), it follows that the precipitation amount fallen during the forecast interval is:

$$P = (\overline{\nabla^2 Z_0} - \nabla^2 Z_0) \cdot k(s), \qquad (1)$$

where k varies with the maximum specific humidity s.

$\overline{\nabla^2 Z_0}$ is the spatial mean of $\nabla^2 Z_0$, Z_0 being the altitude of the 1.000 mb surface.

If s is assumed to vary linearly with the pressure, one gets $k(s)$ under the form:

$$k(s) = \frac{32m^2(p_0 - p_L)^2}{\pi^2 d^2 l^2} \cdot \left(\frac{\Delta s}{\Delta p}\right)_t,$$

wherefrom it follows, in the last analysis, that k depends on the mean temperature of the layer between the 1000 mb and 500 mb levels and implicitly on the mean value of the relative geopotential during the forecast interval.

This dependence has been established by means of a pseudo-adiabat computed for the south-eastern part of Europe (DONEAUD, BESLAGA, STOIAN & CRISTODOR).

The precipitation amount is expressed in liters per square metre.

The other two experimented methods used for the precipitation determinations a calculus of the vertical velocities at several levels, fact which allows for their better ascertainment. In comparison with the preceding method, a series of restrictive assumptions referring to the temperature and air humidity characteristics is done away with.

As in the case of the preceding method, it is considered that the condensed water amount is equal to that of the precipitated one, neglecting the vapour amount which contributes to the cloud formation.

In comparison to the first method, the atmospheric processes are no longer considered as being adiabatic, including in the calculus the individual variations of the air temperature within the layer between 1 and 3 km, (BACIURINA, 1962; BACIURINA & TURKETTI, 1955).

Likewise, during the forecast period, the atmosphere is no longer assumed as being saturated beginning with the ground level. The vertical velocities computed by these methods are due to the convergence of the air currents, which in its turn is produced by both the friction at the ground level and the unstationary character of the motion.

As before, the atmosphere is assumed to be hydrostatic, quasi-geostrophic and barocline, and the Coriolis parameter is kept constant during the forecast period (ZVEREV, 1956).

Both methods make use of the fact that the vertical velocity at the top level of a layer in the free atmosphere is made up of the velocity at its lower level, to which is added a term depending on the unstationary character of the motion in the considered layer (ZVEREV, 1956).

At the ground level the vertical velocity is assumed to be equal to zero. The manner of computation, however, differs from one method to another. One method estimates the vertical velocities at the 850 and 750 isobaric levels, using the value of the wind divergence within the lower layer and the continuity equation written as: $\partial w/\partial p = -(\partial u/\partial x + \partial v/\partial y)$. In the light of the above-mentioned approximation and considerations, the following relationships are established:

$$w_{850} = -\int_{p=1000}^{p=850} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dp, \qquad (2)$$

$$w_{700} = w_{850} - \int_{p=850}^{p=700} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dp, \qquad (3)$$

where T_{850} and T_{700} represent the vertical velocities at the 850 and 700 mb levels; u and v being the horizontal components of the wind (BACIURINA & TURKETTI, 1955). The vertical velocities calculated by the above formulae, assume the form $T = dp/dt$, where p is the pressure.

In order to assess the right-hand member of (2), the wind velocity divergence $D = \partial u/\partial x + \partial v/\partial y$ at the 850 mb level is assumed to keep constant within the 850/1000 mb layer. Since the computation of the divergence in the wind field may introduce significant errors (PETTERSSEN, 1956), the geopotential field gradients have been resorted to in computing the magnitude of D. With this aim in view, use has been made of the 850 mb surface chart. By means of a

$X0Y$ rectangular system of coordinates with the origin in the very point for which the determination is being performed, the value of D is computed by using finite differences and expressing the value of the wind components on the basis of the geopotential field gradients.

The calculus is considerably simplified if the $0X$ axis is set by the tangent to the contour line in the considered point. Integrating then from $p = 1000$ mb to $p = 850$ mb, one gets the value of T 850, the vertical velocity.

The calculus of the vertical velocity from the isobaric level of 700 mb is performed in a similar way, using the 700 mb surface chart and summing the result obtained after the integration to the value of the vertical velocity at the 850 mb level.

The vertical velocities are expressed in millibars per 24 hrs.

The other method makes use, in computing the vertical velocities, of the Laplacian operators of the ground-level pressure and of the geopotential at the 850 mb levels.

Taking into account the above-mentioned considerations, the computation formulae finally assume the form:

$$\delta p_{850} = a_1 \overline{\nabla^2 p_0} + a_2 \frac{d}{dt} \nabla^2 p_0, \tag{4}$$

$$\delta p_{700} = \delta p_{850} + a_3 \left(\frac{d}{dt} \nabla^2 H_{850} + \frac{d}{dt} \nabla^2 H_{700} \right), \tag{5}$$

where: δp 850 and δp 700 represent the vertical velocities at the 850 and 700 mb levels expressed in millibars per 12 hrs;

$\overline{\nabla^2 p_0}$ is the 12 hr mean of the Laplacian operator computed at the ground level along the trajectory described by the air particle;

$(d/dt)\nabla^2 p_0$, $(d/dt)\nabla^2 H_{850}$, $(d/dt)\nabla^2 H_{700}$ represent the individual variations of the baric field Laplacian operators at the ground, 850 mb and 700 mb levels, for one and the same period.

The computation of the Laplacian operators is carried out by means of finite differences using a square grid with a step of 500 km.

The coefficients in the above formulae depend on the chosen units. In (4) the first term of the right-hand member, the so-called "frictional convergence", is taken into account. Its coefficient a_1 depends above all on the selection of $c = \nu/h$, ν being the kinematic viscosity coefficient and h the height above which $c \approx$ constant. Taking $\nu = 7.5$ m^2/s and $h = 40$ m and taking into consideration the length of the grid step and that of the forecast interval, it follows that $a_1 \approx a_2 = -3.5$ and $a_3 = -2.1$ (Zverev, 1956).

The particle trajectory is determined by means of the 850 mb constant pressure chart for the first formula, and of the 700 mb chart for the second.

It follows from (4) and (5) that the ascending vertical motions occur in case of a cyclonic curvature of the isobars ($\overline{\nabla^2 p_0} > 0$), owing to the air currents convergence brought about by friction as a result of the fact that during the forecast interval the air particle moves in a cyclonic field. Ascending motions occur as well in case of the deepening of the baric field both at the ground level and in the free atmosphere $[(d/dt)\nabla^2 p_0 > 0, (d/dt)\nabla^2 H_{850} > 0, (d/dt)\nabla^2 H_{700} > 0]$ as a result of the unstationary character of the processes.

In order to estimate the precipitation amounts, both methods assume in the same way the factors which the humidity of the moving air depend upon, that is: air temperature and its saturation deficit. With this end in view, there has been computed the mean temperature advection value of the 850/1000 mb and 750/850 mb layers, and, as a function of this, the air temperature variation during the forecast period (individual variation) by means of a regression graph (Baciurina & Turketti, 1955).

This is added to the advected value of the mean saturation deficit per layer. Then is calculated the useful vertical velocity, i.e. that part of the vertical velocity which is actually used in the condensation process, after the air has reached the saturation stage.

Finally, by means of the useful vertical velocities and of the mean temperature of the 850 and 700 mb isobaric surfaces, there are calculated the precipitation amounts for each layer, in the considered point. These computations are carried out using another regression graph (Baciurina & Turketti, 1955); the precipitation amounts are given in litres per square metre.

Experimental results

29 rainy periods have been tested, 24 hrs each, from the cold seasons of 1957, 1958 and

TABLE 1. *Contingency table for the precipitation forecast at Bucharest (no. of cases).*

Forecast values in l per sq. m	Recorded values: Lack of precipitation			0–3			3–10			10–25			Total
	A	B	C	A	B	C	A	B	C	A	B	C	
Lack of precipitation	4			10			—			—			A 14
		2			3			2			—		B 7
			4			13			1			1	C 19
0–3	1			7			2			1			A 11
		2			9			1			—		B 12
			1			3			—			—	C 4
3–10	—			1			1			1			A 3
		1			6			—			2		B 9
			—			1			2			1	C 4
10–25	—			1			—			—			A 1
		—			1			—			—		B 1
			—			—			—			—	C —
Total	5	5	5	19	19	17	3	3	3	2	2	2	85

1962. Separate computations were carried out for the precipitation amounts recorded at Bucharest and Cluj stations. Owing to its geographical situation, the former lies under the influence of the Meridional Carpathians in case of a circulation having a N–S orientation.

The latter station is situated in Transylvania and is surrounded by the Carpathians. The precipitation amounts (in l per sq. m) have been divided into classes of values and listed in contingency tables. In order to compare the actual values with the computed ones, the former have been estimated by averaging the values recorded over a circular area with a radius of 80 km and the centre situated in the very point for which the prognostic values are calculated.

The radius has been chosen in a way which

TABLE 2. *Contingency table for the precipitation forecast at Cluj (no. of cases).*

Forecast values in l per sq. m	Recorded values: Lack of precipitation			0–3			3–10			10–25			Total
	A	B	C	A	B	C	A	B	C	A	B	C	
Lack of precipitation	5			7			3			1			A 15
		2			3			2			—		B 7
			5			5			—			—	C 10
0–3	2			6			4			—			A 12
		6			4			1			—		B 11
			1			1			1			—	C 3
3–10	1			1			—			—			A 2
		—			7			4			—		B 11
			1			7			5			—	C 13
10–25	—			—			—			—			A —
		—			—			—			—		B —
			—			—			1			—	C 1
Total	8	8	7	14	14	13	7	7	7	—	—	—	85

TABLE 3. *The extent to which the forecasts proved successful (in percentage) for each class of forecast values at Bucharest.*

	Recorded values												
	Lack of precipitation			0–3			3–10			10–25			
Forecast values in l per sq. m	A	B	C	A	B	C	A	B	C	A	B	C	Total
Lack of precipitation	28			72			—			—			A 100
		29			42			29			—		B 100
			21			69			5			5	C 100
0–3	9			64			18			9			A 100
		17			75			8			—		B 100
			25			75			—			—	C 100
3–10	—			33			34			33			A 100
		11			67			—			22		B 100
			—			25			50			25	C 100
10–25	—			100			—			—			A 100
		—			100			—			—		B 100
			—			—			—			—	C —

avoids the effects of orography as much as possible.

The data obtained by the three methods have been listed in common tables. The order of listing the methods in tables (A, B, C) corresponds to the order of their presentation in the first part of the paper. (See Table 1 and Table 2.)

Analysing the data from these tables one finds that the prognostic values are generally grouped within that column division representing the same class of observed values, or in those of the neighbouring classes. Moreover as far as "the precipitation cases" are concerned, their forecast values are a little larger than the observed ones, for each of the three methods, as a consequence of the approximations resorted to. Bodolai (BODOLAI, 1959, 1960) has ascertained the mean of this difference to be 3.5 l/sq. m, using determinations carried out on actual situations and being based upon the results of certain researches on the amount of humidity that contributes to the genesis of the frontal cloudiness (ZVEREV, 1957).

Based upon a calculus similar to that used in method A, ESTOQUE (1957) got, on the whole, precipitation values smaller than the actual ones. The difference between his results and those in the present paper are likely to be due to the different way of computation of the $k_{(s)}$ function. (Estoque assumed the atmosphere saturated starting with the cloud base determining the pseudo-adiabatic values for North America.)

As Estoque has shown commenting upon his results, the differences between the observed and the forecasted amounts were due, to a great extent, to the convective activity as well.

In the present study, the cases were selected so as to reduce the effects due to convection as much as possible.

Likewise, Thompson and Collins in their calculus of precipitation amounts obtained a little smaller amounts as against the real ones (THOMPSON & COLLINS, 1953). They made use, in their study, of an atmosphere divided into several layers, and performed the calculus of the ascending air on a large scale, by expressing the divergence value from the real wind data taken at several weather stations. They assumed the ascending air as being pseudo-adiabatic, making certain modifications meant to compensate the nonstationary initial conditions.

As Thompson and Collins have shown, the difference between the observed and the computed precipitation amounts can be accounted for by the neglect of both the diabatic heat

TABLE 4. *The extent to which the forecasts proved successful (in percentage) for each class of forecast values at Cluj.*

Forecast values in l per sq. m	Recorded values: Lack of precipitation			0–3			3–10			10–25			Total
	A	B	C	A	B	C	A	B	C	A	B	C	
Lack of precipitation	33			47			20			—			A 100
		29			42			29			—		B 100
			50			50			—			—	C 100
0–3	17			50			33			—			A 100
		55			36			9			—		B 100
			33			33			34			—	C 100
3–10	50			50			—			—			A 100
		—			64			36			—		B 100
			8			54			38			—	C 100
10–25	—			—			—			—			A —
		—			—			—			—		B —
			—			—			100			—	C 100

exchange between the cloud elements and the ambiant air, and the fact that only the mechanism of the large scale precipitation has been taken into consideration. Likewise, the utilization of the real wind in the calculus of divergence could bring in a series of errors.

In the present study, the computations performed by means of the B and C methods took also into account the diabatic heat exchanges between the moving air mass and the environment. As before, in determining the vertical velocities over restricted areas, have been used for the calculus of wind divergence and of the Laplacian operators, the geopotential fields. These reasons, together with others expounded in the paper, may explain the obtained results of B and C methods.

A comparative analysis of the three methods shows that, in the case of the Bucharest station, the forecast precipitation values below 3 l/sq. m present a better grouping for the B method. If one takes into account also the neighbouring classes of observed precipitations, better results are lent by the C method. At Cluj the results obtained by the C method were good in both cases. (See Table 3 and 4).

The data presented in Table 3 show that the B and C methods have a better percentage for the forecast precipitations below 3 l/sq. m.

The forecast precipitations falling into the 3–10 l/sq. m class show better results when the C method was used.

From the data in Table 4 one can draw the conclusion that, the 0–3 l/sq. m class excepted, in which case the A method lends a better percentage, the other classes are better forecasted by the C method.

Analysing the percentage of the extent to which the forecasts proved successful with respect to the same class of observed values or to the neighbouring classes, one finds that the best results are obtained by using the A and C methods (93 per cent) for Cluj.

From the synoptic analysis of the forecast periods it can be deduced that, in the case of the rapid baric waves and with a short wavelength, it is necessary, for the computation of the Laplacian operators, to shorten the forecast period and use a grid with a shorter step.

The precipitation amounts are well estimated in case of strong cyclonic developments, by the A and especially C methods, for the area over which the majority of the precipitations is due to the large scale upward motions.

The precipitation amounts are overestimated within the warm sector of the cyclones, where, nevertheless, the B and C methods which assume the atmosphere as diabatic, lend better results. In the rear of the cold fronts as well as within the cold sector of the stationary fronts the C, and especially the A methods are less effective than the B one. In these situations the vertical velocity variation with height, and that of the maximum specific humidity, exhibit areas of discontinuity which require their computation for several layers.

Conclusions

The methods used for the computation of the precipitation amounts give satisfactory results in the case of cyclonic developments especially for the area in which the vertical velocities are due to the surface friction and to the unstationary character of the motion, provided the proposed classes of values are considered as sufficient. Better results are obtained when the vertical velocities are calculated by using the Laplace operator to the baric field (C method), and provided one takes into account the individual variation of the air temperature and of the specific humidity.

It is advisable to interpret the results as a function of the synoptic situations and especially to take into account the orographic effects and the travelling speed of the baric waves. In the case of rapid waves the forecast period as well as the step of the grid have to be reduced.

The division of atmosphere into narrower layers and the computation of the elements required in the forecast activity for each layer allow the getting of better results especially in the case of a varied relief and of the discontinuous variations with height of the considered meteorological elements.

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