On the energetics of the mean and eddy circulations in the lower stratosphere[1]

By ABRAHAM H. OORT,[2] *Dept. of Meteor., Mass. Inst. of Tech., Cambridge*

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ABSTRACT

A hemispheric network of radiosonde stations is used in order to study the energetics of the lower stratosphere during the IGY period July 1957 through June 1958. For a hemispheric polar cap with 30 and 100 mb as top and bottom boundaries the balance equations of zonal and eddy kinetic energy, and zonal and eddy available potential energy are considered in detail. The eddies appear to build up the kinetic energy of the zonal flow at the expense of the eddy kinetic energy during all seasons. The eddies lose also eddy potential energy to the mean zonal distribution, in agreement with the abnormal upslope direction of the eddy heat transport. Thus, no source of energy for the eddy motions appears to be present *in situ* and the eddies must be forced by the circulation in the adjacent layers, probably by the tropospheric motions.

The energy-cycle in the lower stratosphere is in many respects different from the one usually found in the troposphere, where, as is well known, eddy kinetic energy is destroyed by friction and the energy source is found in the creation of zonal available potential energy by radiation and the subsequent baroclinic processes. For the 100–30 mb layer the necessary kinetic energy is supplied by interaction at the top and/or bottom boundaries, while ultimately the energy is destroyed in the form of zonal available potential energy by radiation.

1. Introduction

The research reported in this paper forms part of an extensive stratospheric study by the M.I.T. Planetary Circulations Project. The basic observations, namely the horizontal components of the wind, the temperature and the height of the pressure levels at 100, 50 and 30 mb, are taken from microcards which were issued by the World Meteorological Organization for the International Geophysical Year (IGY). A selected network of about 240 radiosonde stations, giving a good coverage over the northern hemisphere, is used. The selected time period is July 1957 through June 1958; this year of data was subdivided into four periods of three months each, in order to study possible seasonal variations. The data were reduced for every station separately. Then, after plotting the calculated three-monthly means on hemispheric maps, the latter were analyzed by hand. By this method the mean fields of the basic quantities and of their covariances in time were obtained. Gridpoint values which were read off the maps, form the basic material for calculating the terms in the energy cycle.

This paper deals with the energy budget for a stratospheric layer which is bounded in the vertical by the constant pressure levels 100 and 30 mb and in the horizontal direction by a "wall" at the equator. Our intention is to determine what processes might be responsible for maintaining the mean state of the general circulation in this polar cap. Masswise the layer between 100 and 30 mb is of little importance, since it contains only 1/14 of the total mass of

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[2] The author's present address: Koninklijk Nederlands Meteorologisch Instituut, *De Bilt*, The Netherlands.

the atmosphere, while about 8/10 is concentrated in the troposphere. As we shall see, the general circulation in the lower part of the stratosphere is very different from the one either below in the troposphere or above in the middle stratosphere. In those layers it is evident that the atmospheric disturbances tend to release kinetic energy at the cost of potential and internal energy.

The present work is one of the first attempts in studies of the general circulation to consider the complete energy cycle for a separate layer in the atmosphere. Much of the earlier research of hemispheric scope has been aimed exclusively at the behavior of the atmosphere integrated over all layers in the vertical. There is much to be gained from detailed information on the individual layers of the atmosphere. However, our approach has on this account certain limitations and additional problems, such as the following: (1) boundary effects in the vertical have to be taken into account, (2) the terms at the boundaries cause a certain arbitrariness in the definitions of energy conversions for the layer, and (3) the total mass transport in the layer across a latitude circle is not necessarily equal to zero.

2. Notation

(x, y, p) = coordinates with axes in eastward and northward directions and a pressure coordinate

ϕ = latitude

λ = longitude

H = height isobaric level

u = west-east component of the wind (positive if from the west)

v = south-north component of the wind (positive if from the south)

$c = \sqrt{u^2 + v^2}$

$\omega = dp/dt$ = "vertical velocity"

$\Phi = gH$ = geopotential

T = temperature

θ = potential temperature

α = specific volume

$\gamma = -\partial T/\partial z$ = lapse rate

γ_d = dry adiabatic lapse rate

$dm = a^2 \cos\phi\, d\lambda\, d\phi\, (dp/g)$ = increment of mass

a = radius of the earth

g = acceleration due to gravity

Ω = rotation rate of the earth

$f = 2\Omega \sin\phi$ = Coriolis parameter

R = gas constant

c_p = specific heat at constant pressure

Q = heating rate

F = frictional force

$\overline{(\ )} = 1/(t_2 - t_1)\int_{t_1}^{t_2} (\ )\, dt$ = time average

$(\ )' = (\ ) - \overline{(\ )}$ = deviation from time average

$[(\ )] = 1/2\pi \int_0^{2\pi} (\ )\, d\lambda$ = zonal average

$(\ )^* = (\ ) - [(\ )]$ = deviation from zonal average

$\widetilde{(\ )} = 1/\pi^2 \int_0^{2\pi} \int_0^{\frac{1}{2}\pi} (\ )\, d\lambda\, d\phi$ = hemispheric average

$(\ )'' = (\ ) - \widetilde{(\ )}$ = deviation from hemispheric average

$\overline{[ab]}_E = [\overline{a'b'}] + [\bar{a}^* \bar{b}^*]$

$[\overline{a'b'}]$ = transient eddy covariance of a and b

$[\bar{a}^* \bar{b}^*]$ = standing eddy covariance of a and b

$K_M = K_{M,x} + K_{M,y}$ = zonal kinetic energy

$K_{M,x} = \frac{1}{2}\int [\bar{u}]^2 dm$

$K_{M,y} = \frac{1}{2}\int [\bar{v}]^2 dm$

$K_E = K_{E,x} + K_{E,y}$ = eddy kinetic energy

$K_{E,x} = \frac{1}{2}\int \{[\overline{u'^2}] + [\bar{u}^{*2}]\} dm$

$K_{E,y} = \frac{1}{2}\int \{[\overline{v'^2}] + [\bar{v}^{*2}]\} dm$

$P_M = \frac{1}{2} g \int \dfrac{[\bar{T}]''^2}{\widetilde{T}(\gamma_d - \widetilde{\bar{\gamma}})} dm$ = zonal available potential energy

$P_E = \frac{1}{2} g \int \dfrac{[\overline{T'^2}] + [\bar{T}^{*2}]}{\widetilde{T}(\gamma_d - \widetilde{\bar{\gamma}})} dm$ = eddy available potential energy

$D(K)$ = rate of frictional dissipation of kinetic energy due to the effects of (1) below grid-size eddies inside the volume and (2) eddy stresses at the boundaries by a similar scale of motion

$G(P)$ = rate of generation of available potential energy by diabatic heating

$C(K_E, K_M)$ = rate of conversion from eddy into zonal kinetic energy by the eddy momentum transport

$C(K_{M,y}, K_{M,z})$ = rate of conversion from $K_{M,y}$ into $K_{M,z}$ by mean meridional circulations

$C(P_M, P_E)$ = rate of conversion from zonal into eddy available potential energy by the eddy heat transport

$C(P_M, K_M)$ = rate of conversion from zonal available potential energy into zonal kinetic energy by mean meridional circulations

$C(P_E, K_E)$ = rate of conversion from eddy available potential energy into eddy kinetic energy by large-scale eddy convection

$W_e(K)$ = rate at which work is done on the considered layer by the measured eddy stresses at the boundaries

$W_p(K)$ = rate at which work is done on the considered layer by the pressure forces at the boundaries.

3. Energy budget for 100–30 mb layer

We shall consider a stratospheric layer with the 100 and 30 mb levels (resp. at about 16 and 24 km height) as bottom and top boundaries and a "wall" at the equator as vertical boundary. It is of great interest to establish by what mechanism the time-mean and zonal-mean state of the circulation is maintained in this polar cap and, with this in mind, we shall compute most of the terms in the budgets of zonal and eddy kinetic energy, and zonal and eddy available potential energy.

No attempt will be made to compute the balance of total kinetic and total potential energy. Such calculations were performed by Craig & Lateef (1962) for the kinetic energy during one month in 1957 using only North American stations, and by Barnes (1963) for both forms of energy with a hemispheric network during the first six months of the IGY. It is evident from Barnes' work that with the present data coverage uncertain terms such as those involving transports by mean meridional overturnings, may spuriously dominate the equations. On the other hand, one generally finds that transient and standing eddy transports can be measured more accurately than transports by mean meridional circulations. In view of this, we have decided to consider the equations for the balance of the zonal and eddy forms of energy separately. These calculations may give us in addition valuable and quite reliable information on the redistribution of kinetic and available potential energy among the zonal-mean and eddy forms.

The interpretation of the calculated terms in both the kinetic and the available potential energy equation forms a difficult problem. There is a separation of the terms possible into groups with a distinct physical meaning, but this division is not unique. For example, one may combine some of the terms which indicate a transport, and the conversion term and call the sum "conversion". However, as a rule the name conversion should be reserved for the term which expresses clearly the physical mechanism by which one thinks the transformation takes place. Of course, this conversion term should appear with the opposite sign in the balance equation for the forms of energy from or into which the form of energy in question is converted. In the following discussions we shall adopt the nomenclature which in our opinion makes most physical sense (for the subject of energy transformations see: Miller, 1950; Lettau, 1954; Lorenz, 1955, and Pfeffer, 1957).

The levels of the basic observations are 100, 50 and 30 mb. The assumption is made in the calculations of the integrals that: (1) 100 mb is representative for the layer 100–75 mb, (2) 50 mb is representative for the layer 75–40 mb, and (3) 30 mb is representative for the layer 40–30 mb. Since values of adiabatic vertical motions were available only at 75 and 40 mb, it is further assumed that the calculated vertical transport at 75 mb represents the flux at the lower boundary, while the 40 mb transport is assumed to represent the flux at the upper boundary of our volume. It is possible that the measured transport at 75 mb considerably underestimates the actual transport at 100 mb. In the vertical transports by mean meridional circulations, $\overline{\omega_{ad}}$ is corrected in such a way that the hemispheric average of $\overline{\omega_{ad}}$ vanishes. This is done in order to satisfy continuity of mass. However, it was found afterwards that this correction did not change the mean vertical transports appreciably. No correction has been applied to the eddy transports involving vertical motions, because they are probably only slightly affected by diabatic effects, such as (1) heating or cooling due to radiation and (2) heating due to the release of latent heat. The effects of radiation are slow; thus radiation will not change the computed transient eddy covariances, though it might be important for the standing eddy contributions. Since the amount of water vapor in the lower stratosphere is quite small, the effects of condensation may be neglected.

At this point it is of interest to add some remarks concerning the different scales of motion which we measure in our eddy transports. Let us first consider the contribution of the eddies in space, i.e., the standing eddies. The horizontal resolution in space is limited because of the station distribution and our method of analysis of hemispheric maps, to a scale of motions which is larger than about 500 km. There is practically no resolution in the vertical since we used only data at 100, 50 and 30 mb. The contributions of transient eddies are in general larger than those of the standing eddies; thus, it is of particular interest to look at the time scales involved. In the *vertical* transport by transient eddies (such as the transport of momentum $\overline{u'\omega'}$) we catch only eddies with time scales of more than one day. This is because the adiabatic vertical motions which are used here, were evaluated from two soundings 24 hours apart. They represent some kind of 24 hours smoothed values. On the other hand, the *horizontal* transport by transient eddies (such as the transport of momentum $\overline{u'v'}$) is measured almost instantly and contains, in principle, also the contribution from eddies with shorter time scales from a few hours down to a few minutes. Diurnal effects might be important, but they are not included in the transports, since the observations were taken at approximately the same time of the day. The frictional force is defined as a flux of momentum at the boundaries by eddies with a time or space scale below the scale of our grid. This implies that meso- and micro-scale eddies are included in the frictional force at the top and bottom boundaries, while the frictional force at the vertical wall contains mainly micro-scale, transient eddy contributions and is hence probably of minor importance.

3.1. Zonal kinetic energy

In this section we shall consider the problem of how the zonal kinetic energy is maintained against frictional dissipation inside the volume. The term "zonal kinetic energy" was defined in section 2 as the kinetic energy of the zonal mean and time mean horizontal circulation,[1] i.e., $K_M = K_{M,x} + K_{M,y} = \frac{1}{2}\int \{[\bar{u}]^2 + [\bar{v}]^2\} dm$. Since the mean meridional motions are two orders of magnitude smaller than the mean zonal motions, K_M is very nearly equal to $K_{M,x}$ and we are justified to restrict ourselves in the discussion of the mean motions mainly to the zonal component. Kuo (1951) derived a balance equation for the zonal kinetic energy in the (xyz) coordinate system. A similar equation, rewritten for the pressure coordinate system, was used by Starr (1953, 1959) and Barnes (1963) in their computations of the kinetic energy balance in the northern hemisphere and by Obasi (1963) in his study of the southern hemisphere circulation. In the derivation, the zonal equation of motion is first multiplied by $[\bar{u}]$, next averaged in time over the period considered, and finally integrated in space over the mass of a polar cap. The resulting expression may be written in a form where the effects of eddy and mean motions are separated. To derive equation (1) use is made of the Gauss' divergence theorem and the equation of continuity.

$$0 = C(K_{E,x}, K_{M,x}) + C(K_{M,y}, K_{M,x}) + D(K_{M,x}) + A(K_{M,x}) + W_\epsilon(K_{M,x}) + \text{Seasonal correction.} \quad (1)$$

[1] This definition of zonal kinetic energy is different from the one originally given by Lorenz (1955) who used $K_M = \int\{\overline{[u]^2} + \overline{[v]^2}\} dm$. Also the conversion rates considered here are different from those derived by Lorenz. The present approach is more appropriate in this case because no daily gridpoint data were available.

Equation (1) states that there should be a balance between the following processes:[1]

(*a*) The conversion from eddy into zonal kinetic energy by horizontal and vertical eddies (only x-component of process considered):

$$C(K_{E,x}, K_{M,x}) = \int [\overline{uv}]_E \cos\phi \frac{\partial}{a\partial\phi}\left(\frac{[\bar{u}]}{\cos\phi}\right) dm + \int [\overline{u\omega}]_E \frac{\partial[\bar{u}]}{\partial p} dm.$$

The horizontal part of the integrand of $C(K_{E,x}, K_{M,x})$ consists of the product of the meridional transport of relative angular momentum by eddy motions and the gradient of the angular velocity. If an eddy transport of angular momentum takes place from latitudes where the air has a small relative angular rotation to latitudes where the air rotates more rapidly, the integral indicates a conversion from eddy into zonal kinetic energy.

(*b*) The conversion of energy from the mean meridional motions into the mean zonal motions due to essentially the Coriolis effect:

$$C(K_{M,y}, K_{M,x}) = \int f[\bar{u}][\bar{v}]dm + \int [\bar{v}][\bar{u}]^2 \frac{\tan\phi}{a} dm. \cong \int f[\bar{u}][\bar{v}]dm.$$

Its magnitude is extremely uncertain due to the great difficulty of estimating $[\bar{v}]$ accurately enough.

(*c*) The sum of two processes, which are included in the term "frictional dissipation", i.e., (1) the change of kinetic energy due to the effects of below grid-scale eddy stresses at the boundaries, and (2) the change of kinetic energy due to similar internal friction; this sum being:

$$D(K_{M,x}) = \int [\bar{u}][\bar{F}_x]dm.$$

(*d*) The advection of zonal kinetic energy through the boundaries by mean meridional overturnings:

$$A(K_{M,x}) = \iint_{\phi=0} [\bar{v}]\tfrac{1}{2}[\bar{u}]^2 dx \frac{dp}{g} - \iint_{100\,mb} [\bar{\omega}]\tfrac{1}{2}[\bar{u}]^2 \frac{dx\,dy}{g} + \iint_{30\,mb} [\bar{\omega}]\tfrac{1}{2}[\bar{u}]^2 \frac{dx\,dy}{g}.$$

(*e*) The work performed on our volume by the measured eddy stresses at the boundaries:

$$W_e(K_{M,x}) = \iint_{\phi=0} [\bar{u}][\overline{uv}]_E dx \frac{dp}{g} - \iint_{100\,mb} [\bar{u}][\overline{u\omega}]_E \frac{dx\,dy}{g} + \iint_{30\,mb} [\bar{u}][\overline{u\omega}]_E \frac{dx\,dy}{g}.$$

(*f*) The so-called "seasonal correction":

$$-\int [\bar{u}] \frac{\Delta[u]}{\Delta t} dm.$$

This term contains the considered time period Δt in the denominator and, therefore, decreases in importance with time. Aside from the seasonal trend, this correction contains random daily fluctuations superimposed upon the trend. The term thus arises because of our method of time averaging. In principle, one should average over a few cycles of the observed phenomenon in order to be able to define the time mean state.

An often used alternative form for the production of zonal kinetic energy at the cost of the eddy kinetic energy $C(K_{E,x}, K_{M,x})$ is the following:

$$C'(K_{E,x}, K_{M,x}) = C(K_{E,x}, K_{M,x}) + W_e(K_{M,x}) = -\int [\bar{u}] \frac{\partial[\overline{uv}]_E \cos^2\phi}{a\cos^2\phi\,\partial\phi} dm - \int [\bar{u}] \frac{\partial[\overline{u\omega}]_E}{\partial p} dm.$$

Since $W_e(K_{M,x})$ evidently represents an interaction term at the boundaries, we prefer the earlier mentioned form for the conversion where one splits off the contribution of $W_e(K_{M,x})$. Thus, we shall continue to use the expression $C(K_{E,x}, K_{M,x})$ for the conversion. For the entire atmosphere the boundary term vanishes and $C(K_{E,x}, K_{M,x}) = C'(K_{E,x}, K_{M,x})$.

In Table 1 one finds the estimated terms in equation (1). The values presented in this table permit us to make the following remarks:

(*a*) The action of horizontal eddies, both transient and standing, is throughout the year a significant source for the zonal kinetic energy at the expense of the eddy kinetic energy. In all

[1] In this symbolic form of the balance equation for the zonal kinetic energy no term indicating the rate of change of zonal kinetic energy appears on the left hand side of the equation. Because the balance of the time-mean state is considered, the term which contains the product of $[\bar{u}]$ and $\Delta u/\Delta t$ has a slightly different meaning and is written on the right hand side of the equation as "seasonal correction".

TABLE 1. *Zonal kinetic energy balance for 100–30 mb layer (northern hemisphere).*

Units: 10^{18} erg sec^{-1}, except for K_M and K_E. t.e. = transient eddy contribution; s.e. = standing eddy contribution.

I = July–September 1957; II = October–December 1957; III = January–March 1958; IV = April–June 1958; V = July 1957–June 1958.

		I	II	III	IV	V (year)
$C(K_{E,x}, K_{M,x})$ horizontal part	t.e	9.0	29.8	5.8	14.7	16.1
	s.e	4.5	14.1	24.2	3.2	3.6
$C(K_{E,x}, K_{M,x})$ vertical part	t.e	−0.2	1.7	4.1	3.2	1.9
	s.e	0.5	0.9	2.4	0.0	0.4
$C(K_{M,y}, K_{M,x})$		(−3)[a]	(7)	(−298)	(−214)	(−94)
$W_e(K_{M,x})$ at equatorial wall		−0.4	−0.3	0.5	0.2	0.1
$W_e(K_{M,x})$ at top and bottom boundaries		−0.4	−1.2	−6.4	−1.1	−1.4
$A(K_{M,x})$		(−2)[a]	(−5)	(4)	(−4)	(−4)
"Seasonal" correction		9.2	−26.5	12.6	−0.4	−0.2
				(Units: 10^{25} erg)		
$K_{M,x} \simeq K_M$		6.45	8.44	12.65	3.74	4.10
$K_{E,x}$	t.e.	3.20	5.01	5.96	5.17	4.69
	s.e.	1.40	1.80	1.96	0.61	1.27

[a] Numbers in parentheses are considered to be uncertain since they involve mean meridional circulations, which are difficult to measure.

four periods the direction of this process is the same and extracts energy from the smaller-scale motions in favor of the zonal flow. This indicates a "negative" coefficient of eddy viscosity. The same more familiar process dominates in the troposphere and is one of the most interesting phenomena in meteorology.

(*b*) The large-scale vertical eddies tend to increase the energy of the zonal flow, but their action is of minor importance since their contribution only amounts to about 10 % of the contribution of the horizontal eddies (see also STARR & DICKINSON, 1963).

(*c*) On the other hand one notices that the mean meridional circulations give during three of the four seasons a negative contribution through the term $C(K_{M,y}, K_{M,x})$. This is probably due to the dominating indirect circulation in middle latitudes. STARR (1959) computed the same integral for the atmosphere of the entire northern hemisphere for the two years 1950 and 1951, and found a small negative value in both years. The present measurements give also a negative conversion in the 100–30 mb layer, but the magnitude appears to be too large, probably due to the difficult estimation of $[\bar{v}]$.

(*d*) The work done by the measured eddy stresses at the boundaries is negligible. This has the implication that $C(K_{E,x}, K_{M,x})$ is approximately equal to $C'(K_{E,x}, K_{M,x})$ for the considered volume. An independent calculation of the two expressions confirmed this fact.

(*e*) The seasonal correction can be neglected in the balance for the year. For a three-months period this term gives a definite contribution, but it does not dominate the equation. For the computation one needs the difference of the mean zonal wind at the first and at the last day of the considered period. This difference was computed from the U.S. Weather Bureau Stratospheric Daily Maps (1960).

If one assumes as the southern boundary a vertical wall not at the equator but one at a higher latitude, the work term $W_e(K_{M,x})$ at the vertical wall becomes important and the eddy conversion $C(K_{E,x}, K_{M,x})$ does not contribute significantly anymore. The Coriolis term $C(K_{M,y}, K_{M,x})$ gives a positive contribution for a polar cap north of 60° N, since the mean meridional motions are towards the north at high latitudes.

3.2. EDDY KINETIC ENERGY

In the preceding discussion we have looked at the energy in the zonal mean wind field. We

TABLE 2. *Eddy kinetic energy balance for 100–30 mb layer (northern hemisphere).*

Units: 10^{18} erg sec^{-1}. t.e. = transient eddy contribution; s.e. = standing eddy contribution.

I = July–September 1957; II = October–December 1957; III = January–March 1958; IV = April–June 1958; V = July 1957–June 1958.

		I	II	III	IV	V (year)
$C(K_E, K_M)^a$	t.e.	8.8	31.5	9.9	17.9	18 0
	s.e.	5.0	15.0	26.6	3.2	4.0
$C(P_E, K_E)$	t.e.	− 13.7	− 12.2	− 13.1	− 26.3	− 16.3
	s.e.	− 6.0	− 1.4	− 31.8	− 8.0	− 9.2
				Units: 10^{25} erg		
K_M		6.45	8.44	12.65	3.74	4.10
K_E	t.e.	4.99	8.25	9.60	7.21	7.25
	s.e.	1.69	2.29	2.94	0.89	1.70

[a] In the computation $C(K_E, K_M)$ is approximated by $C(K_{E,x}, K_{M,x})$.

observed the existence of an important interaction with the eddy field in the sense of a buildup of the zonal motions at the cost of the eddying motions. In view of this, it is worthwhile to examine how the eddy field is maintained. In the case of the eddy kinetic energy, it is not sufficient to consider only the east–west motions. There is also a considerable amount of energy in the eddy components of the north–south motions; from the present data it appears that $K_{E,y} \cong \frac{1}{2} K_{E,x}$ in the lower stratosphere (compare estimates in tables 1 and 2). The balance equation for the eddy kinetic energy is derived in an analogous fashion as for the zonal kinetic energy. Thus, the zonal equation of motion is multiplied by $\bar{u}^* + u'$ and the meridional equation of motion with $\bar{v}^* + v'$ these two equations are added together, next averaged in time over the considered period and finally integrated in space over the mass of a polar cap. We will write the result in the form:

$$0 = -C(K_E, K_M) + C(P_E, K_E) + D(K_E) + A(K_E) + W_p(K_E) + W_e(K_E) + \text{Seasonal correction.} \quad (2)$$

where:

$$K_M = \tfrac{1}{2} \int \{[\bar{u}]^2 + [\bar{v}]^2\}\, dm$$

and

$$K_E = \tfrac{1}{2} \int \{\overline{[u^2]_E} + \overline{[v^2]_E}\}\, dm.$$

Similarly as in the balance equation for the zonal kinetic energy the "seasonal correction" which contains the time change of the horizontal wind components, is written on the right hand side of the equation. According to equation (2) there is a balance between the following processes:

(a) The conversion from eddy into zonal kinetic energy by horizontal and vertical eddies:

$$C(K_E, K_M) = \int \overline{[uv]_E} \cos\phi \frac{\partial}{a\partial\phi}\left(\frac{[\bar{u}]}{\cos\phi}\right) dm + \int \overline{[u\omega]_E} \frac{\partial[\bar{u}]}{\partial p} dm + \int \overline{[v^2]_E} \frac{\partial[\bar{v}]}{a\partial\phi} dm. + \int \overline{[\omega v]_E} \frac{\partial[\bar{v}]}{\partial p} dm - \int [\bar{v}]\, \overline{[u^2]_E} \frac{\tan\phi}{a} dm.$$

The last three terms in this expression can be neglected with respect to the first two terms.

(b) The conversion from eddy available potential into eddy kinetic energy:

$$C(P_E, K_E) = -\int \overline{[\omega\alpha]_E}\, dm.$$

(c) The dissipation of eddy kinetic energy due to friction:

$$D(K_E) = \int \overline{[uF_x]_E}\, dm + \int \overline{[vF_y]_E}\, dm.$$

(d) The advection of eddy kinetic energy through the boundaries by mean meridional overturnings:

$$A(K_E) = \iint_{\phi=0} [\bar{v}]\tfrac{1}{2}\{\overline{[u^2]_E} + \overline{[v^2]_E}\}\, dx \frac{dp}{g}$$

$$- \iint_{100\,mb} [\bar{\omega}]\tfrac{1}{2}\{\overline{[u^2]_E} + \overline{[v^2]_E}\}\frac{dx\,dy}{g}$$

$$+ \iint_{30\,mb} [\bar{\omega}]\tfrac{1}{2}\{\overline{[u^2]_E} + \overline{[v^2]_E}\}\frac{dx\,dy}{g}.$$

(e) The change of eddy kinetic energy due to the work permed on the layer by pressure forces at the boundaries:

$$W_p(K_E) = \iint_{\phi=0} \overline{[vH]_E}\,dx\,dp$$

$$- \iint_{100\,mb} \overline{[\omega H]_E}\,dx\,dy + \iint_{30\,mb} \overline{[\omega H]_E}\,dx\,dy.$$

(f) The change of eddy kinetic energy due to the work performed on the layer by the measured eddy stresses at the boundaries:

$$W_e(K_E) = \iint_{\phi=0} \{[\bar{u}^*\,\overline{u'v'}^*] + [\bar{v}^*\,\overline{v'^2}^*]$$

$$+ [\bar{v}^*\tfrac{1}{2}(\overline{u'^{2*}} + \overline{v'^{2*}})]\}\,dx\frac{dp}{g}$$

$$- \iint_{100\,mb} \{[\bar{u}^*\,\overline{u'\omega'}^*] + [\bar{v}^*\,\overline{v'\omega'}^*]$$

$$+ \bar{\omega}^*\tfrac{1}{2}(\overline{u'^{2*}} + \overline{v'^{2*}})]\}\frac{dx\,dy}{y}$$

$$+ \iint_{30\,mb} \{[\bar{u}^*\overline{u'\omega'}^*] + [\bar{v}^*\overline{v'\omega'}^*]$$

$$+ [\bar{\omega}^*\tfrac{1}{2}(\overline{u'^{2*}} + \overline{v'^{2*}})]\}\frac{dx\,dy}{g}.$$

(g) The "seasonal correction":

$$- \int \overline{\left[u\frac{\partial u}{\partial t}\right]_E} dm - \int \overline{\left[v\frac{\partial v}{\partial t}\right]_E} dm.$$

The values of the measured terms in this equation are given in Table 2. The conversion from eddy to zonal kinetic energy $C(K_E, K_M)$ is for all practical purposes equal to the integral $C(K_{E,x}, K_{M,x})$, which was discussed in section 3.1. Thus, eddy kinetic energy is converted into zonal kinetic energy. The dissipation of eddy kinetic energy $D(K_E)$ is analogous to the dissipation of zonal kinetic energy by friction (section 3.1). It contains the effects of (1) below grid-size eddies inside the volume, which probably destroy kinetic energy, and (2) eddy stresses at the boundaries by a similar scale of motions, which could conceivably contribute positively to the eddy kinetic energy balance. The conclusion is that one cannot say beforehand what sign $D(K_E)$ should have. The interaction between the motions in the lower stratosphere and in the surrounding layers of the atmosphere gives rise to three forms of boundary terms: (1) the advection of K_E by mean motions: $A(K_E)$, (2) the work done by pressure forces at the boundaries: $W_p(K_E)$, and (3) the work done by the measured eddy stresses at the boundaries: $W_e(K_E)$. These interaction terms only redistribute the energy. Therefore, a gain of a certain amount of eddy kinetic energy by the layer must be accompanied by an equal loss of eddy kinetic energy by the surrounding atmosphere.

The term $C(P_E, K_E)$ apparently represents in the (x, y, p) coordinate system the important conversion process from eddy potential into eddy kinetic energy by convection. It forms the connecting link between on one side the eddy potential and internal energy and on the other side the eddy kinetic energy. In the troposphere this conversion is large and positive. It supplies the kinetic energy for the eddies and, through the interaction of the eddies with the mean flow, also the kinetic energy for the zonal currents. The transformation process is often visualized as the rising of warm air masses and the sinking of cold air masses, but this does not necessarily represent the complete picture. It is more correct, as was pointed out by Starr, to say that the warmer air has a systematic tendency to be correlated with motions towards lower pressure ($\omega < 0$) in the troposphere and colder air with motions towards higher pressure ($\omega > 0$). This description of the energy conversion leaves the possibility open that the process occurs for an important part horizontally and not predominantly in the vertical direction as is implied by the terms "rising" and "sinking". In the present study vertical motions computed according to the "adiabatic" method (i.e. from the first law of thermodynamics neglecting diabatic heating), are used in the evaluation of $C(P_E, K_E)$. It would be better to include here the effects of diabatic heating, but this is not yet possible since heating rates are not known on a day to day basis. WIIN-NIELSEN (1959) has estimated that the influence of a reasonable mean heating pattern could not be neglected in the standing eddy contribution to $C(P_E, K_E)$. However, we expect that the inclusion of heating will not substantially alter the more important contribution of the transient eddies. Most other hemispheric computations of the conversion term (see e.g. JENSEN (1961) who used "adiabatic" vertical velocities, or WIIN-NIELSEN (1959) and SALTZMAN & FLEISHER (1960, 1961) who used vertical motions from the ω-equation) suffer also from the deficiency that diabatic heating is neglected. HOLOPAINEN (1963) has argued that especially "adiabatic" vertical velocities would lead to bad results for the conversion rate. This is only true if one keeps the static stability in the denominator of the expression for the vertical velocity constant. Therefore, great care has been taken to retain the variability of the static stability. The assumption of a constant static stability would change the meaning of the conversion integral. To avoid other difficulties, only actual wind data have been used in all computations. EDDY (1963) used a direct approach in the calculation of $C(P_E, K_E)$. He computed vertical velocities from the smoothed divergence field of the wind, and correlated them with simultaneous temperatures for a limited time period over North America. Eddy's results agree with those of Jensen and support the "adiabatic" method.

The computed values of $C(P_E, K_E)$ in our layer in the lower stratosphere are shown in Table 2. During all seasons a conversion from eddy kinetic into eddy potential energy takes place; this is in a direction opposite to the one usually found in the troposphere (see also WHITE & NOLAN, 1960).

For the January–March period the standing eddy contribution is larger than the transient eddy contribution both in $C(K_E, K_M)$ and in $C(P_E, K_E)$. In all other periods the transient eddies play a more important role.

Source of energy for the eddy motions in the lower stratosphere

The measurements show clearly that throughout the year: (1) the eddies lose kinetic energy to the mean flow, and (2) the eddies convert part of their kinetic energy into potential energy. Since the seasonal correction is not of much importance, unavoidably the conclusion must be drawn that the lower stratosphere receives its kinetic energy from the surrounding atmosphere and is consequently *forced*. The forcing presumably takes place at the bottom boundary, i.e. at 100 mb in this case, through the work done by pressure forces and/or eddy stresses.[1]

WHITE (1954) discovered that there occurs in the lower stratosphere a countergradient eddy flux of heat. This abnormal heat transport was found in our data during all seasons; only in winter at 30 mb a downgradient heat flux was observed, which probably indicates that the top of the abnormal layer is lower in the winter season (OORT, 1963). Also this process can only be explained if the eddies in the lower stratosphere are forced to act as a *refrigerating* mechanism causing heat to flow from a cold region at low latitudes to a warm one at high latitudes. Therefore, by two independent methods the phenomenon of the necessary forcing has been shown to be present.

From the available data the interaction in the vertical direction could be determined at the 75 and 40 mb levels. It enabled us to estimate the work done by eddy stresses and pressure forces at the top and bottom boundaries (see Table 3). This work is given by the terms:

$$W_e(K_M) + W_e(K_E) = -\tfrac{1}{2}\iint_{100\,mb} \overline{[\omega c^2]_E}\,\frac{dx\,dy}{g} + \tfrac{1}{2}\iint_{30\,mb} \overline{[\omega c^2]_E}\,dx\,dy$$

and

$$W_p(K_E) = -\iint_{100\,mb} \overline{[\omega H]_E}\,dx\,dy + \iint_{30\,mb} \overline{[\omega H]_E}\,dx\,dy.$$

The first term was not separated into the contribution from the balance of zonal kinetic energy, $W_e(K_M)$, and from the balance of eddy kinetic energy, $W_e(K_E)$. Still, the numbers will give us an idea of the magnitude of the measured interaction at the boundaries and may be compared with the deficit in the balance of the eddy kinetic energy. Table 3 shows, generally, an upward transport of energy at 75 mb. All eddy terms added together amount to about -3×10^{18} erg sec^{-1} for the year. Only during July–

[1] The advection by mean motions is probably small.

TABLE 3. *Estimates of interaction terms at the boundaries, computed with the aid of adiabatic vertical velocities.*

Units: 10^{18} erg sec^{-1}.

I = July–September 1957; II = October–December 1957; III = January–March 1958; IV = April–June 1958; V = July 1957–June 1958.

		I	II	III	IV	V (year)
$\frac{1}{2}\iint [\overline{\omega c^2}]_E \frac{dx\,dy}{g}$	75 mb	0.7[a]	−1.8[a]	5.0	−0.0	−0.1[a]
	40 mb	—	—	−0.2	−0.3	—
$\iint [\overline{\omega H}]_E\, dx\, dy$	75 mb	1.0	−2.6	−11.1	−3.3	−2.7
	40 mb	—	—	−1.3	0.8	—

[a] Only contribution due to transient eddies included.

September is there a small downward transport. The transports around 40 mb appear to be small. If one compares the annual mean transport at 75 mb with the amount 48×10^{18} erg sec^{-1} which is needed in order to give a balance for the eddy kinetic energy for the year, a large discrepancy is in evidence. This discrepancy could arise due to the fact that the vertical eddy transport was measured at 75 mb and not at 100 mb. HANSROTE & LAMBERT (1960) found, for example, for one month of data at 150 mb a much larger value for the vertical eddy transport of energy than we measured at 75 mb. Another solution to the difference between the balance requirements and the values in table 3 could be that meso- and/or micro-scale eddies transport considerable amounts of eddy kinetic energy from the troposphere into the lower stratosphere. The eddies which we measure with the adiabatic method, have a time scale of over one day and it appears possible that eddies with shorter time scales accomplish an important part of the vertical transport of this energy.

3.3. ZONAL AVAILABLE POTENTIAL ENERGY

In general circulation studies it is often convenient to deal with the sum of the potential and internal energy, since for a vertical column extending throughout the depth of the atmosphere the two forms of energy are proportional by the factor R/c_v. Only a small part of the potential and internal energy is really available for conversion into kinetic energy. One may consider the difference between the actual total potential plus internal energy and the potential plus internal energy if the mass were redistributed adiabatically in such a way that the isothermal surfaces would be parallel to the isobaric levels, as a measure of the "available potential energy" (LORENZ, 1955). To the approximation that the pressure surfaces are horizontal, this is a good definition for the energy available for conversion into kinetic energy.

In the case of the kinetic energy, we considered eddy and zonal kinetic energy separately. It will be of advantage to make a similar division for the available potential energy. In the case of a strictly zonal symmetric distribution of temperature, no eddy available potential energy is present and it is necessary to disturb first the zonal symmetry in order to make potential energy available for conversion into eddy kinetic energy. Once the zonal symmetry is distorted, the eddy heat fluxes can redistribute the available potential energy between the zonal and the eddy forms. The early studies of MARGULES (1903) concerning the potential energy of a fixed atmospheric volume were extended by LORENZ (1955). In his paper Lorenz gives an analysis of the concepts of zonal and eddy available potential energy. In Lorenz' formulation available potential energy is approximated by the variance of potential temperature on a constant pressure level, weighted by a factor containing the hemispheric average of the static stability. Difficulties come up if one tries to derive the balance equations for the zonal and eddy available potential energy. Aside from the terms which represent the expected energy transformations, extra terms enter into the balance equations. These terms cannot be reduced to boundary integrals and therefore represent some kind of transformation of energy. They depend mainly upon the variation of the mean static stability with pressure and are probably caused by the approximations made in deriving the expressions for the available potential energy. It is not possible here to go into further detail since this would require an extensive theoretical study in itself. Despite the above mentioned difficulties, we shall use the formulae derived by Lorenz. Terms involving the vertical derivatives of measures of the mean static stability are left out and are thus explicitly *assumed* to be small. Neither shall we include boundary terms in the discussion of the balance equations.

One can derive the balance equation for the zonal available potential energy from the first law of thermodynamics in the form:

$$\frac{\partial\theta}{\partial t} = -u\frac{\partial\theta}{a\cos\phi\,\partial\lambda} - v\frac{\partial\theta}{a\partial\phi} - \omega\frac{\partial\theta}{\partial p} + \frac{1}{c_p}\frac{\theta}{T}Q. \quad (3)$$

In this equation Q is the diabatic heating rate, which is equal to the sum of the frictional heating and the radiational flux convergence. Effects of condensation and evaporation may be neglected in the stratosphere. Equation (3) is multiplied by $F(p)[\bar{\theta}]''$, where:

$$F(p) = -(T/\theta)(R/p)(\partial\widetilde{\bar{\theta}}/\partial p)^{-1} = \left(\frac{T}{\theta}\right)^2 \frac{g}{\widetilde{\bar{T}(\gamma_d-\bar{\gamma})}},$$

a stability parameter. The resulting equation is averaged in time over the considered period and next integrated in space over a stratospheric polar cap. Triple correlations are neglected. By using the equation of continuity and the ideal gas law, the balance equation for the zonal available potential energy may be derived in the form:

$$0 = -C(P_M, P_E) - C(P_M, K_M) + G(P_M) + \text{Seasonal correction } (+\text{Boundary terms}). \quad (4)$$

Here: $$P_M = \tfrac{1}{2}g\int\frac{[\bar{T}]''^2}{\widetilde{\bar{T}(\gamma_d-\bar{\gamma})}}dm \quad \text{and}$$

$$P_E = \tfrac{1}{2}g\int\frac{[\overline{T^2}]_E}{\widetilde{\bar{T}(\gamma_d-\bar{\gamma})}}dm.$$

Equation (4) expresses that there should be an approximate balance between:

(*a*) The conversion from zonal into eddy available potential energy by horizontal or vertical eddy processes:

$$C(P_M, P_E) = -g\int\frac{[\overline{vT}]_E}{\widetilde{\bar{T}(\gamma_d-\bar{\gamma})}}\frac{\partial[\bar{T}]''}{a\partial\phi}dm - g\int\frac{\theta}{T}[\overline{\omega T}]_E''\frac{\partial}{\partial p}\left(\frac{[\bar{T}]''}{\widetilde{\bar{\theta}(\gamma_d-\bar{\gamma})}}\right)dm.$$

The conversion $C(P_M, P_E)$ depends upon the transport of sensible heat along the gradient of temperature. Thus $C(P_M, P_E)$ may be compared with the conversion from eddy to zonal kinetic energy $C(K_E, K_M)$, which depends upon the transport of angular momentum along the gradient of angular velocity. If the eddies transport heat down the temperature gradient, zonal available potential energy is converted into eddy available potential energy.

(*b*) The direct conversion from zonal potential energy into zonal kinetic energy by mean meridional circulations:

$$C(P_M, K_M) = -\int[\bar{\omega}]''[\bar{\alpha}]''dm.$$

(*c*) The generation of zonal available potential energy by diabatic heating:

$$G(P_M) = \frac{g}{c_p}\int\frac{[\bar{T}]''[\bar{Q}]''}{\widetilde{\bar{T}(\gamma_d-\bar{\gamma})}}dm.$$

(*d*) The "seasonal correction":

$$-g\int\frac{[\bar{T}]''\dfrac{\Delta[\bar{T}]''}{\Delta t}}{\widetilde{\bar{T}(\gamma_d-\bar{\gamma})}}dm.$$

A similar term arising in the zonal kinetic energy balance was discussed before in section 3.1.

A qualitative argument can be given regarding the sign of the generation term $G(P_M)$. In the troposphere, low latitudes where the temperatures are high, are heated by the surplus of incoming, short wave over outgoing, long wave radiation and high latitudes where lower temperatures prevail, are cooled relatively. This action results in a large positive value of the hemispheric covariance $[\bar{T}]''[\bar{Q}]''$ and consequently in a large generation of zonal available potential energy. The presented picture applies well to the generation integrated over the entire

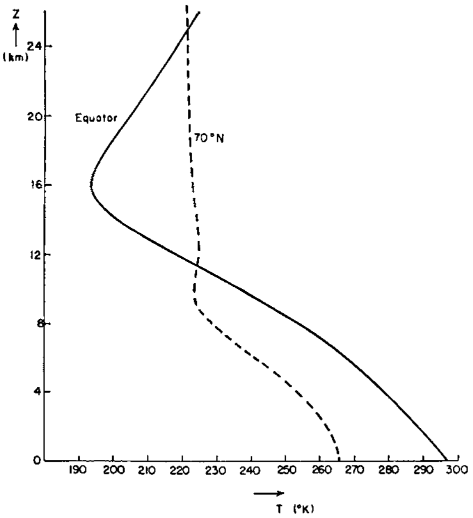


FIG. 1. Comparison of the temperature distribution with height at the equator and at 70° N. The tropospheric data are taken from PEIXOTO (1960) for the year 1950; the stratospheric data are for the year July 1957–June 1958.

atmosphere, but if one considers the lower stratosphere alone, the picture is not correct. Here, the temperature distribution has a maximum in high latitudes in winter and this maximum shifts even to polar latitudes during the summer season. In Fig. 1 curves are plotted of the temperature distribution with height at the equator and at 70° N. If one assumes that there is an excess of absorbed over emitted radiation at all levels at low latitudes, and a deficit at high latitudes, then it is clear that radiation tends to increase the meridional temperature gradient from the ground up to 12 km and above 24 km. However, radiation tends to decrease the meridional temperature gradient in a layer around 16 km height (see also BOVILLE, 1961). This layer contains our volume between 100 and 30 mb and it can be concluded that for the lower stratosphere $G(P_M) < 0$.

We shall compute $C(P_M, P_E)$ for the 100–30 mb layer and determine to what degree this conversion can balance the destruction of zonal available potential energy by radiation. The results are shown in Table 4. The seasonal correction was computed with the aid of temperatures which were read off the U.S. Weather Bureau Stratospheric Daily Maps (1960). Eddy available potential energy is converted into zonal available potential energy throughout the year, as might be anticipated from the counter-gradient heat transport (WHITE, 1954; OORT, 1963). The effect of the standing eddies is about 1/2 to 1/3 of the effect of the transient eddies. The vertical eddies, expressed in the vertical part of $C(P_M, P_E)$, give a slight negative contribution, but they are unimportant compared with the horizontal eddies.

The generation of zonal available potential energy due to the radiation flux convergence can be calculated directly from radiation estimates combined with our measured temperatures. We used the net cooling rates, obtained by DAVIS (1963), and computed for the rate of generation:

TABLE 4. *Zonal available potential energy balance for 100–30 mb layer (northern hemisphere).*

Units: 10^{18} erg sec^{-1}. t.e. = transient eddy contribution; s.e. = standing eddy contribution.

I = July–September 1957; II = October–December 1957; III = January–March 1958; IV = April–June 1958; V = July 1957–June 1958.

		I	II	III	IV	V (year)
$C(P_M, P_E)$ horizontal part	t.e.	−26.6	−35.6	−42.0	−37.1	−35.6
	s.e.	−11.8	−14.1	−11.5	−25.5	−15.9
$C(P_M, P_E)$ vertical part	t.e. + s.e.	−1.9	0.4	−7.6	−5.2	−1.6
"Seasonal correction"		18.8	4.2	−21.3	−8.1	—
				Units 10^{25} erg		
P_M		26.4	18.2	25.7	30.2	23.6
P_E	t.e.	3.30	4.83	7.31	3.99	4.86
	s.e.	1.74	2.39	3.01	1.27	2.10

	Jul.–Sept. 1957	Oct.–Dec. 1957	Jan.–Mar. 1958	Apr.–Jun. 1958	Year
$G(P_M)$	−221	−127	−201	−230	−187
		Units: 10^{18} erg sec^{-1}			

The radiation estimates used by, for example, MURGATROYD & SINGLETON (1961), would give even larger destruction rates for the year: $G(P_M) = -4.8 \times 10^{20}$ erg sec^{-1}. The three-monthly mean vertical motions which are computed by the adiabatic method, are not appropriate to estimate the contribution of the term $C(P_M, K_M)$, since diabatic influences may alter these mean vertical motions significantly. Other investigators, as, e.g., REED *et al.* (1963), find evidence of a conversion from zonal kinetic to zonal potential energy in the lower stratosphere based upon data for a two week period. The discrepancy between the large destruction of P_M by radiation and the insufficient creation by the eddies, tends to support an additional conversion from K_M to P_M through the mean meridional circulations. On the other hand, the radiation estimates are still quite uncertain and this could also be the dominant factor in the discrepancy.

There is evidence of a seasonal variation of the zonal and eddy available potential energy in Table 4. The zonal available potential energy reaches a maximum in summer around April–June, and a minimum in the winter period October–December. The zonal kinetic energy (see Table 1) showed on the other hand a maximum value in winter and a minimum in summer. Both the eddy kinetic (Table 2) and the eddy available potential energy (Table 4) reach a maximum in the winter season.

3.4. EDDY AVAILABLE POTENTIAL ENERGY

With the same sort of approximations used in deriving the zonal balance equation, one can derive the balance for the eddy component. The first law of thermodynamics (3) is multiplied by $F(p)(\bar{\theta}^* + \theta')$, where $F(p)$ is the same stability parameter as the one used in the derivation of the balance of zonal available potential energy. Next the equation is averaged in time and then integrated in space, resulting in the balance equation for the eddy available potential energy for a polar cap:

$$0 = C(P_M, P_E) - C(P_E, K_E) + G(P_E)$$
$$+ \text{Seasonal correction } (+\text{Boundary terms}). \quad (5)$$

In sections 3.2 and 3.3 we considered $C(P_M, P_E)$ and $C(P_E, K_E)$ in detail. The estimated values are repeated in Table 5. It seems that the eddies lose more of their potential energy in building up the zonal energy than they gain by "negative" convection from their kinetic energy.

The rate of generation of eddy available potential energy by diabatic heating is given by

$$G(P_E) = \frac{g}{c_p} \int \frac{[\overline{TQ}]_E}{\tilde{T}(\gamma_d - \tilde{\gamma})} dm.$$

TABLE 5. *Eddy available potential energy balance for 100–30 mb layer (northern hemisphere).*

Units: 10^{18} erg sec^{-1}. t.e. = transient eddy contribution; s.e. = standing eddy contribution.

I = July–September 1957; II = October–December 1957; III = January–March 1958; IV = April–June 1958; V = July 1957–June 1958.

		I	II	III	IV	V (year)
$C(P_M, P_E)$ horizontal part	t.e.	−26.6	−35.6	−42.0	−37.1	−35.6
	s.e.	−11.8	−14.1	−11.5	−25.5	−15.9
$C(P_M, P_E)$ vertical part	t.e + s.e.	−1.9	0.4	−7.6	−5.2	−1.6
$C(P_E, K_E)$	t.e.	−13.7	−12.2	−13.1	−26.3	−16.3
	s.e.	−6.0	−1.4	−31.8	−8.0	−9.2

Very little is known about the magnitude of this generation. Even the sign of $G(P_E)$ is uncertain and cannot be inferred through a simple reasoning similar to the one used for $G(P_M)$. In the case of the troposphere, the studies of WIIN-NIELSEN & BROWN (1960), BROWN (1963) and KRUEGER, WINSTON & HAINES (1963) all indicate an eddy destruction of available potential energy probably due to the tendency of warmer air masses to be cooled and colder air masses to be heated.

GODSON (1960) and other investigators have found a high positive correlation between the temperature in the lower stratosphere and the amount of total ozone. Let us assume that under certain circumstances a high concentration of ozone is present in the 100–30 mb layer, then more short wave radiation will be absorbed in this layer. The effect will be equivalent to an extra heating of warm air and, consequently, a creation of eddy available potential energy $G(P_E)$, if in the layer the heating is determined mainly by the absorption of short wave radiation by ozone; this is certainly true at about 20 km and even more so at greater heights (see e.g. MANABE & MÖLLER, 1961). A creation of eddy available potential energy between 100 and 30 mb is supported by our measurements provided that the boundary terms in equation (5) are not very important.

4. The energy cycle

In the first part of this chapter we shall give a short description of the energy cycle as it is observed in the troposphere. Since the troposphere contains about 8/10 of the total atmospheric mass, this picture will be representative also for the cycle in the entire atmosphere. In section 4.2 we shall present the very different picture of the energy cycle of a layer in the lower stratosphere which was determined in the present study. Finally the contributions of different layers in troposphere and stratosphere will be compared in section 4.3.

4.1. ENERGY CYCLE FOR THE TROPOSPHERE

At low latitudes the earth-atmosphere system gains more energy per unit area by the absorption of short wave radiation from the sun than it looses to space by the emission of long wave radiation; the reverse is true at high latitudes. Due to the excess of incoming over outgoing radiation at low latitudes and the deficit at high latitudes, a meridional temperature gradient is created in the troposphere (this is equivalent to a creation of zonal available potential energy). The zonal currents resulting from the meridional temperature gradient are unstable for certain baroclinic wave disturbances. Maximum instability is found at about the wavelength of typical cyclones and anticyclones. These eddy circulations ("eddies") cause an east-west temperature gradient and thus convert zonal into eddy available potential energy. As the result of vertical motions the eddy potential energy is partly converted into eddy kinetic energy. The eddies appear to lose further potential energy through the combined action of radiation, condensation, evaporation and heat fluxes near the ground. The kinetic energy of these large-scale eddies is subject to the probable destructive influence of smaller-scale eddies which transfer the kinetic energy to still smaller eddies. This cascade of the kinetic energy to smaller and smaller scales, results finally, at the molecular scale in the transformation of the kinetic energy into heat. A significant percentage of the large-scale eddy kinetic energy is in this way destroyed. The remaining part of this energy is converted up the scale into kinetic energy for the zonal current and maintains thus the zonal current against the frictional dissipation of the smaller eddies. Finally, the conversion of zonal potential into zonal kinetic energy brings us back to the beginning of the cycle, i.e., the zonal available potential energy. This conversion has been proved to be small and has even probably a negative value, i.e. zonal kinetic energy is converted into zonal potential energy.

As was pointed out by SALTZMAN (1961), the zonal motions may be conceived of as strong free wheeling circulations with relatively small viscous damping or forcing.[1] The important process of the eddies supplying angular momentum for the zonal currents was first suggested by JEFFREYS in 1926. Jeffreys' ideas on the angular momentum balance have been extended to the eddy effects on the zonal kinetic energy, notably by KUO (1951), VAN MIEGHEM (1952), ARAKAWA (1953) and STARR (1953). Earlier concepts were based on the hypothesis of the existence of a direct mean meridional

[1] This result is based upon the large observed ratio of $C(P_E, K_E)$ and $C(K_E, K_M)$.

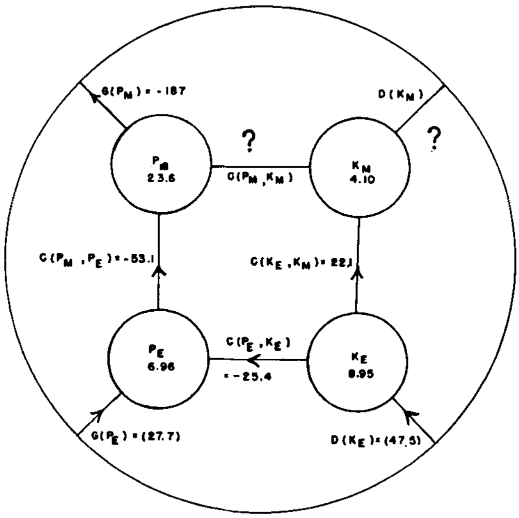


FIG. 2. Energy flow diagram during the year (July 1957–June 1958) for 100-30 mb layer. Energy transformation units: 10^{18} erg sec^{-1}. Energy units: 10^{25} erg. Numbers in parentheses are balance requirements as inferred from other observations. $G(P_M)$ is computed from radiation data by Davis (1963).

circulation. Starr realized as one of the first the importance of Jeffreys' suggestion. Through years of study of actual data on a hemispheric scale at the University of California (Los Angeles) under Jacob Bjerknes and principally at the Massachusetts Institute of Technology under Victor P. Starr, the important role of the eddy processes has been shown and thus the foundation has been laid for the modern concepts on the general circulation (see e.g. STARR, 1958).

4.2. THE ENERGY CYCLE IN THE LOWER STRATOSPHERE

A summary of the measured energy transformations and the total amount of each form of energy present is given in a diagram (Fig. 2), which was originally introduced by LORENZ (1954). Small circles represent the four forms of energy P_M, K_M, P_E and K_E. A line connecting two circles indicates a conversion between the two forms of energy. The interaction with the environment is given by a line connected with the large circle (environment) which contains the four smaller circles. The number in each circle represents the time-mean amount of the specific form of energy in units of 10^{25} erg. The conversions are labeled in units of 10^{18} erg sec^{-1}.

We should always keep in mind that we are dealing with a layer and not with the entire atmosphere. This implies that top and bottom boundary effects cannot be neglected without further consideration. In the diagram the generation $G(P)$ and dissipation $D(K)$ will thus contain these effects at the boundaries.

It is apparent from the energy diagram (Fig. 2) that pressure forces and/or eddy stresses at the top and bottom boundaries (presumably mainly at the bottom boundary with the troposphere) force the energy-cycle to be in a direction more or less opposite to the one in the troposphere.[1] This source of kinetic energy ultimately supplies the necessary balance for the loss of potential energy by radiation. The direct observations show the following special features of the circulation in the 100–30 mb layer:

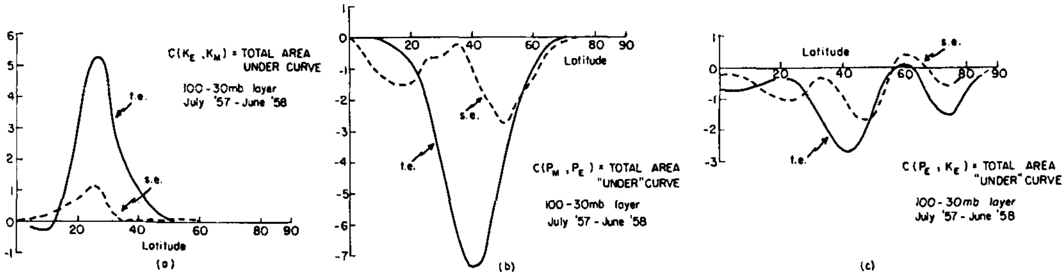
1. A "sinking" of warm air and a "rising" of cold air, which destroys eddy kinetic energy and converts it into eddy potential energy.
2. An eddy transport of heat against the mean temperature gradient, which destroys eddy available potential energy and converts it into mean zonal available potential energy.
3. A destruction of zonal available potential energy by radiation.

Indirectly, the observations imply or suggest by necessary balance requirements that:

1. Interaction with the surrounding atmosphere is a source rather than a sink of energy for the eddies and—through the eddies—also for the zonal flow.
2. Eddy available potential energy could be created by radiation.

The seasonal fluctuations of the three-monthly mean conversion and generation rates are small and no change in sign occurs. In this connection, it is of interest to look at the results of REED, WOLFE & NISHIMOTO (1963) for a 14 day ʳ ·d during the stratospheric sudden warr...ıg oı early 1957. They computed energy flow diagrams at 50 mb on a day to day

[1] In the troposphere, the energy cycle proceeds from P_M, via P_E and K_E, to K_M, i.e. in a counterclockwise direction in the diagram; thus the tropospheric source of energy is found in the generation of zonal available potential energy by radiation and the sink in the destruction of kinetic energy by friction.



FIGS. 3*a*, *b* and *c*. The contributions at the different latitudes to (*a*) the conversion from eddy into zonal kinetic energy $C(K_E,K_M)$, (*b*) the conversion from zonal into eddy available potential energy $C(P_M,P_E)$ and (*c*) the conversion from eddy potential into eddy kinetic energy $C(P_E,K_E)$ by transient eddies (*t.e.*) and standing eddies (*s.e.*). All quantities are computed for the 100-30 mb layer and for the period July 1957–June 1958. The values of hemispheric integrals are as follows:

t.e. $C(K_E,K_M) = 16.1 \times 10^{18}$ erg sec^{-1};
$C(P_M,P_E) = -35.6 \times 10^{18}$ erg sec^{-1};
$C(P_E,K_E) = -16.3 \times 10^{18}$ erg sec^{-1}.

s.e. $C(K_E,K_M) = 3.6 \times 10^{18}$ erg sec^{-1};
$C(P_M,P_E) = -15.9 \times 10^{18}$ erg sec^{-1};
$C(P_E,K_E) = -9.2 \times 10^{18}$ erg sec^{-1}.

basis for the period January 25–February 9, 1957. A marked difference was found in the energy cycle between the first 10 days of the warming period and the next 5 days. The first 10 days showed a cycle similar to the one usually found in the troposphere, while the last period of 5 days might be compared with our results for the three-month period. Thus it appears that the energy flow in the lower stratosphere is in an "abnormal" direction just after the sudden warming sets in, but returns later to its normal direction.

The contribution at the different latitudes to the conversion integrals $C(K_E, K_M)$, $C(P_M, P_E)$ and $C(P_E, K_E)$ is sketched in Figs. 3*a*, *b* and *c*. The transient eddies contribute most effectively in middle latitudes to the conversion from eddy potential and eddy kinetic energy into, respectively, zonal potential and zonal kinetic energy. The contribution from the standing eddies is of less importance. The conversion from eddy potential into eddy kinetic energy for the 100–30 mb layer has almost at every latitude a negative value.

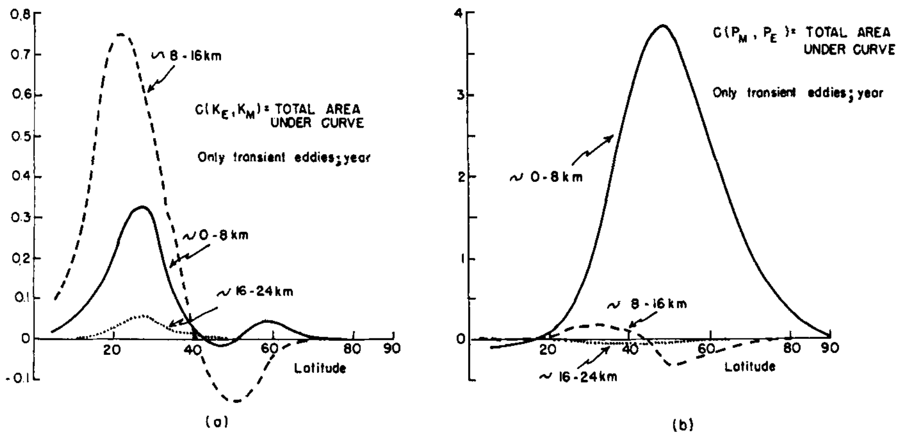
4.3. Comparison tropospheric and stratospheric conversions

In order to avoid any distortion of the picture of energy conversions in the atmosphere by an over-emphasis on the stratospheric processes, graphs (Figs. 4*a* and *b*) have been prepared of the transient eddy contributions to the total integrals $C(K_E, K_M)$ and $C(P_M, P_E)$ of three atmospheric layers about 8 km thick, i.e., (1) lower troposphere 1000–400 mb ($\sim$0–8 km), (2) upper troposphere 400–100 mb ($\sim$8–16 km), and (3) lower stratosphere 100–30 mb ($\sim$16–24 km). The data for 1000–100 mb have been taken from the studies by BUCH (1954) of the wind and by PEIXOTO (1960) of the temperature conditions during the year 1950. Our data for the year July 1957–June 1958 are used for the 100–30 mb layer. Let us emphasize some important properties of the three layers:

1. The main contribution to the conversion from zonal to eddy available potential energy comes from the 0–8 km layer. From 8–16 km and 16–24 km the contribution is negative. The energy source for the eddies is therefore mainly situated in the lowest 8 km.
2. The main part of the conversion from eddy to zonal kinetic energy takes place in the upper troposphere. There is no reversal in the upper layers.
3. The contribution of the lower stratosphere to the hemispheric integrals is about 10 % in the kinetic energy and only 2–3 % in the (negative) potential energy conversion.

5. Some final comments

It is perhaps good to emphasize that in the present paper we have been interested in the maintenance of the energy of (*a*) the *time mean* and *zona lmean* state and (*b*) the transient and standing eddies; this approach was ap-



Figs. 4*a* and *b*. The contributions at the different latitudes from 3 layers about 8 km thick to (*a*) the conversion from eddy into zonal kinetic energy $C(K_E, K_M)$ and (*b*) the conversion from zonal into eddy available potential energy $C(P_M, P_E)$, by the horizontal transient eddy processes. Data for 1000-100 mb are taken from the studies by Buch (1954) of the wind and by Peixoto (1960) of the temperature conditions during the year 1950. 100-30 mb data are for year July 1957–June 1958:

Layer 1000-400 mb ($\sim$0–8 km):
$C(K_E, K_M) = 1.27 \times 10^{20}$ erg sec^{-1};
$C(P_M, P_E) = 22.9 \times 10^{20}$ erg sec^{-1}.
Layer 400–100 mb ($\sim$8–16 km):
$C(K_E, K_M) = 2.53 \times 10^{20}$ erg sec^{-1};
$C(P_M, P_E) = -0.56 \times 10^{20}$ erg sec^{-1}.
Layer 100-30 mb ($\sim$16–24 km):
$C(K_E, K_M) = 0.20 \times 10^{20}$ erg sec^{-1};
$C(P_M, P_E) = -0.36 \times 10^{20}$ erg sec^{-1}.

propriate because of our method of analyzing maps of seasonal mean variables and of their covariances in time. If on the other hand we had computed the terms in the energy budget strictly on a *daily basis*, it would have been more appropriate to solve another problem, namely the question of how the energy of (*a*) the zonally averaged state and (*b*) the zonal inequalities (eddies) are maintained. This was the problem considered by Lorenz (1955) and in this case one should therefore use also the same expressions as in Lorenz' paper, and not the modified formulae as presented in this article. The numerical results for the comparable transformations would presumably be somewhat different.

The concept that there are no energy sources in the lower stratosphere itself, has been confirmed by this study and has been put on a firm basis. The conclusion must be drawn that interaction at the top and/or bottom boundaries supplies the necessary kinetic energy for maintaining the eddy circulations and indirectly also the zonal flow. It is to be expected that the influx of energy for the lower stratosphere occurs at the bottom boundary, i.e., from the underlying troposphere; this was already suggested by Starr (1960). The upward transport does not need to be large compared with the average energy in the troposphere since the contribution of the lower stratosphere to the energy transformations for the entire atmosphere has been shown to be less than 10 %.

From the data at the individual levels it appears that the forced layer generally extends from about 200 to 30 mb; the top boundary sinks below 30 mb during the winter season.

It is further of interest to look at an example of a probable forced regime of motion in the ocean. It appears from a fairly large sample of data on the Gulf Stream that the meanders in the surface layers of this current draw their energy neither from the kinetic energy of the main stream (Webster, 1961) nor from the available potential energy connected with the temperature gradient between the slope water and the water of the Sargasso Sea (Oort, 1964). This would imply again an outside source of energy for these meanders. It is possible that also the eddies in the surface layers of the Gulf Stream are forced by the circulation in the lower atmosphere.

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REFERENCES

ARAKAWA, H., 1953, On the maintenance of westerlies and tropical revolving wind systems. *J. Meteor. Soc. of Japan*, **31**, pp. 195–205.

BARNES, A. A., JR., 1963, Kinetic and potential energy between 100 mb and 10 mb during the first six months of the IGY. Final Rept., Contract No. AF19(604)-5223, Planetary Circulations Project, Dept. of Meteor., Mass. Inst. of Tech., pp. 8–131.

BOVILLE, B. W., 1961, A dynamical study of the 1958–59 stratospheric polar vortex. Sci. Rept. No. 9, Contract No. AF19(604)-3865, Arctic Meteorology Research Group, Dept. of Meteor., McGill University, Montreal, pp. 134.

BROWN, J. A., JR., 1963, A diagnostic study of tropospheric diabatic heating and the generation of available potential energy (unpublished manuscript).

BUCH, H. S., 1954, Hemispheric wind conditions during the year 1950. Final Rept., Part 2, Contract No. AF19-122-153, General Circulation Project, Dept. of Meteor., Mass. Inst. of Tech., pp. 126.

CRAIG, R. A., and LATEEF, M. A., 1962, The stratospheric warming over North America in 1957. Final Rept., Contract No. AF19(604)-5471, Dept. of Meteor., Florida State University, pp. 36.

DAVIS, P. A., 1963, An analysis of the atmospheric heat budget. *J. Atmos. Sci.*, **20**, pp. 5–22.

EDDY, A., 1963, Kinematic divergence and large-scale energy conversion. Sci. Rept. No. 8, Contract No. AF19(604)-8431, Arctic Meteorology Research Group, Dept. of Meteor., McGill University, Montreal, pp. 104.

GODSON, W. L., 1960, Total ozone and the middle stratosphere over arctic and sub-arctic areas in winter and spring. *Quart. J. Roy. Meteor. Soc.*, **86**, pp. 301–317.

HANSROTE, L. S., and LAMBERT, J. K., 1960. Energy interaction between the troposphere and stratosphere, SM thesis, Dept. of Meteor., Mass. Inst. of Tech., pp. 47.

HOLOPAINEN, E. O., 1963, On the dissipation of kinetic energy in the atmosphere. *Tellus*, **15**, pp. 26–32.

JEFFREYS, H., 1926, On the dynamics of geostrophic winds. *Quart. J. Roy. Meteor. Soc.*, **52**, pp. 85–104.

JENSEN, C. E., 1961, Energy transformation and vertical flux processes over the northern hemisphere. *J. Geophys. Res.*, **66**, pp. 1145–1156.

KRUEGER, A. F., WINSTON, J. S., and HAINES, D. A., 1963, unpublished manuscript.

KUO, H. L., 1951, A note on the kinetic energy balance of the zonal wind systems. *Tellus*, **3**, pp. 205–207.

LETTAU, H., 1954, Notes on the transformation of mechanical energy from and to eddying motion. *J. Meteor.*, **11**, pp. 196–201.

LORENZ, E. N., 1954, The basis for a theory of the general circulation. Final Rept., part I, Contract No. AF19-122-153, General Circulation Project, Dept. of Meteor., Mass. Inst. of Tech., pp. 522–534.

LORENZ, E. N., 1955, Available potential energy and the maintenance of the general circulation. *Tellus*, 7, pp. 157–167.

MANABE, S., and MÖLLER, F., 1961, On the radiative equilibrium and heat balance of the atmosphere. *Monthly Weather Review*, **89**, pp. 503–532.

MARGULES, M., 1903, Über die Energie der Stürme. *Jahrbuch Kaiserlich-Königlichen Zentralanstalt Meteor.* (translated in *The Mechanics of the Earth's Atmosphere* by C. Abbe, Smithson. Misc. Coll., **51**, 1910).

MILLER, J. E., 1950, Energy transformation functions. *J. Meteor.*, **7**, pp. 152–159.

MURGATROYD, R., and SINGLETON, J., 1961, Possible meridional circulations in the stratosphere and mesosphere. *Quart. J. Roy. Meteor. Soc.*, **87**, pp. 125–135.

OBASI, G. O. P., 1963, Atmospheric momentum and energy calculations for the southern hemisphere during the IGY. Sci. Rept. No. 6, Contract No. AF19(604)-6108, Planetary Circulations Project, Dept. of Meteor., Mass. Inst. of Tech., pp. 354.

OORT, A. H., 1963, On the energy cycle in the lower stratosphere. Rept. No. 9 Contracts No. AT(30-1) 2241 and AF19(604)-5223, Planetary Circulations Project, Dept. of Meteor., Mass. Inst. of Tech., pp. 122.

OORT, A. H., 1964, Computations of the eddy heat and density transports across the Gulf Stream. *Tellus*, **16**, pp. 55–63.

PEIXOTO, J. P., 1960, Hemispheric temperature conditions during the year 1950. Sci. Rept. No. 4, Contract No. AF19(604)-6108, Planetary Circulations Project, Dept. of Meteor., Mass. Inst. of Tech., pp. 211.

PFEFFER, R. L., 1957, On the physical significance of energy transformation functions. Final Rept., Contract No. AF19(604)-1000, General Circulation Project, Dept. of Meteor., Mass. Inst. of Tech., pp. 42–65.

REED, R. J., WOLFE, J. L., and NISHIMOTO, H., 1963, A spectral analysis of the energetics of the stratospheric sudden warming of early 1957. *J. Atm. Sci.* **20**, pp. 256–275.

SALTZMAN, B., 1961, The zonal harmonic representation of the atmospheric energy cycle—a review of measurements. The Travelers Research Center Inc., Report No. 9 (TRC-9), pp. 19.

SALTZMAN, B., and FLEISHER, A., 1960, The modes of release of available potential energy in the atmosphere. *J. Geophys. Research,* **65**, 1215–1222.

SALTZMAN, B., and FLEISHER, A., 1961, Further statistics on the modes of release of available potential energy. *J. Geophys. Research,* **66**, 2271–2273.

STARR, V. P., 1953, Note concerning the nature of the large-scale eddies in the atmosphere. *Tellus,* **5**, pp. 494–498.

STARR, V. P., 1958, What constitutes our new outlook on the general circulation? *J. Meteor. Soc. of Japan,* **36**, 5, pp. 167–173.

STARR, V. P., 1959, Further statistics concerning the general circulation. *Tellus*, **11**, pp. 481–483.

STARR, V. P., 1960, Questions concerning the energy of stratospheric motions. *Arch. Met. Geoph. Biokl. A*, **12**, pp. 1–7.

STARR, V. P., and DICKINSON, R. E., 1963, Large scale vertical eddies in the atmosphere and the energy of the mean zonal flow. *Geofisica pura e applicata*, **55**, pp. 133–136.

U.S. Weather Bureau, 1960, Daily 100-millibar and 50 millibar and three times monthly 30-millibar synoptic weather maps of the IGY period. U.S· Department of Commerce, Washington, D.C.

VAN MIEGHEM, J., 1952, Comment on a note on the kinetic energy balance of zonal wind systems. *Tellus*, 4, pp. 68–70.

WEBSTER, F., 1961, The effect of meanders on the kinetic energy balance of the Gulf Stream. *Tellus,* **13**, pp. 392–401.

WHITE, R. M., 1954, The counter-gradient flux of sensible heat in the lower stratosphere. *Tellus*, **6**, pp. 177–179.

WHITE, R. M., and NOLAN, G. F., 1960, A preliminary study of the potential to kinetic energy conversion process in the stratosphere. *Tellus*, **12**, pp. 145–148.

WIIN-NIELSEN, A., 1959, A study of energy conversion and meridional circulation for the large-scale motion in the atmosphere. *Monthly Weather Review,* 87, 319–332.

WIIN-NIELSEN, A., and BROWN, J. A., 1960, On diagnostic computations of atmospheric heat sources and sinks and the generation of available potential energy. Proc. Internat. Symposium on Numerical Weather Prediction, Nov., 7–13, 1960, Tokyo, Meteorological Society of Japan, pp. 593–613.