On the formation and development of layer clouds

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ABSTRACT

Empirical results indicating similar turbulent transfer for cloud droplets and for water vapour or sensible heat have been used in order to derive a set of prognostic equations for the total moisture content and for the equivalent potential temperature. With given external conditions, especially imposed vertical velocity the equations can be used for forecasting the formation and development of layer clouds. An example of practical application is discussed.

1. The equations for turbulent transfer of heat and moisture

The processes of cloud formation are influenced by the transfer of water vapour in the atmosphere as well as by changes in the heat content (temperature) of the air. At a fixed point the variations in moisture and heat content are caused by vertical motion turbulent mixing, horizontal advection as well as by radiation. If the total specific moisture content, i.e. the mass of water vapour, of water drops and ice crystals contained in 1 g of air is denoted by S and maximum specific humidity by S_m, then in real atmospheric conditions the specific water content (i.e. the mass of water drops and ice crystals contained in 1 g of air) of a cloud under formation is equal to the difference:

$$\delta = S - S_m. \quad (1)$$

Mathematically the process of moisture and heat transfer in the atmosphere, resulting in cloud formation, is described by a system of differential equations:

$$\frac{\partial s^*}{\partial t} + u\frac{\partial s^*}{\partial x} + w\frac{\partial s^*}{\partial z} = \frac{\partial}{\partial z} k \frac{\partial s^*}{\partial z} - \frac{m}{\varrho}, \quad (2)$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + w\frac{\partial \theta}{\partial z} = \frac{\partial}{\partial z} k \frac{\partial \theta}{\partial z} + \frac{Lm}{C_p \varrho}, \quad (3)$$

where s^* is the specific humidity of the air; θ the potential temperature; u, w horizontal and vertical wind components; k the turbulent exchange coefficient; ϱ density of the air; C_p specific heat of air under constant pressure; L latent heat of evaporation; m the absolute rate of water vapour condensation, i.e. the mass of water drops and ice crystals formed per cm^3 of air in one second.

The systems (2)–(3) are extremely complicated. SHWETZ (1956) has shown that the direct method of solving this system involves formidable difficulties. In order to overcome these the author has developed a method which essentially simplifies the mathematical part of the problem and at the same time gives a possibility to obtain sufficiently general results.

The physical basis for the method is that cloud elements (water drops and ice) are almost completely taking part in the turbulent motion responsible for the general transfer mechanism.

On the basis of research conducted by UDIN (1946), FUKS (1955), LEVITSCH (1954), EAST & MARSHALL (1954), who developed the main ideas of KOLMOGOROV (1941) and OBUKHOV (1941) on the statistical theory of turbulence, one may obtain a quantitative evaluation of the relative extent to which cloud elements of different radii are carried along in the turbulent motion. The following results,

Droplet radius, μ	Extrainment in per cent
$r \leqslant 40$	100
$r \simeq 100$	65–85
$r \simeq 1000$	4–15

have been found by the author using data on the micropulsation of the vertical velocity and

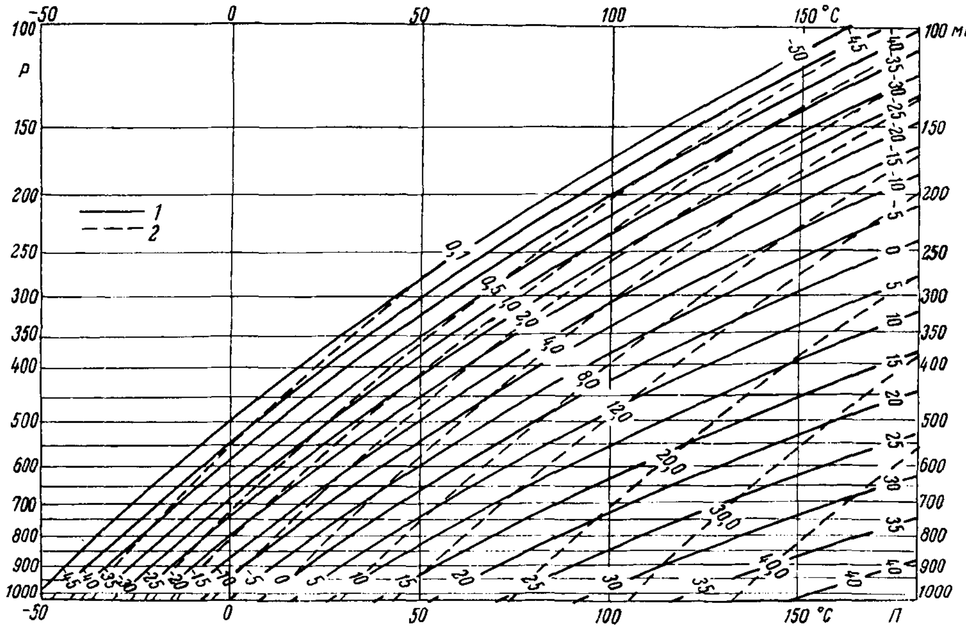

FIG. 1. Nomogram for the computation of specific humidity and temperature within cloud (the scale is about 1:3 as compared to the size of a working form). 1, isotherms, 2, isolines of maximum specific humidity.

of the temperature of the air, collected during flights in clouds of various forms.

Since the radius of cloud elements as a rule does not exceed 40 μ (the maximum frequency of cloud drops is in general found at 7–10 μ) it follows that the specific moisture content, s, in a non-precipitating cloud should approximately satisfy the same differential equation as the specific humidity or

$$\frac{\partial S}{\partial t}+u\frac{\partial S}{\partial x}+w\frac{\partial S}{\partial z}=\frac{\partial}{\partial z}k\frac{\partial S}{\partial z}. \tag{4}$$

A second differential equation is obtained by multiplying eq. (2) by L/Cp and adding the result to eq. (2), or

$$\frac{\partial \prod}{\partial t}+u\frac{\partial \prod}{\partial x}+\frac{\partial \prod}{\partial z}=\frac{\partial}{\partial z}k\frac{\partial \prod}{\partial z}, \tag{5}$$

where $\prod$ denotes the function

$$\prod(x, z, t)=\theta(x, z, t)+\frac{L}{C_p}s^*(x, z, t). \tag{6}$$

The system of differential equations (4)–(5) is simpler than the systems (2)–(3), since the latter is complicated by the quantity m.

Sufficiently general solutions of equations (4) and (5) under given boundary and initial conditions have been obtained by the author (1959), and are given here as known functions f_1 and f_2 of the independent variables x, z, t:

$$S(x, z, t)=f_1(x, z, t), \tag{7}$$

$$\Pi(x, z, t)=f_2(x, z, t). \tag{8}$$

For a cloud with its water vapour in a state of saturation the last equations take the form

$$S_m(\theta, P)+\delta=f_1, \tag{9}$$

$$\theta+\frac{L}{C_p}S_m(\theta, P)=f_2, \tag{10}$$

where S_m is the maximum specific humidity at given temperature and pressure, or

$$S_m=0.622\frac{E(T)}{P}. \tag{11}$$

Equation (10) may be solved with regards to

S_m, and θ (or T). Using the Clausius-Clapeyron formula

$$\ln\frac{E}{E_0} = \frac{L}{AR_n}\left(\frac{1}{T_0} - \frac{1}{T}\right), \qquad (12)$$

(where E_0 is the maximum water vapour pressure at temperature T_0, $AR_n = 0.111$ cal/gm degrees), equation (11) and the definition of θ, we obtain from (10)

$$b\frac{S_{m0}}{S_m} - \frac{a\left(\frac{P_0}{P}\right)^{\frac{\varkappa-1}{z}}}{\ln\frac{S_m}{S_{m0}} + \ln\frac{P}{P_0} - \frac{a}{T_0}} = f_2 \qquad (13)$$

$$\text{or} \quad T\left(\frac{P_0}{P}\right)^{\frac{\varkappa-1}{\varkappa}} + b\cdot\frac{P_0}{P}\exp\left[a\left(\frac{1}{T_0} - \frac{T}{1}\right)\right] = f_2 \quad (14)$$

where $a = L/AR_n$; $b = LS_{m0}/C_p$, $\varkappa = C_p/C_v$ are constants, and S_{m0} is the maximum specific humidity at temperature T_0 and pressure P_0.

The evaluation of the transcendental equations (13) and (14) can be performed with the help of a special nomogram (Fig. 1). The function Π and the logarithm of pressure P are the coordinates, the two families of curves are isolines for maximum specific humidity (S_m = const) and for temperature (T = const).

If S_m and T are determined from known Π and P with the help of the nomogram, then by using (9) we may find (knowing $s = f_1$) the most important variable in the problem considered—the specific water content of the cloud, δ.

Computation with the help of the nomogram is performed for all levels of positive water content. The upper and the lower cloud boundaries are determined by constructing a curve for the variation of the water content with altitude (pressure). The points where the water content of a cloud becomes zero are

$$\delta(x, z, t) = f_1(x, z, t) - S_m(x, z, t) = 0 \qquad (15)$$

and give the lower and upper boundaries for a newly formed cloud.

For levels outside the cloud the potential temperature is computed with the use of equation

$$\theta(x, z, t) = f_1(x, z, t) - \frac{L}{C_p} f_1(x, z, t), \qquad (16)$$

Since here $\delta = 0$; $s^* = S = f_1$; $\prod = \theta + L/Cp\,S$.

Having made the computations for several time intervals (e.g. for 6, 12, 18, 24 hours after initial time), we may follow the evolution of the water content, and of the humidity and temperature as well as the variation of the cloud boundaries with time at various points.

Another method to compute the temperature with the aid of the known function Π is more useful, if the integration of (5) is made on electronic computers. If $\Delta\Pi_i = \Pi_i - \Pi_{i-1}$, i denoting, the number of time steps, then the cloud temperature increment $\Delta T_i = T_i - T_{i-1}$ at pressure P is expressed by:

$$\Delta\prod_i = \left\{\left(\frac{1000}{P}\right)^{\frac{\varkappa-1}{\varkappa}} + 0.622\frac{L^2}{C_pP}AR_n\cdot T_{i-1}^2\right\}\Delta T_i,$$

where E_{i-1} is the saturation water vapour pressure at the temperature T_{i-1}. Calculating ΔT_i with the aid of this relation the cloud temperature at time-step i can be directly determined.

2. The exchange coefficient

The most important quantities affecting the process of heat and moisture transfer are the vertical velocity (w) and the coefficient (k). These values also determine the process of cloud formation. The value of the vertical velocity for the scale of motion considered here (horizontally of the order of hundreds and thousands of km) varies from fractions of cm/sec to several cm/sec. Data on the values of the exchange coefficient in cloud as well as in the free atmosphere in general (above the boundary layer) were not available until recently. We shall here give experimental data obtained from aircraft accelerometer measurements (see MATVEJEV, 1958*a* and *b*) in several flight expeditions conducted with the participation of the author in various parts of the country including arctic regions.

Table 1 gives mean values of k in various types of cloud (in temperate latitudes and in the Arctic), Tables 2 and 3 give the frequency of these values.

For temperate latitudes the value of k in clouds does on the average considerably exceed the value found for cloudfree air as can be seen from Table 2.

In the Arctic where the flights were conducted mostly within subinversion clouds no essential

TABLE 1. *Mean values of k (m^2/sec) in clouds of various forms.*

Temperate latitudes	As	Ns	Ac	Sc	Cu–Cb
(winter)	27	56	55	59	66
The Arctic	St	Sc	Cu hum		
(summer)	26	39	33		

difference was observed between the exchange coefficient in clouds and out of clouds.

From dimensional analysis and similarity theory the author has derived the following formula for the exchange coefficient

$$k = \frac{u^2}{\beta} f(\mathrm{Ri}),$$

where u is the wind velocity; β the vertical shear and Ri the Richardson's number.

The functional form of $f(\mathrm{Ri})$ has been determined from data obtained in carefully conducted experiments using modern airplanes. During the flights vertical accelerations were registered and synchronous wind and temperature soundings made of the atmosphere. Having the values k, u, β and Ri it is found that the data may be satisfactorily described by a function of the form

$$f(\mathrm{Ri}) = a_1 - b_1 \lg \mathrm{Ri},$$

where a_1 and β_1 are constants. The values of these have been determined by the least square method for a series of flights made in summer over the European part of the U.S.S.R. giving $a_1 = 17.9$ and $b_1 = 11.6$.

3. Some computational results

Let us now consider some particular cases where the values of the coefficients as well as the initial and boundary conditions are varied in order to demonstrate the effect of various factors in the cloud formation process.

As a first case the specific humidity and the temperature be known at the ground

$$z = 0: S = S_1(t), \ T = T_1(t), \tag{17}$$

and at the tropopause

$$z = H: S = S_2(t), \ T = T_2(t). \tag{18}$$

At initial time the temperature is assumed

TABLE 2. *Frequency (in per cent) of different values of the exchange coefficient (temperate latitudes, winter).*

n = number of occasions.

Variation interval k(m^2/sec)		≤ 10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	≥ 80
In clouds	%	2.1	—	2.1	13.0	28.4	26.2	13.0	6.5	8.7
	n	1	—	1	6	13	12	6	3	4
In cloud-free air	%	—	53.9	15.4	9.6	7.7	1.9	1.9	5.8	3.8
	n	—	28	8	5	4	1	1	3	2

TABLE 3. *Frequency (in per cent) of different values of the exchange coefficient (the Arctic, summer).*

n = number of occasions.

Variation interval k (m^2/sec)		≤ 10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
Below cloud	%	1.0	—	—	12.6	25.2	18.5	23.3	11.8	5.8	1.9
	n	1	—	—	13	26	19	24	12	6	2
In cloud	%	1.3	1.3	9.0	19.2	33.2	14.1	15.4	2.6	2.6	1.3
	n	1	1	7	15	26	11	12	2	2	1
Above cloud	%	—	—	—	15.9	36.4	15.9	13.6	9.1	6.8	2.3
	n	—	—	—	7	16	7	6	4	3	1

to decrease linearly with height $T(z, 0) = T - \gamma z$, and the humidity to decrease exponentially as

$$S(z, 0) = \varphi_1(z) = S_0 \exp(-\alpha z). \qquad (19)$$

The solution to eq. (4) under the conditions (17)-(19) is obtained as

$$s(z, t) = \frac{2}{H} \exp\left(\frac{w}{2k} z - \frac{w^2}{4k} t\right) \sum_{n=1}^{\infty} \exp\left(-\frac{k\pi^2 n^2}{H^2} t\right)$$

$$\times \sin \frac{\pi n}{H} z \left\{ \int_0^H \exp\left(-\frac{w}{2k} z'\right) \cdot \varphi_1(z') \cdot \sin \frac{\pi n}{H} z' dz' \right.$$

$$+ \frac{k\pi n}{H} \int_0^t \left[\exp\left(\frac{w^2}{4k} \tau\right) s_1(\tau) + (-1)^{n-1} \right.$$

$$\left. \times \exp\left(\frac{w^2}{4k} \tau - \frac{w^2}{2k} H\right) \cdot s_2(\tau) \right] \exp\left(\frac{k\pi^2 n^2}{H^2} \tau\right) d\tau. \qquad (20)$$

If S_1 bnd S_2 are constants, then (20) in transformed into

$$S(z, t) = S_1 - \frac{S_1 - S_2}{r - 1} (r^{\frac{z}{H}} - 1) + 2S_1 \exp\left(\frac{w}{2k} z - \frac{w^2}{4k} t\right)$$

$$\times \sum_{n=1}^{\infty} C_n \exp\left(-\frac{k\pi^2 n^2}{H^2} t\right) \sin \frac{\pi n}{H} z, \qquad (21)$$

where $$C_n = \pi n \left[\frac{1}{\left(\frac{wH}{2k} + \alpha H\right)^2 + \pi^2 n^2} - \frac{1}{b_n} \right]$$

$$\cdot \left[1 + (-1)^{n-1} \frac{S_2}{\sqrt{r} \cdot S_1} \right],$$

$$r = \exp\left(\frac{w}{k} H\right); \quad b_n = \frac{w^2 H^2}{4k^2} + \pi^2 n^2.$$

From (21) it follows that the stationary distribution of specific moisture content with altitude is described by the following formula

$$S_{st}(z) = S_1 - \frac{S_1 - S_2}{r - 1} (r^{\frac{z}{H}} - 1). \qquad (22)$$

Let us now consider a more general case when the specific humidity and the temperature vary with time at the boundaries. Integration by parts transforms (21) into

$$S(z, t) = S_1(t) - \frac{S_1(t) - S_2(t)}{r - 1} (r^{\frac{z}{H}} - 1) \Big)$$

$$+ 2 \exp\left(\frac{w}{2k} z - \frac{w^2}{4k} t\right) \sum_{n=1}^{\infty} \exp\left(-\frac{k\pi^2 n^2}{H^2} t\right)$$

$$\times \sin \frac{\pi n}{H} z \cdot \left\{ S_1(0) \cdot C_n - \frac{\pi n}{bn} \int_0^t \exp\left(\frac{k b_n}{H^2} \tau\right) \right.$$

$$\left. \left[a_1 + (-1)^{n-1} a_2 \exp\left(-\frac{w}{2k} H\right) \right] d\tau \right\}, \qquad (23)$$

where $a_1 = ds_1/dt$, $a_2 = ds_2/dt$ is the rate of change of values S_1 and S_2 with time. The constants z, b_n and C_n have the same meaning as above (in the expression for C_n the values S_1 and S_2 are equal to $S_1(0)$ and $S_2(0)$ respectively).

If we approximate the variations of the specific humidities S_1 and S_2 by linear functions of time (such an assumption is sufficiently satisfactory for aperiodic processes of an advective type), the values a_1 and a_2 can be regarded as constants, approximately, and the time integration on the right-hand side of (23) is considerably simplified since the series contained in (23) converges rapidly due to the multiplier

$$\exp\left(-\frac{k\pi^2 n^2}{H^2} t\right).$$

A similar formula is also obtained for the function $\Pi(x, z, t)$ substituting Π_1 and Π_2 on the right-hand side of (23) for S_1 and S_2 respectively.

In order to determine the importance of various factors in the formation of clouds, s and Π have been computed from (23) and the corresponding equation for Π, using values obtained from a sounding in Moscow, July 14, 1953, 1730 h as initial data for s and Π. The initial vertical distribution of the specific humidity has been approximated by an exponential function while the initial vertical distribution of the temperature is approximated by a linear function. The time variation of temperature and specific humidity at the upper and lower boundaries (1 and 11 km) are described by linear functions. The time integration was carried out over the periods 14, 22.4 and 36.4 hours. For k the value 50 m²/sec was taken for w two different values, 1 and 2 cm/sec.

Initial data and the results are given in Table

TABLE 4. *Distribution of cloud temperature and cloud water content after 36.4 hours.*

		1	2	3	4	5	6	7	8	9	10	11
	z, km	1	2	3	4	5	6	7	8	9	10	11
	T initial, °C	14.4	8.0	2.4	−2.3	−7.4	−13.5	−20.2	−27.4	−34.3	−42.6	−50.3
	ς initial g/kg	5.0	3.4	2.6	1.8	1.2	0.8	0.5	0.3	0.2	0.1	0.0
Temperature T, °C												
$w=0$		12.1	8.5	2.6	−3.3	−8.7	−14.3	−20.8	−26.0	−31.8	−37.5	−42.0
$w=1$ cm/sec	Without regarding condensation	12.1	6.6	0.0	−6.5	−12.4	−18.9	−25.0	−30.2	−34.5	−38.2	−42.0
	With regard for condensasion	12.1	6.6	2.6	−4.7	−9.9	−16.5	−22.5	−27.7	−34.0	−38.2	−42.0
$w=2$ cm/sec	Without regarding condensation	12.1	5.5	−2.5	−9.0	−16.0	−23.3	−29.4	−34.8	−39.6	−43.0	−42.0
	With regard for condensation	12.1	8.2	−1.3	−6.7	−12.1	−18.9	−26.9	−31.8	−37.9	−41.4	−42.0
Specific water content g/kg												
$w=1$ cm/sec		—	—	0.1	0.6	0.4	0.4	0.3	0.2	0.1	—	—
$w=2$ cm/sec		—	0.1	0.8	1.5	1.8	1.6	1.3	1.1	0.7	0.3	—
Absolute water content g/m³												
	$w=1$ cm/sec	—	—	0.09	0.51	0.30	0.27	0.18	0.11	0.05	—	—
	$w=2$ cm/sec	—	0.10	0.76	1.27	1.35	1.07	0.78	0.58	0.32	0.12	—

4 and in Figs. 2 and 3. These are rather self-explanatory but a few characteristic features may be pointed out. It is seen from Table 4 that the turbulent transfer of heat, the vertical velocity and the release of latent heat are all important factors in determining the final state and that none of these can be neglected.

Comparing the curves in Fig. 2 it is seen that an increase in vertical velocity from 1 cm/sec to 2 cm/sec results—as could be expected—in a considerable charge in the cloud water content and in the position of the lower and upper boundaries of the cloud.

Fig. 3 shows that the formation of clouds in the upper and middle layers precedes the formation of low level clouds, and also that the maximum water content is at all times found close to the cloud base.

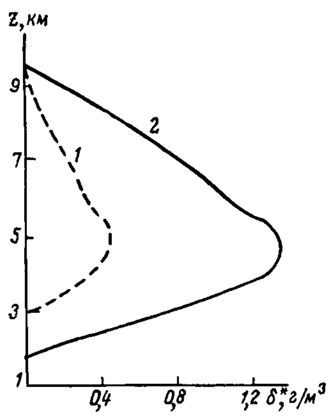

FIG. 2. Distribution of absolute water content with altitude in 36.4 hrs after the initial moment with $w=1$ cm/sec (1) and with $w=2$ cm/sec (2).

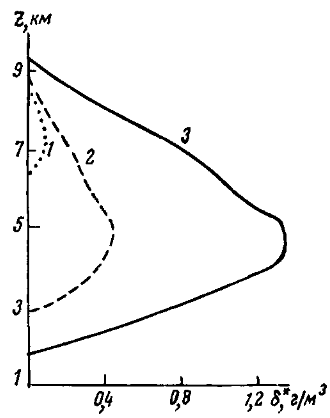

FIG. 3. Distribution of absolute water content with altitude with $w=2$ cm/sec in 14 hrs (1), in 22.4 hrs (2) and in 36.4 hrs (3).

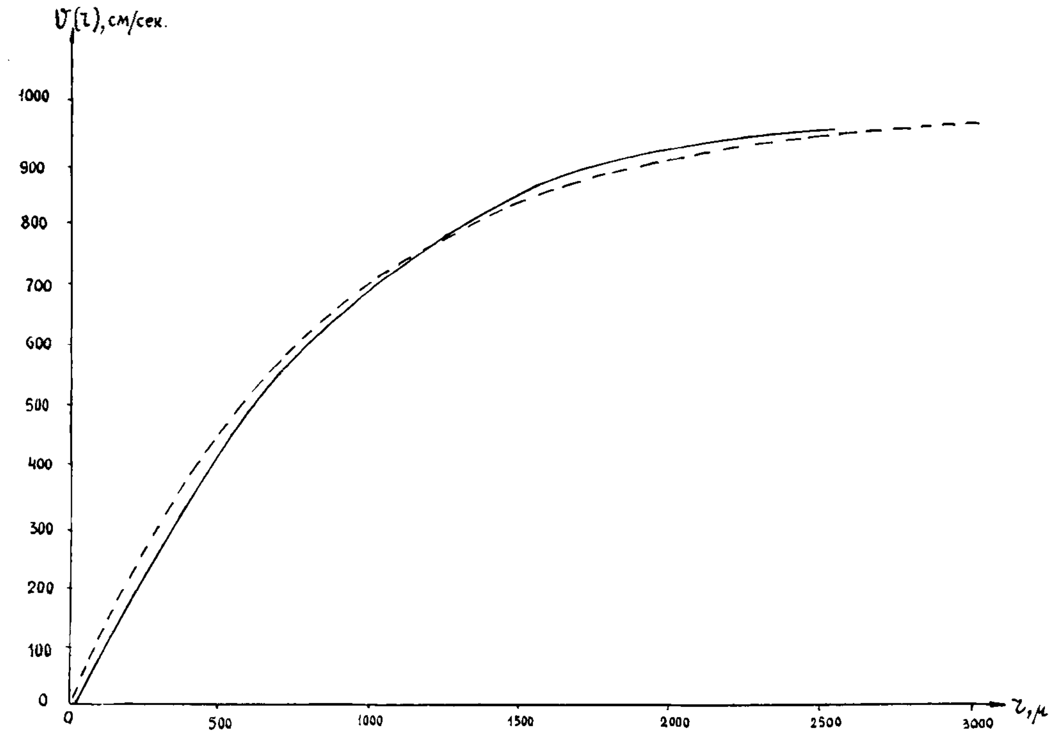

FIG. 4. Velocity of fall of water droplets. —— experimental, --- theoretical.

4. The calculation of the fall of cloud elements influenced by gravity

When the cloud elements become so large that the cloud precipitates the moisture transfer equation may be written

$$\frac{\partial S}{\partial t} + u\frac{\partial S}{\partial x} + w\frac{\partial S}{\partial z} = \frac{\partial}{\partial z}k\frac{\partial S}{\partial z} - \frac{1}{\varrho}\frac{\partial Q_k}{\partial z}, \qquad (24)$$

where Q_k is the flux of the water drops and ice crystals (g/cm² sec) in the gravitational field. If $v(r)$ is the fall velocity of a drop with radius r and $f(r)$ the estimated drop size distribution then in the interval r to $r+dr$ the flow of water per sec and cm² is

$$n \cdot f(r)\,dr\tfrac{4}{3}\pi\varrho_k r^3 \cdot v(r),$$

where n is the total number of drops per cm³, ϱ_k the drop density. Summing over all radii, we have

$$Q_k = -\tfrac{4}{3}\pi n\varrho_k \int_0^\infty r^3 f(r)\,v(r)\,dr, \qquad (25)$$

where downward flow is considered negative. On the other hand the following relation holds for the liquid water content

$$\delta = S - S_m = \tfrac{4}{3}\pi\varrho_k \frac{n}{\varrho}\int_0^\infty r^3 f(r)\,dr, \qquad (26)$$

where n/ϱ is the number of drops per 1 g of air. From (24) and (25) it follows

$$Q_k = -\varrho(S - S_m)\cdot\tilde{v}, \qquad (27)$$

where $\tilde{v}$ is a weighted average velocity of falling drops, or

$$\tilde{v} = \frac{\int_0^\infty r^3 f(r)\,v(r)\,dr}{\int_0^\infty r^3 f(r)\,dr} \qquad (28)$$

Solving (27), eq. (24) takes the following from

$$\frac{\partial S}{\partial t} + u\frac{\partial S}{\partial x} + w\frac{\partial S}{\partial z} = \frac{\partial}{\partial z}k\frac{\partial S}{\partial z} + \frac{1}{\varrho}\frac{\partial \varrho\tilde{v}(S - S_m)}{\partial z}.$$

From analysis of experimental data GUNN & KINZER (1949) it is found that the fall velocity may be approximated with good accuracy by:

$$v(r) = v_\infty[1 - \exp(-cr)] \tag{29}$$

The constants v_∞ and c have been determined by the method of least squares with the result

$$v_\infty = 995 \text{ cm sec}^{-1} \quad c = 12 \cdot 10^{-4}\ \mu.$$

Fig. 4 shows the agreement between computed data and those given by GUNN & KINZER (1949), It is seen that the computed fall velocity is somewhat too high at small r and somewhat too low at large r. In calculating the weighted average velocity ($\tilde{v}$) these errors will, however, tend to compensate. With regard to the drop size distribution function, $f(r)$ occurring in (28) it is known that experimental data are in good agreement with the normal-logarithmic formula by A. A. Kolmogoroff. However, since the use of this formula would entail considerable numerical computations, the following form for $f(r)$ has been used

$$f(r) = \frac{\beta^{\beta+1}}{r_m^{\beta+1}\Gamma(\beta+1)} \cdot r^\beta \exp \cdot \left(-\frac{\beta r}{r_m}\right) \tag{30}$$

where r_m is the drop radius where $f(r)$ is maximum, β is a parameter to be determined empirically and $\Gamma(\beta+1)$ is the gamma function. This frequency function is known as the gamma-distribution formula (DJATCHENCO, 1952). With $\beta = 2$ eq. (30) becomes

$$f(r) = \frac{4r^2}{r_m^2} \exp\left(-\frac{2r}{r_m}\right). \tag{31}$$

which according to KHRGIAN & MASIN (1952) is in reasonable agreement with experimental data.

Using the expressions (29) and (31) in (28) we obtain after integration

$$\tilde{v} = v_\infty\left[1 - \frac{64}{(2 + cr_m)^6}\right]. \tag{32}$$

For different values of r_m this equation gives the following values of $\tilde{v}$

r_m, μ	5	10	15	20	50
$\tilde{v}$, cm/sec	18	34	52	72	161

r_m, μ	100	200	5000	1000
$\tilde{v}$, cm/sec	292	480	790	935

It should here be noted that r_m depends on the liquid-water content of the cloud and on the intensity of the precipitations.

The method presented in this article has been developed in order to make forecasts on the formation and evolution of layer clouds where the horizontal dimension considerably exceeds the vertical. It seems, however, possible that the method could be applied to convective clouds, if in equations (2) and (3) terms relating to horizontal turbulent mixing are included.

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