On the mean meridional circulation in the presence of a steady state, symmetric, circumpolar vortex

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ABSTRACT

The steady state equations governing the "forced" (in the sense of KUO, 1956) mean meridional circulation in the presence of a circular vortex are presented, and it is shown that a constraining relation must exist between the sources of heat and momentum (eddy convergences, diabatic heating, and friction) and the parameters of the zonal mean state (zonal vertical wind shear, meridional temperature gradient, absolute vorticity, and static stability).

In the case of general heat and momentum forcing, solutions for vertical motion ω are obtained explicitly. These solutions involve integrals along the characteristics of the equations, which are isolines of potential temperature and absolute angular momentum.

In the special case of no heat or momentum forcing present, it is shown that a necessary but not sufficient condition for the existence of a "free" mean meridional circulation is that $f = Z$ Ri (called resonance, after KUO 1956) where Z is the absolute vorticity and Ri is the Richardson number. Equivalently, the characteristics of angular momentum and potential temperature must coincide. It is shown by means of the characteristics that the existence of a region in which the absolute vorticity, measured on an isobaric surface, is negative, may be a sufficient condition. The necessary condition implies that the static stability is also negative in such a region. The conclusion that such free circulations in the atmosphere cannot exist in the mean state agrees with Kuo, but the criterion is somewhat different.

In the general forcing case, a possible constraining relationship is derived, and its usefulness considered. Simpler formulae are presented which hold along lines on which one of the components of the meridional circulation vanishes. By considering also the ratio of the components of the meridional circulation along lines on which either the momentum or the heat source functions vanish, a consistency picture between the positioning of the mean meridional cells and the sources and sinks of momentum and heat is developed. In particular, it is shown that lines of vanishing heat or momentum source cannot, in general, be in phase. Such lines must cross at the "center" of each cell (where both components of the circulation are zero). The lines of zero momentum source must slope generally upward toward the equator, in general agreement with observation. For the constraints to be consistent with the observed counter-gradient transport of heat in the lower stratosphere, the lowest layer of cells must extend into the lower stratosphere. Equatorward counter-gradient transport of heat in the troposphere in very low and very high latitudes is also suggested.

The implications for observational and numerical model studies of the general circulation are briefly discussed.

1. Introduction

In diagnostic studies of the general circulation, it is usually very useful to assume that the atmosphere possesses a long term steady state flow pattern. All partial derivatives with respect to time in the time averaged equations of motion may then be put equal to zero. In such a case, we can speak of the mean flow patterns as being "forced" by the eddy convergences of heat and momentum, plus all other heat and momentum sources, such as radiation, condensation, and friction. Further, the mean flow patterns can be broken into symmetric and asymmetric parts—the zonally averaged flow and the deviation from it. The symmetric flow is then considered to be forced by the zonally averaged part of the momentum and heat sources, while the asymmetric mean flow is forced by the asymmetric momentum and heat sources. The question of the symmetric circulation has been treated theoretically principally by ELIASSEN (1952), and KUO (1956). The asymmetric circulation has been treated primarily by CHARNEY & ELIASSEN (1949) and SMAGORINSKY (1953), and more recently by SALTZMAN (1961, 1962, 1963). This paper will be concerned only with the symmetric flow.

Eliassen considered the problem of the types of mean meridional circulation which could arise in the presence of a symmetric balanced vortex due to point sources of momentum and heat. Kuo showed that for the earth's atmosphere, the mean meridional circulations that do arise *must* be forced by the sources of heat and momentum. That is, the resonance condition for a free mean meridional circulation can never be reached. This paper will show that for the steady state case, Kuo's proof is not strictly correct, if friction and heat conduction are also considered as forcing terms. In addition, several diagnostic conditions between the heat and momentum sources and the parameters of the mean state (such as vertical wind shear, meridional temperature gradient, etc.) will be derived. Finally, a picture of the consistency conditions required between the mean meridional motion and the heat and momentum sources will be put forth.

2. Symbols

The following, in most cases conventional, symbols will be used. Other, less conventional ones will be defined as they appear in the text.

p = pressure, in mb
ϕ = latitude (from 0° to +90°)
λ = longitude
u = zonal wind (positive toward east)
v = meridional wind (positive toward north)
$\omega = dp/dt$
Ω = angular velocity of the earth
f = Coriolis parameter $= 2\Omega \sin \varphi$
g = acceleration of gravity
a = radius of the earth
χ = frictional force/unit mass
$Z = f - (a \cos \phi)^{-1}(\partial/\partial\phi)[\bar{u}] \cos \phi$ = mean zonal vorticity = dynamic stability
ϱ = density
T = temperature
θ = potential temperature
C_p = specific heat of air at constant pressure
Γ = static stability $= -T\,(\partial/\partial p) ln\,\Theta$
Φ = geopotential

$\overline{(\quad)} = \dfrac{1}{t_2 - t_1}\displaystyle\int_{t_1}^{t_2} (\quad)\,d\xi$ = time average

$(\quad)' = (\quad) - \overline{(\quad)}$ = deviation from time average

$[(\quad)] = \dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} (\quad)\,d\lambda$ = zonal average

$(\quad)^* = (\quad) - [(\quad)]$ = deviation from zonal average

3. The equations

A complete derivation of the time averaged equations will not be given. The reader is referred to SALTZMAN (1961) for details. When time and zonal averages are applied, the steady state zonal equation of motion, the continuity equation, and the thermodynamic equation, written in spherical coordinates assuming hydrostatic equilibrium, become, respectively,

$$\omega \frac{\partial u}{\partial p} - Zv = G, \tag{1}$$

$$(a \cos \phi)^{-1} \frac{\partial}{\partial \varphi} v \cos \phi + \frac{\partial \omega}{\partial p} = 0, \tag{2}$$

$$v \frac{\partial T}{a \partial \phi} - \Gamma \omega = H. \tag{3}$$

Here the $[\overline{(\quad)}]$ operator has been dropped from u, v, ω, T, for simplicity. G represents all the momentum sources, and H all the heat sources. That is,

$$G = -(a \cos^2 \phi)^{-1} \frac{\partial}{\partial \phi} (\overline{[u'v']} + [\bar{u}^* \bar{v}^*]) \cos^2 \phi - \frac{\partial}{\partial p} (\overline{[u'\omega']} + [\bar{u}^* \bar{\omega}^*]) + [\bar{\chi}]$$

and

$$H = \frac{[\bar{Q}]}{C_p} - (a \cos \phi)^{-1} \frac{\partial}{\partial \phi} (\overline{[v'T']} + [\bar{v}^* \bar{T}^*]) \cos \phi - \frac{\partial}{\partial p} (\overline{[\omega'T']} + [\bar{\omega}^* \bar{T}^*]) + \frac{R}{C_p p} (\overline{[\omega'T']} + [\bar{\omega}^* \bar{T}^*]),$$

where χ is the frictional force per unit mass, and Q the diabatic heating, including thermal conduction and frictional heating. A positive G or H thus means a source of momentum or heat.

It is immediately evident from equations (1) to (3) that if we consider only the mean meridional circulation, represented by v and ω, as the dependent variable, there must exist a constraining relationship between the eddies and the parameters of the mean state so that all three equations may hold simultaneously. If the zonal wind is assumed to be in approximate geostrophic balance, i.e.

$$\left(f + \frac{\delta u}{a} \tan \phi\right) \frac{\partial u}{\partial p} = \frac{R}{pa} \frac{\partial T}{\partial \phi} \tag{4}$$

then we have a complete set of equations, and we may solve for v, ω and the constraints.

It is instructive to consider all the equations for one component of the meridional circulation, say ω, possible from (1) to (3). There are three, though of course only two are independent. They are

$$\frac{\partial \omega}{\partial p} + r \frac{\partial \omega}{\partial \phi} + L\omega = M, \tag{5}$$

$$\frac{\partial \omega}{\partial p} + s \frac{\partial \omega}{\partial \phi} + J\omega = K, \tag{6}$$

$$\left(\frac{\partial T}{\partial \phi} \frac{\partial u}{\partial p} - a\mathbf{Z}\Gamma\right) \omega = a\mathbf{Z}H + \frac{\partial T}{\partial \phi} G, \tag{7}$$

where
$$r = \frac{\Gamma}{\dfrac{\partial T}{\partial \phi}}; \quad s = \frac{1}{a\mathbf{Z}} \frac{\partial u}{\partial p}, \tag{8}$$

$$L, J = (\cos \phi)^{-1} \frac{\partial}{\partial \phi} (r, s) \cos \phi, \tag{9}$$

$$M, K = (\cos \phi)^{-1} \frac{\partial}{\partial \phi} (H, -G) \cos \phi. \tag{10}$$

r and s, then, are the slopes of the characteristic curves of (5) and (6) respectively. It is simple to show, then, that (5) has isotherms of potential temperature as characteristics, while those of (6) are isolines of absolute angular momentum. With the slopes as given above, a latitude wall has a slope of zero, and an isobar has a slope of $\pm \infty$. It is evident from (5) to (7) that if we know G and H, it is easiest to find ω from (7), and find v via continuity. If we know only G (or H), we may find ω from (6) (or 5) and solve for H (or G) from (7). To solve for ω, we may write

$$\frac{d}{dp} = \frac{\partial}{\partial p} + r, s \frac{\partial}{\partial \phi},$$

where d/dp is the derivative with respect to p taken along the characteristic. Then the solutions to (5) and (6) are (easily found by variation of parameters),

$$\omega = \omega_0 \exp\left(-\int L, J dp'\right) + \exp\left(-\int L, J dp'\right) \times \int M, K \exp\left(\int L, J dp''\right) dp'. \tag{11}$$

Generally speaking, we can measure more of G more easily than we can H. In addition, in (6) the characteristics can be said to start at the "top" of the atmosphere, so that in the solution of (6) we may set $\omega_0 = 0$. Also, the characteristics of (6) are very nearly lines of constant latitude. Therefore, it is desirable to use the solution of (6) for ω. In that case, we have,

$$\omega = \exp\left(-\int_0^P J dp'\right) \int_0^P \exp\left(\int_0^{P'} J dp''\right) K dp'. \tag{12}$$

Actual calculations of the mean meridional circulation in the southern hemisphere using the above formula and the heat sources inferred by it from (7) will be presented in a later paper. We will not, unfortunately, be able to infer the various parts of H.

Finally, to find the constraints on the mean state parameters, we may substitute for ω from (7) into (6) or (5). This will be done in section 5.

4. The case of no momentum or heat sources

Let us consider first the idealized case when there are no sources of heat or momentum in the system ($G \equiv 0$, $H \equiv 0$). Under what conditions is a meridional circulation possible? From (7), and solving (1) and (3) for v, we have

$$\left(\frac{\partial T}{\partial \phi} \frac{\partial u}{\partial p} - a\mathbf{Z}\Gamma\right) \omega, v = 0. \tag{13}$$

Obviously, then, a *necessary* condition for the existence of this "free" meridional circulation is that

$$\frac{\partial T}{\partial \phi} \frac{\partial u}{\partial p} - a\mathbf{Z}\Gamma = 0. \tag{14}$$

Or in terms of a Richardson number, assuming the thermal wind relation (4) holds (and neglecting the $(\delta u/a)\tan\phi$ term), we have

$$f = Z \, \mathrm{Ri}. \tag{15}$$

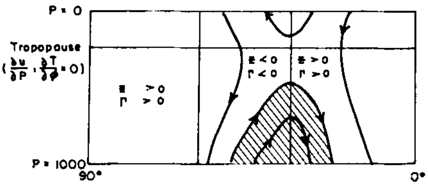

FIG. 1. Schematic drawing of characteristic curves (lines with arrows). Arrows indicate possible direction of integration. A free meridional circulation could occur within shaded area.

Equation (14) implies $r = s$, and (5) and (6) become identical. Therefore a necessary condition for the existence of a mean meridional circulation in the absence of any heat or momentum sources is that the potential temperature isotherms coincide in some region with the angular momentum isolines. That this is *not sufficient* can be seen from (5) or (6). They both say that

$$\frac{d}{dp}\left(\omega \exp\left(\int J dp\right)\right) = 0 \tag{16}$$

or that $\omega \exp(\int J dp)$ is conserved along a characteristic. Now from (15) it is evident that when $Z > 0$ we must have $\Gamma > 0$, and when $Z < 0$ we must have $\Gamma < 0$. Then for the case $Z > 0$ everywhere (implying $\Gamma > 0$ everywhere), the integral $\int J dp$ is everywhere bounded, and we can fill the meridional cross-section with characteristics which reach the top of the atmosphere ($p = 0$). Since we must require that $\omega = 0$ at $p = 0$, $\omega = 0$ everywhere, and therefore there is no meridional circulation. If, however, $Z < 0$ in some region (implying $\Gamma < 0$ in that region), we can show that some characteristics never reach the line $p = 0$, and therefore, since $p = 0$ is the only line along which we may definitely say $\omega = 0$, we cannot guarantee the non-existence of a mean meridional circulation. Fig. 1 shows an example of such a case. Strictly speaking, of course, our equations do not apply to the case of $\Gamma < 0$, since then the hydrostatic assumption is no longer valid. (The same objection applies to Kuo's equations.)

Now, Kuo's criterion for the existence of a free mean meridional circulation was

$$\mathrm{Ri} \leqslant \frac{f}{\mathbf{Z}} \tag{17}$$

if we consider friction and thermal conduction as forcing terms. But from (15) it is evident that only the equality sign can hold. In this case, Kuo would obtain a parabolic rather than a hyperbolic equation for the meridional stream function and with a closed region such as the meridional cross section, the problem would be overdetermined. It is evident, then, that the actual conditions for the existence of a free mean meridional circulation are $\mathrm{Ri} = f/Z$, and $Z < 0$ over some finite region (implying $\Gamma < 0$ in that region), where Z is measured on an *isobaric* surface. It is clear that such free mean meridional circulations cannot in fact exist, since even if at some time during the period of averaging $\Gamma < 0$ and $Z < 0$ in some region, the atmosphere would have responded to the convective and dynamic instabilities with motion at least until $\Gamma > 0$ and $Z > 0$ again. In the mean state, then, with or without momentum and heat sources, the atmosphere can never reach "resonance",[1] given by $\mathrm{Ri} = f/Z$, over a finite region.

5. Momentum and heat sources present

The constraining relationship between the momentum and heat sources and the parameters of the mean state may be found most easily by substituting for ω from (7) into (6). Rewriting (7) in terms of the Richardson number, we have

$$\omega = \frac{\mathrm{Ri}}{\mathrm{Ri} - 1} F, \tag{18}$$

where $$F = \frac{H}{\Gamma} + \frac{G}{a\mathbf{Z}r}.$$

Letting $\beta = \mathrm{Ri}/(\mathrm{Ri} - 1)$, substitution into (6) gives

$$\frac{\partial}{\partial p}\beta F + s\frac{\partial}{\partial \phi}\beta F + \beta F J = K \tag{19}$$

the solution of which is

$$\beta F = (\beta F)_0 \exp\left(-\int J dp'\right)$$
$$+ \exp\left(-\int J dp'\right)\int K \exp\left(\int J dp''\right) dp'. \tag{20}$$

[1] Here we mean resonance in the general sense of an infinite response of the meridional circulation to any forcing, which would occur if $\mathrm{Ri} = f/Z$.

Now, except when $\partial u/\partial p$ is very large (just above the jet stream) or when Z is very small (very close to the equator), the factors $\exp(\pm \int J dp) \approx 1$. (In any case, these factors in the second term on the right of (20) tend to cancel.) We then have

$$\frac{H}{\Gamma} + \frac{G}{aZr} = \frac{\mathrm{Ri}_0}{\mathrm{Ri}} \frac{\mathrm{Ri}-1}{\mathrm{Ri}_0 - 1}\left(\frac{H_0}{\Gamma_0} + \frac{G_0}{a\mathbf{Z}_0 r_0}\right) + \frac{\mathrm{Ri}-1}{\mathrm{Ri}}\int_{p_0}^{p} K dp'. \quad (21)$$

In this case, it is most convenient to begin the integration at the ground. Very near the tropopause, (21) simplifies still further, since $\mathrm{Ri}/(\mathrm{Ri}-1) \to 1$ as $\mathrm{Ri} \to \infty (r \to -\infty)$. In that case

$$\frac{H_t}{\Gamma_t} = \frac{\mathrm{Ri}_0}{\mathrm{Ri}_0 - 1}\left(\frac{H_0}{\Gamma_0} + \frac{G_0}{a\mathbf{Z}_0 r_0}\right) + \int_{p_0}^{p_t} K dp, \quad (22)$$

where the subscript t refers to the tropopause level.

Equation (21) is probably most useful if knowing all the relevant parameters at the ground, and knowing the momentum sources and the parameters of the mean state elsewhere, we wish to infer the heat sources. An analogous formula would be derived from (7) and (5) for inferring the momentum sources, though to use numerically it would require more work, due to the less simple paths of the potential temperature characteristics.

Equation (21) gives the constraints in the general case, but much simpler formulae may be derived for particular regions in the atmosphere. Near a line along which $v = 0$, eliminating ω from (1) and (3) gives

$$H\frac{\partial u}{\partial p} = -\Gamma G. \quad (23)$$

This formula would hold between mean meridional cells, and along an approximately horizontal line through their centers (defined as the point where $\omega = v = 0$). Equation (23) then implies that along such lines, in the troposphere, there must be sources of both momentum and heat, or sinks of both. In the stratosphere (except near the polar night jet), there must be either a momentum source and a heat sink, or a momentum sink and a heat source.

Near a line along which $\omega = 0$, (7) yields the relation

$$H\mathbf{Z} = -\frac{1}{a}\frac{\partial T}{\partial \phi} G. \quad (24)$$

This formula would then hold along approximately vertical lines through the center of the mean meridional cells, and along a horizontal line between different layers of cells, if more than one layer exists. It is evident, then, that also along $\omega = 0$ lines in the troposphere there must be sources of both heat and momentum, or sinks of both, and in the stratosphere, either a heat sink and a momentum source, or a heat source and a momentum sink. Equations (23) and (24) then together say that in any region bounded by $v = 0$ and $\omega = 0$ lines, there must either be line of zero heat source *and* a line of zero momentum source, or no such zero lines.[1]

It is also evident from (23) and (24) that $H = 0$ and $G = 0$ lines may cross only where $v = 0$ and $\omega = 0$. The only possible points in the interior of the cross-section are at the centers of meridional cells, and at the stagnation points between layers of cells, if more than one layer exists.

It is possible to find more exactly the relative positions of the meridional cells and the heat and momentum sources by considering the ratio of v to ω along lines where G or H vanish. Along a line on which $H = 0$

$$\left(\frac{v}{\omega}\right)_{H=0} = r. \quad (25)$$

Along a line $G = 0$,

$$\left(\frac{v}{\omega}\right)_{G=0} = s \quad (26)$$

the ratio of the ratios is given by

$$\frac{\left(\frac{v}{\omega}\right)_{H=0}}{\left(\frac{v}{\omega}\right)_{G=0}} = \frac{\mathbf{Z}\mathrm{Ri}}{f} {}^{(2)}. \quad (27)$$

[1] The above conclusion must be modified slightly if the line of maximum wind and the line of vanishing meridional temperature gradient do not coincide, as is probable equatorward of the mid-latitude jet. Since $\partial T/\partial \phi = 0$ at a lower pressure than where $\partial u/\partial p = 0$, the conclusion still holds in the troposphere.

[2] Strictly speaking, we are talking here of a

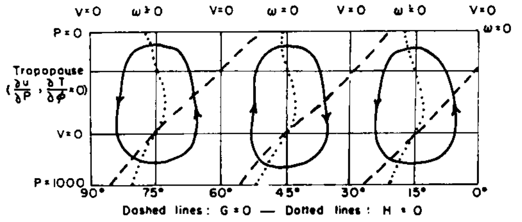

FIG. 2. Idealized configuration of mean meridional cells, momentum and heat source zero lines.

Equation (27) then implies that, in general, the $H=0$ and $G=0$ lines are not parallel, including where they cross, as resonance would be achieved if they were. Now in the troposphere, $r<s<0$ in general. Therefore, in the troposphere, $G=0$ and $H=0$ lines must be confined to regions of downward and equatorward, or upward and poleward motion. In addition, since $r<s$, the $H=0$ lines must make a smaller angle with $\omega=0$ lines than do the $G=0$ lines. If the $\omega=0$ lines are approximately vertical, then the $H=0$ lines will be much more closely vertical than will the $G=0$ lines.

In the stratosphere, $r>s>0$ in general, and therefore, $H=0$ and $G=0$ lines must be confined to regions of downward and poleward, or upward and equatorward motion. Here again, if the $\omega=0$ lines are nearly vertical, then the $H=0$ lines will be much more nearly vertical than the $G=0$ lines.

Both $G=0$ and $H=0$ lines must cross from the troposphere to the stratosphere. (25) implies that $H=0$ lines will cross only where $\omega=0$, $v \neq 0$, since $r \to -\infty$ at the tropopause. In the case of a single layer of cells, the $H=0$ lines would therefore cross the tropopause above the middle of each cell. (26) implies that $G=0$ crosses where $v=0$, $\omega \neq 0$, which must be between cells.[1]

Since there is very good evidence for a maximum of momentum transport poleward around 30° (cf. BUCH (1954), WIDGER, (1949), STARR and WHITE (1954)), and a maximum of heat transport northward between 40° and 50° (cf. PEIXOTO, 1960) we can show from (1) and (3) that the circulation must be thermally indirect. Similarly, if there exist maxima of equatorward transport of heat and momentum at some low and some high latitude (the evidence for the low latitude maxima is quite strong, for the high latitude maxima, not so strong), there we must have direct circulation in these regions. Thus the existence of the often claimed "three cell" meridional circulation pattern, entirely within a single hemisphere, requires three $H=0$ and three $G=0$ lines, the middle one representing maximum poleward transport, the other two representing essentially maximum equatorward transport.

For the simple stratosphere considerd ($\delta T/\delta\phi>0$ everywhere), the lowest layer of cells must extend into the lower stratosphere to be consistent with the observed poleward countergradient transport of heat in that region. Otherwise the constraints would require that there be a maximum of heat transport poleward at low levels, and a maximum of heat transport equatorward at high levels, in middle latitudes (contrary to observations).

To obtain a clearer picture of the above results, let us assume for simplicity that each cell has a latitudinal extent of 30°. Then the constraining relations derived above give rise to the orientation of meridional cells, $G=0$ and $H=0$ lines seen in Fig. 2. Fig. 2 was drawn assuming the centers of the cells to be midway between the edges. This probably gives rise to too great an equatorward slant to the $G=0$ lines. This can be lessened by placing the centers to the equatorward side of each cell.

6. Comparison with observations

At the present time, the state of our observational knowledge of the mean meridional cross-section is really not good enough to verify *independently* the positions of the mean meridional cells relative to the lines of zero momentum and heat sources. That is, we do not know the shape of mean meridional cells accurately enough from direct measurements. Obviously a mean meridional circulation obtained indirectly by inference from equations (1) to (3) from the momentum and heat sources (to the degree that we can measure *them* ac-

mean Richardson number along an isobar between the two lines, which, for adjacent H and G lines, will not greatly alter the result. The relation holds exactly where the $H=0$ and $G=0$ lines cross.

[1] Here we have assumed that both $\partial u/\partial p$ and $\partial T/\partial\phi$ change sign at the tropopause. If the mean state is considered for the winter season only, the configuration would be altered somewhat by the presence of the polar night jet as $\partial u/\partial p$ would change sign much higher up.

curately) will give rise to a picture not unlike Fig. 2. What observational information is available (cf. BUCH, 1954) indicates that at least the transient eddy convergence part of G gives the proper slope for middle latitude $G = 0$ line. As for the heat sources, data of PEIXOTO (1960) for the eddy transports of heat show approximately the right latitude for the $H = 0$ line in middle latitudes, but the slope is in doubt. There is also some evidence for a maximum of equatorward heat transport (another $H = 0$ line) in the tropics. Peixoto's data show rather little evidence for the existence of a maximum of equatorward transport in the polar regions. The existence of a high latitude direct cell may therefore be somewhat more in doubt, though data of SALTZMAN & FLEISHER (1960) do indicate its presence, at least for the winter season. In the southern hemisphere, recent calculations (OBASI, 1963; GILMAN, 1963) show a very pronounced polar direct cell, for which there is a $G = 0$ line. The position of the $H = 0$ line is not known as yet, as the heat transports have not been calculated.

It is to be noted that figure 2 implies, to the degree that the horizontal convergences are dominant in determining the zero lines of H, there will be a counter-gradient transport of heat in the *troposphere* in the tropics and near the poles. That this does occur, at least in the tropics, has recently been pointed out by STARR and WALLACE (1964).

What is really needed is an analysis of the better data available now for a much longer time period, say for 5 years. Such a study is under way at M.I.T. in conjunction with the Travelers Research Center, Inc. When the statistics of this study are compiled, the above consistency requirements obtained theoretically can be compared with the numerical results. The degree to which the lines of zero heat and momentum sources superimposed on the directly measured mean meridional circulation agree with Fig. 2 will be an indication of the accuracy of the numerical results, and the validity of the steady state assumption. It is evident that the consistency conditions can also be applied to statistics derived from multilevel general circulation models.

In conclusion, it should be noted that over the last fifteen years, there has been considerable argument over the relative roles of the mean meridional circulation and the horizontal eddies in the momentum and heat budgets. It should be evident from the results of this paper that for a differentially heated rotating atmosphere with turbulent exchange processes present, the two mechanisms are inseparable. Both are required for consistency. The size, shape and number of the meridional cells is determined by the positions of the lines dividing momentum and heat source regions from sink regions. Conversely, if the mean meridional cellular circulations were known very well (which they are not), the source and sink regions of momentum and of heat could be inferred with confidence.

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