SHORTER CONTRIBUTION

Travelling Planetary Waves

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(Manuscript received October 1, 1963)

The oscillation of longitudinal position of the very long atmospheric waves, of wave-number 1, 2, and 3 has been described by KUBOTA and IIDA (1954), ELIASEN (1958), and others. In this note the motions of these waves are further examined, the data being Fourier zonal harmonics of the 500 mb height at latitudes 40°, 50°, and 60° N for the period October 1, 1951, to March 31, 1952.

In the following, the phase-angle represents displacement west of Greenwich, so that it decreases for a progressive, easterly moving wave.

When the amplitude and phase of the long waves are plotted on polar diagrams the wave-vectors frequently exhibit a roughly circular motion, most commonly anticlockwise. To illustrate this behavior, polar diagrams for wave-numbers 1 and 2, each for one month, are shown in Fig. 1. Provisionally, we will consider this relation between amplitude and position changes to be due to the presence of a retrogressive "travelling planetary wave" superimposed on a "stationary wave" (which varies in position).

The amplitudes of the cosine and sine components of a travelling wave fluctuate, the fluctuations of the cosine component being a quarter-period ahead of those of the sine component for westward movement (anticlockwise rotation on the wave-vector diagram). The most direct numerical way of showing such a relationship in a sequence of data is to calculate the quadrature spectrum (see PANOFSKY and BRIER, 1958). The quadrature spectra for the amplitudes of the cosine and sine components of the harmonics for $N = 1$, 2, and 3, for latitudes 40°, 50°, and 60° N were computed on a CDC 1604 computer. The spectra were computed from lag correlation functions for five and for fifteen lags. The results, normalized by dividing by the product of the standard deviations of the amplitudes of the cosine and sine components, are shown in Fig. 2. The cospectra are also shown, because of their relevance to the statistical significance of the quadrature.

As an indication of the statistical significance of the spectra, confidence limits were computed from the distribution derived by GOODMAN (1957):

$$P(b) = \frac{\delta^n \Gamma(n+\frac{1}{2})}{\sqrt{\pi}\,\Gamma(n)\,[\delta + (b-\beta)^2]^{n+\frac{1}{2}}}$$

where $P(b)$ is the probability of an estimate (of cospectrum or quadrature), $\Gamma(n)$ is the gamma function, δ is the true value of the coherence (the sum of the squares of cospectrum and quadrature), β is the true value of the cospectrum or quadrature, and n is the number of degrees of freedom, approximately equal to $(2N - m/2)/m$ (N being the number of pairs of data from which the cross-spectra are computed, and m the number of lags used). For 73 degrees of freedom (appropriate to the 5-lag computation) the "5 % level" (which is likely to be exceeded 5 % of the time if the true quadrature is zero) is 0.15, for 23 degrees of freedom (corresponding to 15 lags) it is 0.35. Although the distribution of quadrature estimates depends on the true value of the coherence, and thus of the cospectrum, the 5 % level for the quadrature varies only about 0.01 as 0 or 0.2 is taken as the true value for the cospectrum.

The fluctuations of the 15-lag quadrature spectra, compared to the 5-lag spectra (for which the effective smoothing is over frequency intervals three times broader than for the 15-lag spectra), are apparently not significant. The main feature of the quadrature spectra for $N = 1$ and 2 is that they are significantly positive in the middle of the frequency range correspond-

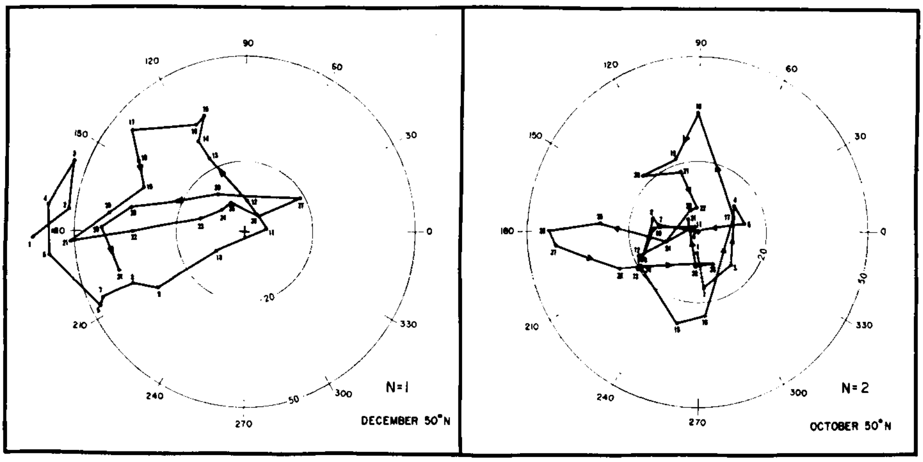

FIG. 1. Wave-vector diagrams, showing the daily variations of amplitude and phase of the first and second 500 mb zonal harmonics, each for one month. The unit of amplitude is 10 geopotential feet.

ing to periods of 2½ to 6 days. (As shown by VAN DER HOVEN (1957) there is a peak in the spectrum of horizontal wind speed near four days. This is of course reasonably ascribed to synoptic-scale disturbances—it seems that it is partly due to planetary waves.) Since a positive quadrature means the cosine component leads the sine component by a quarter-period, this certainly appears to correspond to the rotation of the wave-vectors already described. The wave-number 3 shows systematic negative values, corresponding to forward moving waves, but the values are relatively small, and may be interpreted as due to a slight preponderance of the progressive over the retrogressive waves (both of which are conspicuous in the polar diagrams, which are not reproduced here).

The dominant retrogressive wave-speeds for $N = 1$ and 2 may be estimated from the positions of the maxima of the smoothed spectra. The results are as follows (averaging for the three latitudes):

$N = 1$: 1 cycle/4 days, i.e. 90° long./day (approx.)

$N = 2$: 1 cycle/3 days, i.e. 60° long./day (approx.)

These speeds are about twice those observed by KUBOTA and IIDA (1954) for the difference between the daily harmonic and the mean for the 36-day period they considered. The difference is not surprising in view of the differing procedures, and also the variability from month to month.

The interpretation of this behavior of the planetary waves is not obvious. The observed behavior can be regarded as simply a correlation between wave-speed and amplitude, such that

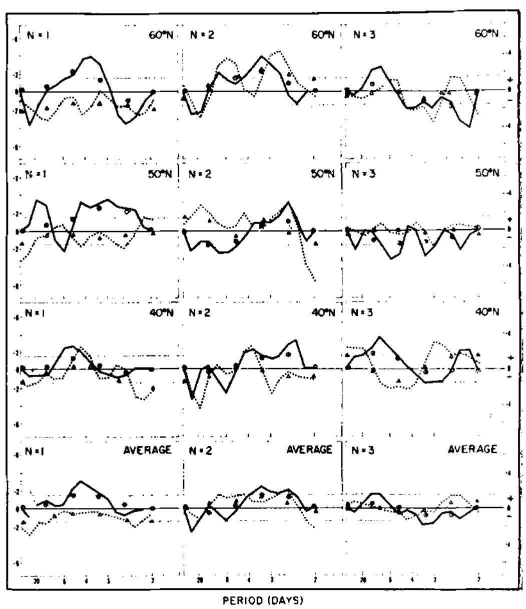

FIG. 2. Normalized quadrature and co-spectra for the amplitudes of cosine and sine components of 500 mb zonal harmonics.

Quadratures: 5 lags ○, 15 lags ——

Cospectra: 5 lags △, 15 lags ......

Thin horizontal lines are 5% confidence limits for 5-lag spectra. Abscissa is linear in frequency: 0, 1/30, 2/30, ... 15/30 day^{-1}.

the waves are retrogressive when the amplitude is large, progressive when it is small. In these terms, we must consider the critical amplitude separating forward and backward moving waves to be variable. The two-component model already mentioned requires, similarly, that the "stationary" component be variable in position and amplitude. The two-component model does, however, suggest a qualitative explanation of the observed behavior. Let us consider the quasi-stationary component as being forced, by orography and the horizontal distribution of heating. This forced wave would fluctuate in phase and magnitude. The travelling planetary wave component can be considered a free wave, arising from the fluctuating forced wave or as a result of baroclinic development.

This study has been supported in part by the U.S. Weather Bureau, Grant WBG10-Project 4. The help of Professor Lyle H. Horn, who computed the zonal harmonics; Mr. Charles Hutchins, who computed the cross-spectra; and Mr. Martin Sponholz, is gratefully acknowledged.

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