On Coriolis force variability in the prognostic schemes

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ABSTRACT

В работе исследуется влияние переменности силы Кориолиса (эффект Россби или β-эффект) на атмосферные процессы. В первой части изучается механизм адаптации. Во второй части строится решение прогностического уравнения с линейным членом $\beta(\partial H/\partial x)$, перенесенным в левую часть. На простой модели показано, что такой способ решения позволяет слегка увеличить шаг по времени по сравнению с обычным способом решения уравнения для $\partial H/\partial t$. Наконец, излагаются некоторые дополнительные результаты и даются выводы.

The influence of the Coriolis force variability (Rossby effect or β-effect) on the atmospheric processes is discussed. In the first part the adaptation process is investigated. In the second part the explicit form of the solution of the prognostic equation with a linear term $\beta(\partial H/\partial x)$ on the left-hand side is estimated. It is shown that, in comparison with the conventional method, the method described allows the time step to be increased slightly. Finally, some additional results and conclusions are given.

1. Interaction between the pressure and wind fields

Investigations of the interaction between the pressure and wind fields in the atmosphere have shown that the adaptation time (i.e. the time of the mutal adaptation of these fields so that balance between Coriolis force and pressure gradient would be established) is of several hours (OBUKHOV, 1949; KIBELJ, 1955; MONIN, 1958). This result was obtained on the assumption that the Coriolis parameter is constant. By accounting for the variability of the Coriolis force–Rossby effect one can make the adaptation mechanism more clear (DOBRISHMAN & NOZADZE, 1958). The numerical forecast schemes of the geopotential field under the quasi-solenoidal approximation deal with quasi-adapted fields. Therefore, before going to the prognostic scheme with detailed accounting of the Coriolis force variability the problem of the adaptation of wind and pressure field is described.

The linearized system of hydrodynamic equation is given in the conventional form:

$$\frac{\partial u}{\partial t} - lv + \frac{\partial H}{\partial x} = 0;$$

$$\frac{\partial v}{\partial t} + lu + \frac{\partial H}{\partial y} = 0;$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} - \frac{1}{c^2}\frac{\partial}{\partial \zeta}\zeta^2\frac{\partial}{\partial \zeta}\frac{\partial H}{\partial t} = 0, \qquad (1.1)$$

where t—time, u, v—components of wind velocity towards axes X, Y respectively, (X—to the East, Y—to the North); l—Coriolis parameter, H—geopotentional, c—parameter with speed dimension and equal to 100–120 m/sec, ζ—dimensionless pressure.

If l is constant, then the stationary solution of (1.1) will be the geostrophic wind:

$$u = -\frac{1}{l}\frac{\partial H(x, y, \zeta)}{\partial y}; \quad v = \frac{1}{l}\frac{\partial H(x, y, \zeta)}{\partial x}.$$

But if $l = l(y) = l_0 + \beta y$, where β is the Rossby parameter, then the stationary solution will be the zonal geostrophic wind:

[1] Presented at the WMHO/IUGG Symposium on Numerical Weather Forecasting, Oslo 1963.

$$u = -\frac{1}{l}\frac{\partial H(y,\zeta)}{\partial y}; \quad v \equiv 0.$$

The more accurate solution of the problem on the spherical Earth gives the same result, for example the stationary solution of the system in the barotropic case

$$\frac{\partial v_\lambda}{\partial t} + 2\omega v_\theta \cos\theta + \frac{1}{r_0 \sin\theta}\frac{\partial H}{\partial \lambda} = 0;$$

$$\frac{\partial v_\theta}{\partial t} - 2\omega v_\lambda \cos\theta + \frac{1}{r_0}\frac{\partial H}{\partial \theta} = 0;$$

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta} v_\theta \sin\theta + \frac{1}{\sin\theta}\frac{\partial v_\lambda}{\partial\lambda} + \frac{1}{c_*^2}\frac{\partial H}{\partial t} = 0$$

is the zonal geostrophic wind:

$$v_\lambda = \frac{1}{2\omega r_0 \cos\theta}\frac{\partial H(\theta)}{\partial\theta}; \quad v_\theta \equiv 0.$$

Here v_λ, v_θ are components of the wind velocity for the axes of the spherical coordinates system; r_0—mean radius of the Earth, ω—angular velocity of the Earth rotation, c^* has the same meaning as c, but differs quantitatively.

In order to make a more careful analysis of the adaptation process let us eliminate the functions u and H from the system (1.1). Then the equation for v will be as follows;

$$\Delta\frac{\partial v}{\partial t} + \frac{1}{c^2}\frac{\partial}{\partial\zeta}\zeta^2\frac{\partial}{\partial\zeta}\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial t^2} + l^2\right)v + \beta\frac{\partial v}{\partial x} = 0, \tag{1.2}$$

which is to be solved under the initial conditions

$$t = 0 \quad v = v^{(0)}(x, y, \zeta);$$

$$\frac{\partial v}{\partial t} = v^{(1)}(x, y, \zeta) = -\frac{\partial H^{(0)}}{\partial x} - l v^{(0)};$$

$$\frac{\partial^2 v}{\partial t^2} = v^{(2)}(x, y, \zeta) = c^2\frac{\partial}{\partial y}\left(\frac{\partial u^{(0)}}{\partial x} + \frac{\partial v^{(0)}}{\partial y}\right) + \frac{\partial}{\partial\zeta}\zeta^2\frac{\partial}{\partial\zeta}\left(l\frac{\partial H^{(0)}}{\partial x} + l^2 v^{(0)}\right) \tag{1.3}$$

and the boundary conditions

$$\zeta = 1 \quad \left(\frac{\partial^2}{\partial t^2} + l^2\right)\left(\zeta\frac{\partial}{\partial\zeta} + \varepsilon_1\right) v = 0;$$

$$\zeta \to 0 \quad \zeta^{1+\varkappa} v \to 0, \quad 0 < \varkappa < \tfrac{1}{2},$$

where $\varepsilon_1 \approx 0.10 - 0.12$ and index (0) designates the values at the initial moment.

It is easily seen that the solution of the equation (1.2) is of a two-dimensional wave type[1] $\zeta^\nu e^{i(\mu x + \nu y - \sigma t)}$. The wave phase velocities are determined from the cubic equation

$$\sigma^3 - \sigma\left[l^2 + \frac{\varrho^2 c^2}{\varepsilon_1(1-\varepsilon_1)}\right] - \frac{\beta c^2 \mu}{\varepsilon_1(1-\varepsilon_1)} = 0;$$

$$\varrho^2 = \mu^2 + \nu^2.$$

For a wavelength less than 10,000 km $\sigma_1 \approx -\sigma_2$, practically this result coincides with one, obtained without taking the Rossby effect into acount. But

$$\sigma_3 \approx \frac{\beta\mu c^2}{\varepsilon_1(1-\varepsilon_1)\left[l^2 + \dfrac{\varrho^2 c^2}{\varepsilon_1(1-\varepsilon_1)}\right]}$$

is due to this effect. From Fig. 1 it is seen that the wave phase velocity of the third type increases when the wave length increases, but $|\sigma_{1,2}|$ decreases under the same conditions.

To obtain the general solution it is convenient to use the operational calculus. For the Laplace-Carson transformation we obtain the nonhomogeneous equation

$$\Delta\bar{v} + \frac{p + l^2}{c^2}\frac{\partial}{\partial\zeta}\zeta^2\frac{\partial}{\partial\zeta}\bar{v} + \frac{\beta}{p}\frac{\partial\bar{v}}{\partial x} = \Delta x^{(0)} + \frac{1}{c^2}\frac{\partial}{\partial\zeta}\zeta^2\frac{\partial}{\partial\zeta}(p^2 v^{(0)} + p v^{(1)}) = \varphi_0 + p\varphi_1 + p^2\varphi = \phi, \tag{1.4}$$

where the symbols φ_k are evident.

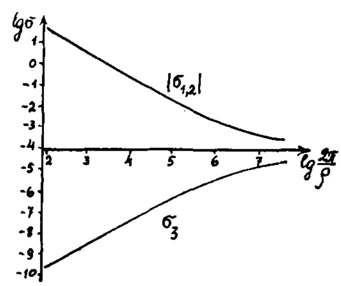

FIG. 1.

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[1] Such waves are studied in detail in DIKIJ (1961).

Finally, we divide the atmosphere into 2 n layers ($\zeta_0 = \zeta_{\frac{1}{2}} = 0$; $\zeta_{2n+1} = 1$) and replace the differential operator with a finite difference one (see NEMCHINOV, 1959). Then, for each level under index k we obtain the equation

$$\Delta \bar{v}_k + \frac{p^2 + l^2}{c^2} L(\bar{v}_k) + \frac{\beta}{p} \bar{v}_k = \phi_k,$$

where

$$L(\bar{v}_k) = \frac{1}{\delta \zeta_k} \left\{ \frac{\zeta_{k+\frac{1}{2}}}{\delta \zeta_{k+\frac{1}{2}}} (\bar{v}_{k+1} - \bar{v}_k) - \frac{\zeta^2_{k-\frac{1}{2}}}{\delta \zeta_{k-\frac{1}{2}}} (\bar{v}_k - \bar{v}_{k-1}) \right\}. \quad (1.5)$$

Now we canonize the system (NEMCHINOV, 1961) using the boundary conditions. Then for the eigenfunction (number j) which is a linear combination of all $\bar{v}_k$, we obtain the following equation:[1]

$$\Delta \bar{w}_j - \lambda_j^2 \bar{w}_j + \frac{\beta}{p} \frac{\partial \bar{w}_j}{\partial x} = f_j,$$

where λ_j^2 are eigenvalue numbers of the following form:

$$\lambda_j^2 = a_j^2 \frac{p^2 + l^2}{c^2}.$$

Here a_j^2 depends only on the method of dividing the atmosphere into layers. In a particular case for a two-level model at 300 mb and 700 mb with boundary conditions taken at 1000 mb and 150 mb we obtain $a_1^2 = 0.964$; $a_2^2 = 4.2$. It should be mentioned that in the case of a barotropic atmosphere $a_1^2 = 1$.

Since for the original equation (1.2) (this equation is of a hyperbolic type) c is proportional to the velocity of the wave front, we obtain the following result: the wave front velocity is proportional to c/a_j for eigenfunction of number j.

The regular solution for $\sqrt{x^2 + y^2} \to \infty$ of the equation (1.6) is

$$\bar{w}_j = -\frac{1}{2\pi} \iint_{-\infty}^{\infty} f_j(x, y: p) \left\{ e^{(-\beta r \cos\theta)/2p} H_0^{(1,2)} \times \left[r \sqrt{\frac{a_j^2}{c^2}(p^2 + l^2) + \frac{\beta^2}{4p^2}} \right] \right\} r\, dr\, d\theta, \quad (1.6)$$

where

$$r = \sqrt{(x - x')^2 + (y - y')^2}; \quad \theta = \text{arc tg} \frac{y - y'}{x - x'}; \quad H_0^{(1,2)}$$

is the Hankel function.

The operational calculus facilitates the study of the behaviour of the Green function since a large value t corresponds to a small value $|p|$ and vice versa.

(a) At the very beginning of the adaptation process, when $|p| \gg l$, i.e. $t < 1/l \approx 2-3$ hours, the Hankel function may be replaced by

$$K_0\left(\frac{rpa_j}{c}\right)$$

McDonald functions, and $e^{(-\beta r \cos\theta)/2p}$ by 1. The originals to $p^m K_0(rpa_j/c)$ for $(m = 0.1)$, (DITKIN, 1951), are expressed as

$$m = 0, \quad \frac{c}{ra_j} \sqrt{t^2 - \frac{r^2 a_j^2}{c^2}} \quad \text{and} \quad m = 1, \quad \frac{1}{\sqrt{t^2 - \frac{r^2 a_j^2}{c^2}}}. \quad (1.7)$$

(b) $\qquad t \sim O(1/l)$.

The influence function here is still axisymmetrical since $e^{(-\beta r \cos\theta)/p} \approx 1$, and $H_0^{(1,2)}$ may be replaced by

$$K_0\left(\frac{r}{ca_j}\sqrt{l^2 + p^2}\right).$$

The corresponding originals will be

$$m = 0 \quad \int_0^t \chi_1(t_1)\, dt_1;$$

$$m = 1 = \chi_1(t) = \frac{\cos l \sqrt{t^2 - \frac{r^2 a_j^2}{r^2}}}{\sqrt{t^2 - \frac{r^2 a_j^2}{c^2}}}. \quad (1.8)$$

These formulae were derived by OBUKHOV (1949).

$$(c) \qquad \frac{la_j}{\beta c} < t < \frac{2}{\beta r} \approx \frac{1.5 \text{ day}}{r_1 (1000 \text{ km})}.$$

The operator $p^m e^{(-\beta r \cos\theta)/2p} K_0(\beta r/2p)$ should be inverted here. For simplicity we give the original here only for $m = 0$:

[1] We discuss here the deduction of the equation (1.5) in detail, since the forecasting equation will be transformed in the same way.

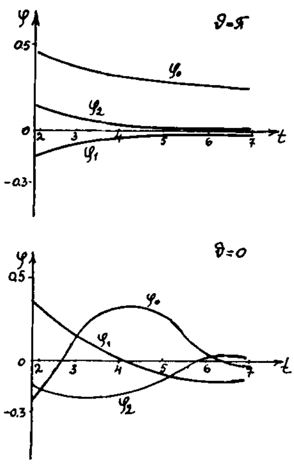

FIG. 2.

$$\frac{2}{\pi}\int_0^t \frac{K_0(\beta r t_1)\cos\sqrt{2(1+\cos\theta)\,\beta r\,(t-t_1)}\,dt_1}{\sqrt{t_1}\,\sqrt{t-t_1}}. \tag{1.9}$$

From further discussion it will be seen that this function is derived as a result of the solution of the prognostic equation.

(*d*) $t \gg 2/\beta r_1$. This time corresponds to the asymptotic case. But $t > r/c$ always. Thus we obtain the following result; at large distances where disturbances from zero arrive at $t = r/c$, the asymptotic behaviour is reached earlier than at the points situated at a shorter distance. It may be shown that the originals in this case are expressed as

$$G_m = \frac{d^m}{at^m}\,\frac{\cos\sqrt{2\beta r(1+\cos\theta)\,t}}{\sqrt{\beta r t}} \quad (m = 0, 1, 2). \tag{1.10}$$

Hence it is clear that Y_m behaves as $1/(\sqrt{t})^{m+1}$ when t becomes infinite.

Thus for any initial discrepancy between the pressure and wind fields $v \to 0$, and the geostrophic zonal wind is established in the limit $t \to \infty$. It should be noted that in cases (*c*) and (*d*) the results do not depend on a_j, i.e. on the method of dividing the atmosphere into layers and the solution does not change its sign to the west of the point of disturbance. As an example, Fig. 2 shows functions Y_m for $r = 2000$ km for two directions $\theta = 0$ and $\theta = \pi$.

According to this short analysis we see that the asymptotic solution will be essentially different for the case when l is constant and $l = l(y)$. What is asymptotic for the first case, is the beginning of the middle stage of the process for the second one.

The solution suitable for all t may be expressed by using an inverse Riemann-Mellin formula. For this purpose it is necessary to cut the plane of the complex variable p through the branch-points of the Hankel's function argument. After some transformation, we obtain:

$$p^m e^{(-\beta r\cos\theta)/2p}\, H_0^{(1,2)}\left[r\sqrt{\frac{a_j^2}{c^2}(l^2+p^2)+\frac{\beta^2}{4p^2}}\right] = -\frac{1}{\pi}\int_{\xi_1}^{\xi_2}\xi^{m-1}R_m(\xi,t)\,d\xi + \frac{2}{\pi^2}$$
$$\times\left[\int_0^{\xi_1}\xi^{m-1}Q_m(\xi,t)\,d\xi + \int_{\xi_2}^{\infty}\xi^{m-1}Q_m(\xi,t)\,d\xi\right], \tag{1.11}$$

where

$$R_m = \sin\eta_1 N_0(\eta_2) - \cos\eta_1 J_0(\eta_2);$$
$$Q_m = \sin\eta_1 K_0(\eta_3); \tag{1.12}$$

$$\eta_1 = \xi t + \frac{\beta r\cos\theta}{2\xi} + \frac{\pi}{2}(m-1);$$
$$\eta_3 = e^{i\pi/2}\eta_2 = \frac{r a_j}{c}\sqrt{\xi^2 - l^2 + \frac{\beta^2}{4a_j^2\xi^2}}, \tag{1.13}$$

ξ_1, and ξ_2 are the arithmetic values of the roots of the equation

$$\xi^4 + l^2\xi^2 + \frac{\beta^2 c^2}{4a_j^2} = 0$$

and J_0 N_0 are the Bessel and Neumann functions respectively.

Fig. 3 shows the influence function for the simplest case of the one-level model $a_1 = 0$ at $m = 1$. This is perhaps the most interesting case

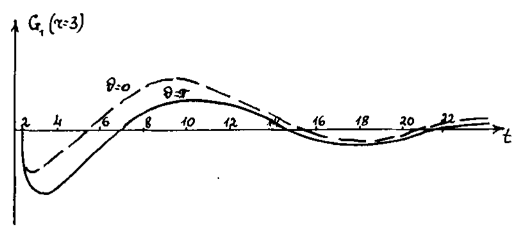

FIG. 3.

since it shows the behaviour of the geostrophic zonal wind deviations in the barotropic case.

In a general baroclinic case, as it is seen from the analysis, the solution is presented as a linear combination of the barotropic models with different parameters a_j^2.

2. The solution of the prognostic equation

In the numerical forecasting schemes of the pressure field based on the idea of quasi-geostrophic or quasi-solenoidal character of the motion the following equation is usually used:

$$M\left(\frac{\partial H}{\partial t}\right) = F(x, y, \zeta, H(x, y, \zeta, t)), \qquad (2.1)$$

where M is a linear differential operator of the elliptic type affecting only the coordinates. The explicit forms of this operator and of the right-hand part depend on the method of treating the original equations (8, 9). In practice, the solution consists in the determination of the inverse operator that affects the coordinates only. In a symbolic form the solution may be expressed as follows:

$$\frac{\partial H}{\partial t} = M^{-1}[F(x, y, \zeta, H)]. \qquad (2.2)$$

Since the purpose of a forecast is to determine H as a function of coordinates and time, the geopotentional field is obtained by time integration of the expression (2.2).

$$H = H^0 + \int_0^t M^{-1}[F(x, y, \zeta, H)]\, dt_1$$

or, since the operators $\int_0^t \dots dt_1$ and M^{-1} are commutative and bounded:

$$H = H^0 + M^{-1}\left[\int_0^t F(x, y, \zeta, H)\, dt_1\right].$$

The right-hand side of (2.1) is in general also a differential operator operating with regard to coordinates only. Therefore the problem of determining H as a function of coordinates and time should be formulated as a Cauchy problem. In (2.1) the function $H = H(x, y, \zeta, t)$ is to be found when known at initial time. However, since the non-linear operator cannot usually be inverted in explicit form it is not here possible to obtain an exact solution. In order to solve (2.1) it is therefore convenient to consider the optimal operator on the left-hand side which operates with regard to coordinates and time and which can be inverted explicitly. Such an expansion operator can be found, for instance, by including the term $\beta\partial H/\partial x$, representing the variation of the Coriolis parameter (THOMPSON, 1961; KOGAN, 1957; GATES, 1953).

The prognostic equation may be written

$$\frac{\partial}{\partial \zeta}\zeta^2\frac{\partial}{\partial \zeta}\frac{\partial H}{\partial t} + m^2\Delta\frac{\partial H}{\partial t} + \beta m^2\frac{\partial H}{\partial x}$$
$$= -\frac{m^2}{l^2}(H, \Delta H) - \frac{1}{l}\frac{\partial}{\partial \zeta}\left[\zeta^2\left(H, \frac{\partial H}{\partial \zeta}\right)\right] \qquad (2.3)$$

and should be solved under the initial condition

$$H = H^{(0)}(x, y, \zeta) \quad (t = 0) \qquad (2.4)$$

and the boundary conditions

$$\zeta = 1 \quad \frac{\partial^2 H}{\partial \zeta \partial t} + \varepsilon_1\frac{\partial H}{\partial t} = -\frac{1}{l}\left(H, \frac{\partial H}{\partial \zeta}\right), \qquad (2.5)$$

$$\zeta \to 0 \quad \lim\left|\zeta^{1+\varkappa}\frac{\partial^2 H}{\partial \zeta \partial t}\right| < \infty \quad (0 < \varkappa < 1). \qquad (2.6)$$

Here m is the parameter determining the influence domain 700–1000 km, and (A, B) denotes a Jacobian.

Equation (2.3) may be solved in the same way as equation (1.2).

Using the Laplace–Carson transformation we obtain the equation

$$\frac{\partial}{\partial \zeta}\zeta^2\frac{\partial}{\partial \zeta}\bar{H} + m^2\Delta\bar{H} + \frac{\beta m^2}{p}\frac{\partial \bar{H}}{\partial x}$$
$$= \frac{1}{p}F + F^0 = F^{(1)}, \qquad (2.7)$$

which should be solved at the boundary conditions

$$\zeta = 1 \quad \frac{\partial \bar{H}}{\partial \zeta} + \varepsilon_1\bar{H} = \frac{1}{p}f + f^{(0)} = f^{(1)},$$

$$\zeta \to 0 \quad \left|\zeta^{1+\varkappa}\frac{\partial \bar{H}}{\partial \zeta}\right| < \infty.$$

Here F and f denote the right-hand part of (2.3) and (2.5) respectively, and

$$F^{(0)} = \frac{\partial}{\partial \zeta} \zeta^2 \frac{\partial}{\partial \zeta} H \big|_{t=0} + m^2 \Delta H \big|_{t=0};$$

$$f^{(0)} = \frac{\partial H}{\partial \zeta} \big|_{t=0} + \varepsilon_1 H \big|_{t=0}.$$

The form of (2.7) is similar to that of (1.4). Therefore it may be solved in the same way. Using the finite-difference scheme with respect to ζ and taking into account the boundary conditions we obtain at level k

$$L(\bar{H}_k) + m^2 \Delta \bar{H}_k + \frac{\beta m^2}{p} \frac{\partial \bar{H}_k}{\partial x} = F_k^{(1)},$$

where L is the operator (1.5).

After canonization we find following equation for the eigenfunction

$$m^2 \Delta \bar{h}_j - \lambda_j^2 \bar{h}_j + \frac{\beta m^2}{p} \frac{\partial \bar{h}_j}{\partial x} = \phi_j^{(1)},$$

where $\varphi_j^{(1)}$ are the linear combinations of the corresponding values at all levels and λ_j^2 are eigenvalues. They are proportional to m^2 and depend on the method of dividing the atmosphere into layers. The regular solution of this equation at $\sqrt{x^2+y^2} \to \infty$ is expressed as:

$$\bar{h}_j = -\frac{1}{2\pi} \int_0^{2\pi} \int_0^{\infty} \left(\frac{1}{p} \psi_j + \psi_j^{(0)} \right)$$

$$\times e^{(-\beta r \cos \theta)/2p} H_0^{(1,2)} \left(r \sqrt{\frac{\beta^2}{4p^2} + \lambda_j^2} \right) r \, dr \, d\theta. \quad (2.8)$$

In order to find the original it is necessary to use the Riemann-Mellin inversion formula. After some transformations the solution in explicit form may be expressed as:

$$\bar{h}_j = -\frac{1}{2\pi} \int_0^{2\pi} \int_0^{\infty}$$

$$\times [\psi_j(r,\theta) G_j^{(1)}(r,\theta) + \psi_j^{(0)} G_j^{(0)}(r,\theta)] \, r \, dr \, d\theta, \quad (2.9)$$

where $G_j^{(1)}$ and $G_j^{(0)}$ are Green functions

$$G_j^{(m)}(r,\theta) = -\frac{1}{\pi} \int_{\beta/2\lambda_j}^{\infty} \xi^{m-1} R_m(\xi, t)$$

$$+ \frac{2}{\pi^2} \int_0^{\beta/2\lambda_j} \xi^{m-1} Q_m \, d\xi. \quad (2.10)$$

R_m and Q_m are the same as in (1.12) but

$$\eta_3' = i \eta_2' = r \sqrt{\frac{\beta}{4\xi^2} - \lambda_j^2}.$$

To clarify the solution the simplest case of a one-level model is discussed here in detail. It is easy to obtain this case by putting $\lambda_j = 0$ in (2.8). (It is possible to start directly from the equation

$$\frac{\partial \Delta H}{\partial t} + \beta \frac{\partial H}{\partial x} = -\frac{1}{l} (H, \Delta H);$$

the result is the same.)

The Green functions in this case may be expressed in a form more suitable for the analysis. It may be shown that in (2.11) the function $K_0(\beta r/2p)$ is to be used instead of

$$H_0^{(1,2)} \left(r \sqrt{\frac{\beta^2}{4p^2} + \lambda_j^2} \right).$$

In order to find the original, tables of Laplace-Carson transformation may be used (DITKIN & KUZNECOV, 1951):

$$\frac{1}{p} e^{(-\beta r \cos\theta)/2p} K_0 \left(\frac{\beta r}{2p} \right) = \frac{4}{\pi \sqrt{\beta r (1+\cos\theta)}}$$

$$\times \int_0^t \frac{K_0(2\sqrt{\beta r t_1}) \sin \sqrt{2\beta r (1+\cos\theta)(t-t_1)} \, dt_1}{\sqrt{t_1}}$$

and

$$e^{(-\beta r \cos\theta)/2p} K_0 \left(\frac{\beta r}{2p} \right)$$

$$= \int_0^t \frac{K_0(2\sqrt{\beta r t_1}) \cos \sqrt{2\beta r (1+\cos\theta)(t-t_1)} \, dt_1}{\sqrt{t_1}\sqrt{t-t_1}}.$$

After introducing the variable $t_1 = t\eta^2$ these influence functions will be expressed in a form more suitable for calculation

$$g_1 = -\frac{4t}{\pi^2}$$

$$\times \int_0^1 K_0(2\sqrt{\beta r t}\,\eta) \sin (\sqrt{2\beta r t (1+\cos\theta)(1-\eta^2)}) \, d\eta;$$

$$g_0 = \frac{4}{\pi}$$

$$\times \int_0^1 \frac{K_0(2\sqrt{\beta r t}\,\eta) \cos (\sqrt{2\beta r t (1+\cos\theta)(1-\eta^2)}) \, d\eta}{\sqrt{1-\eta^2}}. \quad (2.11)$$

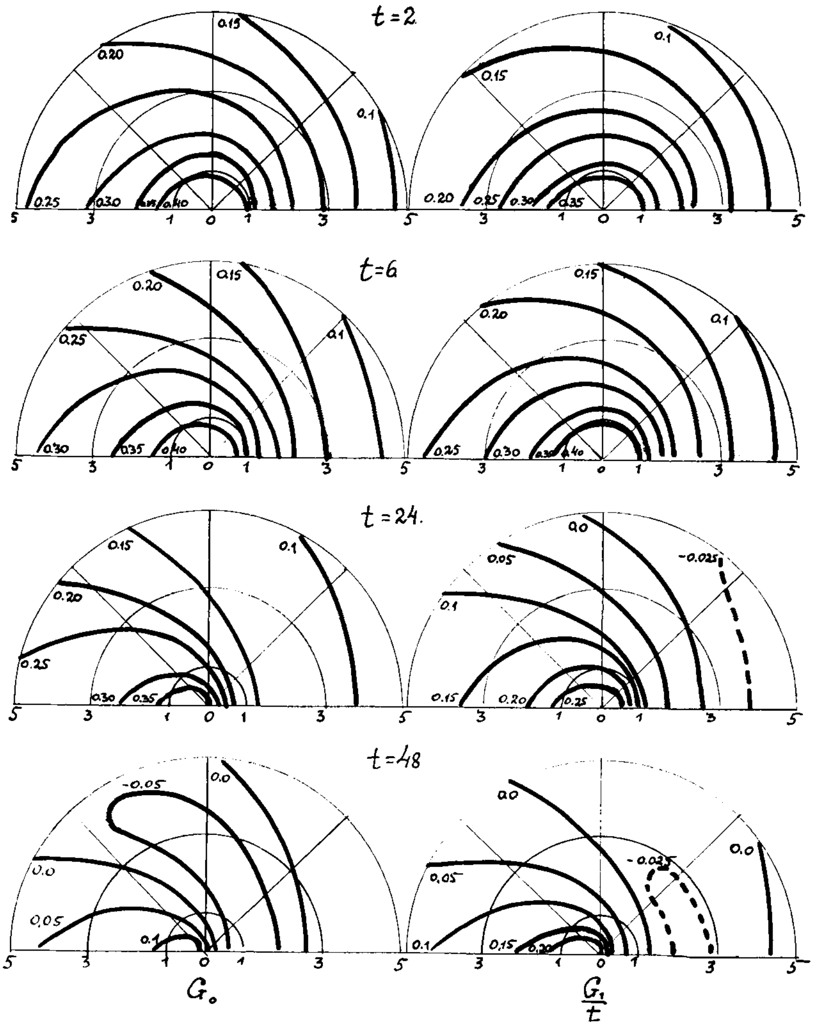

FIG. 4.

With this notation the solution of the prognostic equation for a barotropic model will be expressed as

$$H(x, y, t) = H^0(x, y) + \iint_{-\infty}^{\infty} \left[-\frac{1}{l} (H, \Delta H) g_1 + \Delta H^{(0)} g_0 \right] dx'\, dy'.$$

Fig. 4 shows the values of functions g_0 and g_1/t at different times. (r in 10^3 km).

It is easily seen that in comparison with the conventional method of solving the elliptic equation for $\partial H/\partial t$, the method discussed here, allows for a small increase of the time step. We consider a simple example of the exact solution for a Rossby wave.

$$H = H_0 - Uy + be^{-i[\mu(x-ct)+\nu y]},$$

where $$c = U - \frac{\beta}{\mu^2 + \nu^2}.$$

It is easy to check that the solution of the equation

$$\frac{\partial \Delta H}{\partial t} + \beta \frac{\partial H}{\partial x} = -\frac{1}{l}(H, \Delta H)$$

for the time step δt is of the form:

$$H_0 - Uy + be^{i(\mu x + \nu y)}$$

$$\times \left[1 - \left(1 - U \frac{\mu^2 + \nu^2}{\beta}\right) (1 - e^{-i\beta\mu\delta t/(\mu^2 + \nu^2)})\right].$$

Using the conventional method we obtain

$$H = H_1 - Uy + b$$

$$\times \left[1 - i\mu \left(U - \frac{\beta}{\mu^2 + \nu^2}\right) \delta t\right] e^{i(\mu x + \nu y)}.$$

It is evident that the solution of (2.15) is nearer to the exact solution than that of (2.16), because the wave amplitude in (2.15) is a periodic time function, whereas in (2.10) it increases monotonically, and (2.16) will coincide with the exact solution at $U = 0$. Similar conclusions are true for baroclinic models.

Some examples of forecasts for small areas were computed with the help of the method presented here.

3. Some additional results and conclusions

It was essential to determine the mutual influence of β-effect and orographic effect. The investigations (Hvedelidze, 1961) show that in the case of a short-range forecast of the geopotentional field near small mountain ranges of the Caucasus type the orographic influence is greater than the β-effect. Near the large mountain ranges such as the Rocky Mountains, extending along the meridian, the contribution of both factors is comparable.

The adaptation to the zonal motion at low latitudes, where β is great, is quicker than at high latitudes. In numerical forecast schemes of pressure and wind fields for the areas covering near-equatorial regions, more detailed accounting of the variability of the Coriolis force will lead to more accurate results.

The suggested method may also be used in the case of quasi-solenoidal models in order to account more in detail for the sphericity of the Earth.

REFERENCES

Dikij, L. A., 1961g, Sobstvennye kolebaniia baroklinnoj atmosfery nad sfericheskoj zemlej. *Izvestiia AN SSSR, seriia geofizicheskaia*, No. 5.

Ditkin, V. A., and Kuznecov, P. I., 1951g, Spravochnik po operacionnomu ischisleniiu, GITTL.

Dobrishman, E. M., and Nozadze, R. O., 1958g, O sviazi polej davleniia i vetra v barotropnoj atmosfere. *Trudy CIP*, vap. 78.

Dobrishman, E. M., 1961g, O primerah tochnyh reshenij nelinejnyh prognosticheskih uravnenij. *Izvestiia AN SSSR, seriia geofizicheskaia*, No. 2.

Dobrishman, E. M., 1958g, O reshenii uravneniia dlia izmereniia geopotenciala, *Trudy CIP*, vap. 78.

Gates, L. A., 1953, A method of numerical forecasting by juxtaposition and its application to the equivalent barotropic model. *Journal of Meteorology*, No. 10, No. 2.

Hvedelidze, Z. V., 1961g, Vliianie peremennosti sily Koriolisa na izmenenie polia geopotenciala pri nalichii gor. *Akademiia nauk Gruzinskoj SSR*, t. XXVII, No. 2.

Kibelj, I. A., 1955, O prisposoblenii dvizheniia vozduha k geostraficheskomu. *DAN SSSR*, t. 104, No. 4.

Kibelj, I. A., 1957. Vvedenie v gidrodinamicheskie metoda kratkosrochnogo prognoza pogody. GITTL.

Kogan, S. I., 1957g, O reshenii nelinejnoj zadachi, sviazannoj s kratkosrochnym prognozom polia geopotenciala. *Trudy Instituta fiziki atmosfery AN SSSR*, No. 2.

Monin, A. A., 1958g, Izmenenie davleniia v baroklinnoj atmosfere. *Izvestiia AN SSSR, seriia geofizicheskaia*, No. 4.

Nemchinov, S. B., 1959g, O reshenii uravnennia dlia prognoza polia atmosfernogo davleniia. 12-*vestiia AN SSSR, seriia geofizicheskaia* No. *12*.

Nemchinov, S. V., 1961g, O prognoze polej davleniia i vetra v atmosfere. *Izvestia AN SSSR, seriia geofizicheskaia*, No. 7.

Obuhov, A. M., 1949g, K voprosu o geostroficheskom vetre. *Izvestia AN SSSR, seriia geograficheskaia i geofizicheskaia*, No. 4.

Thompson, P. D., 1961, *Numerical Weather Analysis and Prediction*. New York.