On the orographic influences of the Alps[1]

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ABSTRACT

The orographic influence of the Alps on 500 mb northerly or southerly flow has been studied, using the vorticity equation and a numerically evaluated mountain profile. The effects on trajectory deflection from orography and from the variation in the coriolis parameter have opposite signs for northerly flow but the same sign for southerly flow. Using the most frequent wind speed at 500 mb it is found that in the majority of cases a northerly current will pass the Alps without any significant deflection.

Experience has shown that the orographic influence of the Alps in Central Europe produces certain characteristic features in upper currents crossing the mountain range. The influence of earth's orography on the westerlies has been studied by several authors CHARNEY & ELIASSEN (1949), BOLIN (1950) and it has been shown that the orography of the great continental divide in North America may account for the formation of semi-permanent lee-side troughs. Since the Alps act as a barrier on a north–south current, the problem of its influence on the large scale flow pattern is to some extent different.

For the theoretical treatment we make use of the vorticity and continuity equations in a p-system. The two-dimensional individual change of the absolute vorticity η is

$$\frac{d\eta}{dt} = -\omega\frac{\partial\zeta}{\partial p} + \eta\frac{\partial\omega}{\partial p} - \left(\nabla\omega\times\frac{\partial\mathbf{v}}{\partial p}\right)\cdot\mathbf{k}, \qquad (1)$$

$$A + B - C$$

where ζ is the relative vorticity, $\omega = dp/dt$ and $\mathbf{v}$ the horizontal velocity. A is the vertical vorticity transport, B the divergence term, and C the twisting term. With regard to the three terms on the right hand side of equation (1) it is an empirical fact that in a flow across a mountain the vertical velocity imposed due to the kinematic boundary condition will exceed by at least one order of magnitude the vertical velocity that results from barotropic or baroclinic processes alone.

If the geopotential of the earth's surface is given as

$$gz = k(x,y) \qquad (2)$$

the kinematic boundary condition may be written (approximately) in the form

$$\omega_g = -\varrho_g \mathbf{v}_g \cdot \nabla k, \qquad (3)$$

where the subscript g means that the values are to be taken at the ground and where ϱ_g denotes density.

Using observed data for temperature and wind REUTER & PICHLER (1960*a*) have shown that the term B in (1) is in the majority of cases one order of magnitude larger than the terms A or C. In order to evaluate the term B at 500 mb we assume a power law variation of ω with height

$$\omega = \omega_g\left(\frac{p}{p_g}\right)^b, \qquad (4)$$

where in a previous investigation REUTER & PICHLER (1960*b*) have shown that the value, $b = 4$, gives the best approximation to observations made by gliders of the vertical velocity over mountains.

Using (3) and (4) the term B may be written

$$B \simeq -f\varrho_g\frac{b}{p_g}\left(\frac{p}{p_g}\right)^{b-1} a\mathbf{v}\nabla k \qquad (5)$$

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assuming $\zeta < f$. Here $\mathbf{v}$ is the 500 mb wind and assuming a linear wind-profile $a\mathbf{v}$ gives the wind speed at the ground, $\mathbf{v}_g$. The value of the coefficient a has been determined from the empirical formula

$$\mathbf{v}_g = a\mathbf{v} = \mathbf{v}\left[1 - 0.75\frac{p_g - p}{p}\right]. \qquad (6)$$

Considering now that the expression

$$E^2 = \frac{f\varrho_g b}{p_g}\left(\frac{p}{p_g}\right)^{b-1} a \qquad (7)$$

is essentially positive and varies only little with x and y and furthermore that k is not a function of time, the vorticity equation may be written

$$\frac{d}{dt}(\eta + E^2 k) = 0, \qquad (8)$$

where now the terms A and C in (1) have been neglected.[1]

Eq. (8) has been applied to 500 mb fields in the case of northerly flow with good agreement between observed and computed height variations (REUTER & PICHLER, 1960*b*). The results show that the second term inside the brackets in (8) is necessary in order to account for height changes on the lee-side of the mountain.

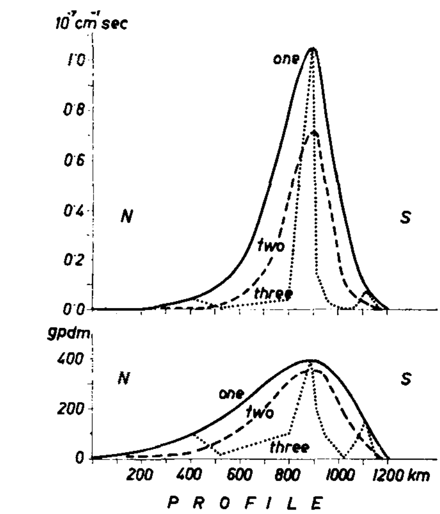

FIG. 1. The profile of the Alps along a meridian (10 degrees east) and the quantity E^2k as a function of distance.

[1] More correctly E^2 should be replaced by $\overline{E^2} = \int E^2 k\, ds / \int k\, ds$.

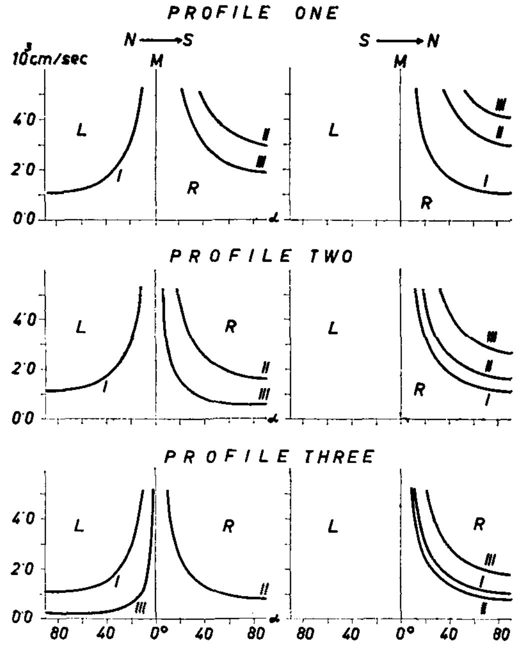

FIG. 2. Deflection angle α as a function of wind-speed for the three profiles shown in Fig. 1 and (I) computed from the coriolisterm, (II) from the orographic term and (III) for the total effect. L means deflection to left, R a deflection to right from the initial direction.

The mechanism of the orographically imposed divergence may best be demonstrated by considering particle trajectories. Integration of eq. (8) along a trajectory yields

$$\zeta + f + E^2 k = \zeta_0 + f_0 = \text{const.}, \qquad (9)$$

where subscript zero refers to an initial point outside the mountains. It is convenient here to consider a steady state case where the wind speed, c, is constant along a trajectory and where the initial point is chosen at an inflection point of the trajectory. Lateral shear will be neglected. Then, if K_s and K_t denote the curvature of a streamline and a trajectory, respectively, we have

$$\zeta = cK_t = cK_s = c\frac{d}{dy}\left\{\frac{dx}{dy}\left[1 + \left(\frac{dx}{dy}\right)^2\right]^{-\frac{1}{2}}\right\} = c\frac{d\sin\alpha}{dy}, \qquad (10)$$

where α is the angle of deflection of the trajectory from the north–south direction.

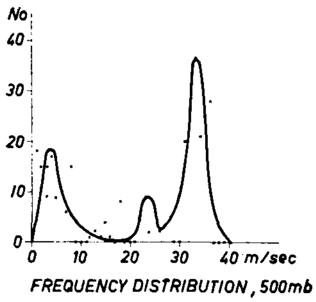

FIG. 3. Frequency distribution of windspeed for currents with north–south direction at the 500 mb surface obtained from radiosonde observations at Munich (West Germany).

Eqs. (9) and (10) give

$$(\sin \alpha)_{y_1} - (\sin \alpha)_{y_0} = \frac{1}{c} \int_{y_0}^{y_1} (f_0 - f) dy - \frac{1}{c} \int_{y_0}^{y_1} E^2 k dy \tag{11}$$

from which it is seen that the first term causes a deflection to the east and the second term a deflection to the west in the case of an initial north-southerly flow.

The integrals in (11) have been evaluated for three different mountain profiles, all corresponding to a cross section along the meridian 10° E but somewhat different with respect to form and to the degree of smoothing, see the lower part of Fig. 1. The variation of the function E^2k is shown in the upper part of Fig. 1.

After passage of the mountain the resulting deflection angle as well as the contributions from each of the two terms is given in Fig. 2 as a function of the wind speed. It is clear that profile one represents an upper and profile three a lower limit for the values of E^2k entering into the computations of the deflection angle. The most realistic case lies between these two extremes and is probably approximately given by profile two.

Since the resulting deflection depends very much on the wind speed a study has been made on the frequency distribution of wind

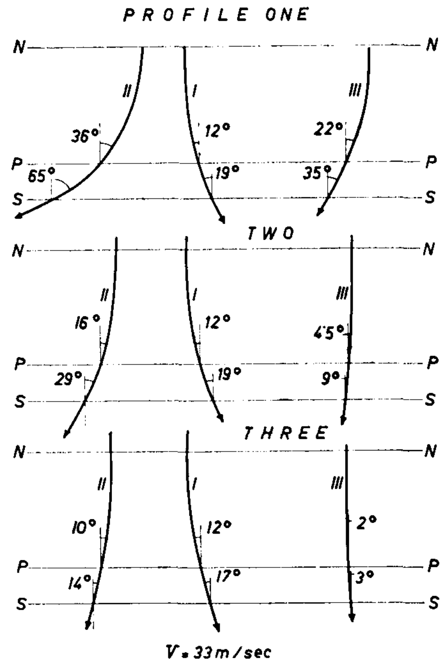

FIG. 4. The deflection angle α as a function of distance (N means the initial point, P the highest point of the profile, S the south point) for a windspeed of 33 m/sec and (I) for the coriolisterm (II) for the orographic term and (III) for the total effect in the case of north–south flow.

speed in straight north-southerly flow at 500 mb over Munich. The result is shown in Fig. 3 where the most frequent wind speed is 33 m sec^{-1}. By using this value the trajectories given in Fig. 4 were constructed. Considering that profile two probably is the most realistic we arrive at the result that in the most frequent cases the flow will pass the Alps with very little or no deflection. This means that the wave formation due to conservation of absolute vorticity will take place at lower latitudes than in a case without mountains.

In southerly flow the orographic and the coriolis terms have the same sign resulting in a larger deflection and thus in a reduction of the amplitude of the wave.

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