The parameters of atmospheric turbidity

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ABSTRACT

The methods for evaluating the atmospheric turbidity parameters, introduced by the present author in 1929–30, are subjected to a critical examination. A method first suggested by M. HEROVANU (1959) is here simplified and expanded, and used for deriving the named parameters in adherence to a procedure described by the present author in a previous paper in this journal (1961). The procedure is applied to the pyrheliometric observations at Potsdam in 1932–36, published by HOELPER (1939) A comparison between the frequency distribution of the coefficient of wave-length dependence α at the high level station Davos and the low level station Potsdam gives results which are discussed in detail. In all the figures of the present paper, where the turbidity coefficients occur, they are multiplied by 10^3.

1. Introduction

The regular radiation measurements within a meteorological net work are subject to considerable limitations. As a rule the observations refer to integrals over rather large intervals of wavelengths, covering for instance the whole energy spectrum of sun radiation, or of terrestrial radiation. Frequently measurements of total sun radiation $\int_0^\infty F(\lambda)d\lambda$ are now supplemented by pyrheliometric measurements with aid of certain internationally accepted glass filters, namely the Schott Filters OG 1, RG 2 and RG 8. Through the use of these filters we are able to evaluate the following integrals within solar radiation:

$$\int_{525}^{\infty} F(\lambda)d\lambda, \int_{630}^{\infty} F(\lambda)d\lambda, \int_{700}^{\infty} F(\lambda)d\lambda \qquad (1)$$

and combining these measurements with measurements of total solar radiation, also the integrals:

$$\int_{0}^{525} F(\lambda)d\lambda, \int_{0}^{630} F(\lambda)d\lambda, \int_{0}^{700} F(\lambda)d\lambda \qquad (2)$$

The corresponding measurements can be executed by aid of rather simple and comparatively inexpensive instruments—pyrheliometers in combination with glass filters—and by observers of qualifications generally represented within the staff of the more advanced meteorological observing stations.

It is evident that the results of the observations obtained in this way have their limitations but also their advantages, compared with the results of a more detailed spectrum analysis of the sun radiation. The former observations are unable to answer many questions concerned with the shape and occurrence of atmospheric absorption lines, and their dependence of for instance temperature and pressure. Minor traces of certain constituents of the atmosphere cannot be discovered through them, and in general, *full use of the simple integral measurements can only be obtained on the basis of results previously obtained with aid of more elaborate spectrometric devices.*

The advantage of the integral measurements, as they are now organized within the meteorological networks, lays chiefly in their possibility to furnish, through the activity of a great number of stations, a synoptic idea of the occurrence of certain parameters, which are correlated to the energy exchange through radiation, and through the use of which the climatological and statistical treatment of radiation data can be simplified and systematized. In addition to the fundamental astronomical parameters: solar constant, solar height and azimuth, these parameters are in the first place: the water

content of the atmosphere, which mainly determines the long wave absorption of radiation within the atmosphere, and *also certain parameters defining the scattering properties of the aerosol above the place of observation.* These lastnamed parameters and the methods for their derivation will be considered in the present note.

From the fundamental results of spectrobolometric investigations it may now be derived, that the value of the integrals [2] above, are all chiefly influenced by the scattering of the air molecules and by the extinction by the aerosol, viz. by the solid and fluid particles, always to some degree contained in the air, but only to a very small degree by selective absorption within certain permanent gaseous constituents of the atmosphere.

If due correction is made for the named absorption and for the scattering by the air molecules, which can be regarded as known within narrow limits, we are thus able to derive, from every one of the integral values [2], a measure of the total amount of energy lost from the primary beam through *the extinction by the aerosol*, within the region covered by the respective integral.

The spectral investigations give good reasons for supposing, that in the general case the extinction by the aerosol—either by scattering or absorption or both together—is a continuous function of wavelength, without selective bands or lines. We may put:

$$p_\lambda = e^{-f(\lambda)m}, \tag{3}$$

where p_λ is the transmission through the aerosole alone, other extinction effects eliminated, $f(\lambda)$ is the extinction function and m is the optical air mass, A practical approach, which covers many cases of a clear atmosphere is obtained by putting:

$$f(\lambda) = \frac{\beta}{\lambda^\alpha}, \tag{4}$$

where β is the extinction coefficient, corresponding to the wavelength 1μ and α is an exponent, closely correlated to the size of the scattering particles and, to the frequency of their size distribution (ÅNGSTRÖM, 1929; 1930). The empirical result has been lifted up on the theoretical plane by the investigations of VAN DE HULST (1951), FOITZIK (1950) and JUNGE (1951, 1952*a*, *b*, *c*). Junge has shown that a condition for that the aerosol extinction can be given for the whole spectrum by the simple expression (4), is that the frequency distribution of the particle size within the aerosol shall be governed by the exponential formula:

$$dN = -\frac{k}{r^\nu} \cdot dr, \tag{5}$$

where dN is the number of particles within the interval dr of the radius r, and K is a constant dependent upon certain physical parameters, characteristic for the particles, and upon their total mass.

The wavelength dependence of the extinction can under this assumption be given in the form:

$$f(\lambda) = \frac{\beta}{\lambda^{\nu-3}}, \tag{6}$$

an expression, which coincides with (4) if we put: $\alpha = v - 3$, viz. $v = \alpha + 3$.

A rather common case, represented especially by the spectrobolometric results from high level stations, is that, where α adopts a value of about 1.0 or slightly higher. This would imply a frequency distribution, where dN is inversely proportional to the fourth power of the radius. Junge (1952) has reported on cases where the frequency distribution of microscopic droplets comes very near to such a pattern. We shall make some further comments to this question after having considered some experimental results concerning the variations to which β and α are subject in given cases.

In a recent paper the present author has presented a simple mathematical procedure for determining as well β as α from measured values corresponding to the integrals [2] given above. Knowing the spectral distribution of the extraterrestrial solar radiation (given by Nicolet on the basis of the Smithsonian results) and also the effect of the Rayleigh scattering of the air molecules, and applying finally a small correction for the absorption by ozon, we may from every one of the three integrals [2] derive a value for β corresponding to a given value of α, let us call it a *standard value* and assume it to be 1.3. If in an actual given case α really has this standard value, the β-values derived from the three integrals on the basis of standard tables will all be equal to a common value β.

The same is also the case if we derive β-values from the differences

$$\int_0^{630} F(\lambda)d\lambda - \int_0^{525} F(\lambda)d\lambda = \Delta_1(\lambda)$$

and $$\int_0^{700} F(\lambda)d\lambda - \int_0^{630} F(\lambda)d\lambda = \Delta_2(\lambda).$$

In fact convenient tables or diagrams for deriving β from the named integrals and their differences are now found in articles by A. Ångström, O. Hoelper, A. Ångström and A. J. Drummond, M. Herovanu, in the Linkes Meteorologisches Taschenbuch and in the IGY instruction Manual, Part VI: Radiation Instruments and Measurements.

I have previously shown (ÅNGSTRÖM, 1961) that for our purpose of determining true values of β and α, we may regard the radiation defined by the integral $\int_0^{530} F(\lambda)\,d\lambda$, as well as the radiation defined by the differential band

$$\int_0^{710} F(\lambda)d\lambda - \int_0^{630} F(\lambda)d\lambda = \Delta_2(\lambda)$$

as monochromatic radiations, viz. treat them mathematically as such. We will denote the β-values derived from them through the indices g and rr, viz. β_g and β_{rr} respectively. For the case $\alpha = 1.3$, we have $\beta_g = \beta_{rr}$ in agreement with the assumptions, on which tables and diagrams generally are based. More generally we have:

$$\alpha = 1.3 - 5.94\,{}^{10}\log\frac{\beta_{rr}}{\beta_g} \tag{7}$$

and for the true β with index (o)

$$\beta_0 = (1.3 - \alpha)\,{}^{10}\log\lambda_1. \tag{8}$$

From (7) we get, as λ_1 is approximately $0.669\,\mu$

$${}^{10}\log\frac{\beta_o}{\beta_{rr}} = (1.3 - \alpha)\,0.175. \tag{9}$$

As a result of recent works concerning the transmission of the standard filters OG 1, RG 2 and RG 8 (ÅNGSTRÖM & DRUMMOND, 1961), the World Meteorological Organization accepted at its New Delhi Meeting 1962, an idealized standardization of 525, 630 and 700 mμ for the center of lower wavelength cutoff of these filters. with application of these standard limits of the integrals, we get instead of (7) and (8) the slightly modified equations:

$$\alpha = 1.3 - 6.02\,{}^{10}\log\frac{\beta_{rr}}{\beta_g} \tag{10}$$

and $${}^{10}\log\frac{\beta_0}{\beta_{rr}} = (1.3 - \alpha)\,0.177. \tag{11}$$

In the case of the differential band:

$$\int_0^{630} F(\lambda)d\lambda - \int_0^{525} F(\lambda)d\lambda = \Delta_1(\lambda)$$

with an effective wavelength of $0.575\,\mu$ combined with the band $\int_0^{525} F(\lambda)d\lambda$, with an effective wavelength of $0.450\,\mu$, we get if the respective β-values are denoted with β_{rg} and β_g, equations in analogy with (10) and (11), namely

$$\alpha = 1.3 - 9.32\,{}^{10}\log\frac{\beta_{rg}}{\beta_g} \tag{12}$$

and $${}^{10}\log\frac{\beta_0}{\beta_{rg}} = (1.3 - \alpha)\,0.24 \tag{13}$$

HEROVANU (1959) has derived values for α and β in a way similar to the one used here, but with application of tabular values for the integrals, instead of founding mathematical expressions on the assumption of monochromatic groups of radiation. From the Tables 5 and 6 of his paper it is evident that our equations [12] and [13], which apply to the same integrals as those considered by Herovanu, give, if modified to suit the terminology of Mr. Herovanu, results which coincide with his, for α within less than 0.1, and for β within less than 2 per cent. (See Table 1.) With due regard to the difficulties, adhering to exact radiation measurements and proper corrections of the same, it seems at present scarcely justified to introduce different constants in the formulas [10] to [13] for different air masses. A correction for air mass would as far as α is concerned only change its value with less than about ± 0.1 in the extreme cases ($1 <$ relative airmass < 4) and the corresponding value of β with less than 2 percent.

The formulas (12) and (13) of the present author then adopts the form:

TABLE 1.

	β'_{rg}/β'_g			β'_o/β'_{rg}	
	Herovanu		Ångström's	Herovanu	Ångström's
α	$M=1$	$M=4$	formula		formula
0.00	1.28	1.24	1.280	1.77	1.73
0.50	1.15	1.13	1.145	1.32	1.32
0.80	1.07	1.05	1.050	1.11	1.11
1.00	0.00	0.00	0.000	1.00	1.00
1.20	0.94	0.96	0.950	0.900	0.895
1.50	0.87	0.88	0.873	0.76	0.76
2.00	0.77	0.81	0.781	0.57	0.575

[*Remarks.* The values β'_{rg} β'_g and β'_o of Herovanu are derived from tables computed by him under the assumption $\alpha = 1$.]

$$\alpha = 1.0 - 9.32\,{}^{10}\log\frac{\beta'_{rg}}{\beta'_g}$$

$$\log\frac{\beta'_o}{\beta'_{rg}} = (1.0 - \alpha)\,0.24.$$

From these formulas the values of the fourth and sixth columns in the table are computed.

Observations on the basis of which an evaluation of α and β can be founded are still rather limited in number. Schüepp and following him P. Valko have tried to make such evaluations with application of a somewhat different procedure. A large number of observations, which for this purpose seem extremely valuable are published by HOELPER (1939), the prominent German meteorologist, early deceased in the time of World War II. On account of the inopportunity of the time of publication, these works of Hoelper have scarcely received the attention which they undoubtedly deserve. In tables collected by Hoelper, values are given of the integrals of radiation, which may be used for evaluation of the fundamental parameters, α and β, according to the method described above. As the chief purpose of the present note is more to draw attention to the value of the informations gained from a knowledge of the named parameters, than to aim at a complete and extensive treatment, for which time is not available, I will here limit myself to a discussion of the α- and β-values, as they can be derived from an extract from the tables of Hoelper.

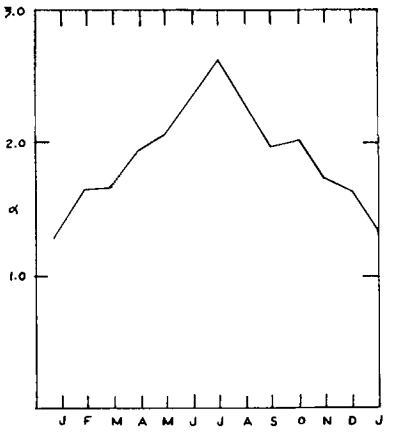

FIG. 1. Annual variation of wavelength exponent α, Potsdam 1932–1936.

2. The turbidity parameter α

Computing mean values of α for Potsdam corresponding to days, where more than one set of observations are available and limiting myself to the period beginning with the Polar Year 1932–33 and extended over 4 years, I derive the annual variation of α, shown graphically in Fig. 1, in which monthly mean values are plotted for each month. A very marked annual variation is apparent from the lastnamed curve, with a maximum value slightly above 2, in the summer, June–July, and an equally pronounced minimum of slightly above 1.0 in January.

From 336 separate values the frequency distribution is derived and shown in Fig. 2. There are, evidently, pronounced "bands" of high frequency alternating with equally pronounced minimum values. A comparison with the frequency distribution derived from the α-

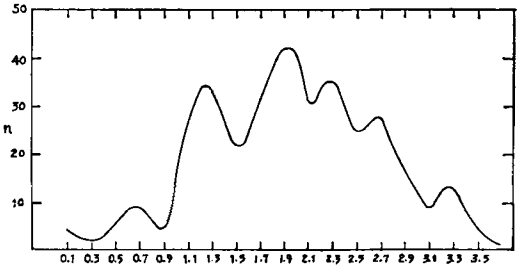

FIG. 2. α-values, frequency, whole year Potsdam 1932–1936, 336 values derived by present author from data of Hoelper.

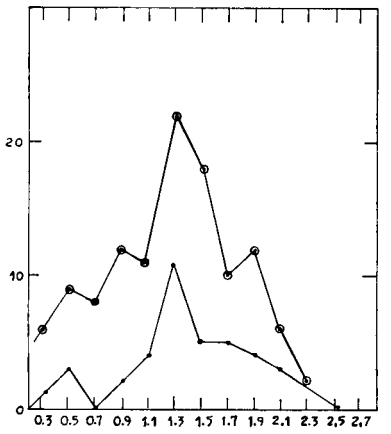

FIG. 3. α-values frequency. ··· Potsdam, winter 1932–1936 (41 values derived by present author from data by Hoelper);

⊙⊙⊙ Davos 20.3.35–26.5.36, 1.1.45– 5.7.46, 2.10.42–20.6.43, 1.4.46– 4.4.46 } 116 values according to Schüepp.

determinations from Davos, published by SCHÜEPP (1949) is instructive. The frequency of the lastnamed α-values is given by a curve of Fig. 3. Apart from minor irregularities which may be ascribed to accidental effects, we find here one single pronounced maximum corresponding to an α-value of 1.3–1.4.

Even if care must be taken not to draw too wide conclusions from the rather limited number of observations, it seems evident that there is in the Potsdam values a rather widespread distribution over a wide range of different α-values. The pronounced maximum within the range 0.9–1.5, which is rather characteristic for high level stations, as Davos and the Smithsonian Mountain stations, is highly reduced in the Potsdam values and instead we find here a rather marked maximum of frequency corresponding to an α-value of 1.9–2.0. Secondary maximums occur, however, as well at $\alpha = 1.3$ to 1.4, closely coinciding with the maximum value at Davos, as also but less marked and probably purely accidental at $\alpha = 2.3$ to 2.4, and at $\alpha = 2.7$ to 2.8.

The interpretation of the frequency spectrum invites to some discussion. Accepting the theory of Junge, we may simply regard a change of α from, let us say 1.0 to 2.0 as a change in the size distribution as expressed through equation [5] above, from having an exponent 4 to an exponent 5, which would mean that the small particles would be more highly represented in the case of the higher α-value. The more frequent occurrence in almost equidistant bands, of certain preferred α-values, seem on the other hand to invite to a question, whether or not the function [5] ought to be replaced by an expression of the type:

$$dN = \frac{K_1}{r_1^{v_1}} dr_1 + \frac{K_2}{r_2^{v_2}} dr_2 + \frac{K_3}{r_3^{v_3}} dr_3 + \ldots \qquad (14)$$

with a sacrifice of the continuous character of [5]. Here r_1, r_2 and r_3 are certain preferred values of the radius of the particles. Also with the application of a formula like [14] we would find that the extinction could, practically, be given through an expression characterized by a λ-dependence, as found by experiments.

3. The turbidity coefficient

We now proceed to compute the actual β-values from equation (13), using the α-values derived from (12) and using the values of β_{rg} and β_g for Potsdam, published by HOELPER. As regards these values we consider especially those which are mean values of two or more individual values, and obtain then the mean values for the various months graphically plotted in Fig. 4 under the heading β_0. These values represent the extinction coefficient of the aerosol at the wavelength 1 μ. For evaluations of the influence of aerosole extinction on the water vapor absorption the extinction coefficient, β_0, thus defined has actuality. Its annual variation is determined by as well the amount and the size of the scattering particles. Their combined influence gives an annual variation during the polar year, with a maximum in the early autumn and a minimum in March or April. More representative for the integral scattering of sun radiation is the aerosole extinction $\varepsilon_{0.5}$ corresponding to the wavelength 0.5 μ. It may be computed from:

$$\varepsilon_{0.5} = \frac{\beta_0}{0.5^{\alpha}} = 2^{\alpha} \cdot \beta_0. \qquad (15)$$

The value of the factor 2^{α} is given in Table 3, under the heading $\varepsilon_{0.5}/\beta_0$. The annual variation of $\varepsilon_{0.5}$ during the polar year at Potsdam is

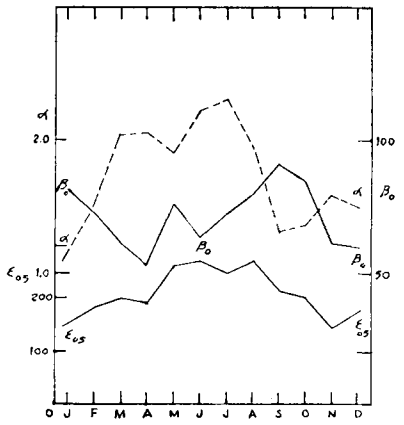

FIG. 4. Potsdam: Polar year (August 1932–July 1933) annual variation of $\beta_0\, \varepsilon_{0.5}$ and α.

demonstrated in Fig. 4. The variation is rather regular showing a flat maximum in the summer, May to August, and a minimum in November to January. The annual trend corresponds rather well to the traditional variation of turbidity under conditions of a constant α. The curve lets us conclude that in the summer, at high sun, not less than about 30 percent of the sun radiation in the visible spectrum is absorbed and scattered by the aerosole. In the winter the extinction referred to unit airmass is considerably less, but as the average airmass is larger in about the same proportion, there is probably a rather constant percentic loss of energy from this reason during the different parts of the year as far as clear sky conditions are concerned.

If we multiply $\varepsilon_{0.5}$ with $^{10}\log e$, viz. with 0.432, we obtain the turbidity coefficient B of Schüepp.

Here a rather remarkable coincidence which practically is of some importance must be pointed out. If we for the individual days of observation in Potsdam compare the B-values, computed in the way just indicated, with the β-values derived by Hoelper, *from measurements with the RG 2-filter*, we find a close agreement, numerically, between both (Fig. 5). In fact the correlation, if some few extreme deviations, evidently arisen from error of observation, are excluded, is as high as 0.95. The relationship between B and β_r can be expressed through the simple linear equation:

$$B = 1.05\, \beta_r, \tag{16}$$

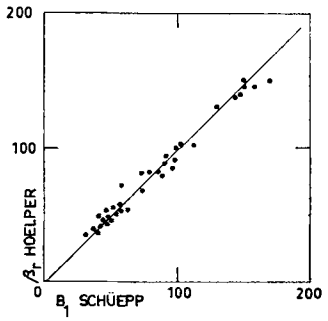

FIG. 5. Turbidity coefficient β_r derived by Hoelper against coefficient B of Schüepp.

where the constant practically seems to be independent of the parameter α. Let us assume a monochromatic radiation of wavelength λ to be used for determining β_1. The transmission, with elimination of Rayleigh scattering and selective absorption may be p_λ. We have under the assumption $\alpha = 1.3$:

$$\log p_\lambda = -\frac{\beta_1}{\lambda^{1.3}}. \tag{17}$$

The real coefficient β_0, however, is given by

$$\log p_\lambda = -\frac{\beta_0}{\lambda^{\alpha}}. \tag{18}$$

From (17) and (18) we get:

$$\frac{\beta_0}{\beta_1} = \frac{\lambda^{\alpha}}{\lambda^{1.3}}. \tag{19}$$

The extinction $\varepsilon_{0.5}$ at $\lambda = 0.5$ is given by

$$\varepsilon_{05} = \beta_0 \cdot 2^{\alpha} \tag{20}$$

and from (20) and (19)

$$\varepsilon_{0.5} = \frac{\beta_1 \lambda^{\alpha} \cdot 2^{\alpha}}{\lambda^{1.3}} \tag{21}$$

and

$$B = \varepsilon_{05}\, {}^{10}\log e = \frac{\beta_1 \lambda^{\alpha} \cdot 2^{\alpha}}{\lambda^{1.3}} \cdot 0.434. \tag{22}$$

If the monochromatic radiation used for determining β_1, has the effective wavelength 0.5, the ratio between B and β_1, is constant and equal to 1.07. Let us replace λ by the effective wavelength of the short wave radiation corresponding to the filter RG 2. This wave-

TABLE 2.

(1) α	(2) β_{rg}/β_g	(3) β_{rr}/β_g	(4) β_0/β_{rg}	(5) β_0/β_{rr}	(6) $\varepsilon_{0.5}/\beta_0$	(7) B/β_{rg}	(8) B/β_{rr}	(9) B/β_r	(10) $B/\varepsilon_{0.5}$	(11) B/β_0
0.0	1.30	1.64	3.05	1.70	1.00	0.89	0.74	1.13	0.434	0.43
0.1	1.35	1.58	1.94	1.63	1.07	0.90	0.76	1.13		0.43
0.2	1.31	1.52	1.84	1.57	1.15	0.92	0.78	1.12		0.50
0.3	1.28	1.47	1.74	1.50	1.23	0.93	0.80	1.12		0.53
0.4	1.25	1.41	1.64	1.44	1.32	0.94	0.82	1.11		0.57
0.5	1.22	1.36	1.55	1.38	1.41	0.95	0.84	1.11		0.61
0.6	1.19	1.31	1.47	1.33	1.52	0.96	0.87	1.11		0.65
0.7	1.16	1.26	1.39	1.28	1.62	0.97	0.89	1.10		0.70
0.8	1.13	1.21	1.32	1.23	1.74	0.99	0.92	1.10		0.75
0.9	1.10	1.16	1.25	1.18	1.87	1.01	0.95	1.09		0.81
1.0	1.08	1.12	1.18	1.13	2.00	1.02	0.98	1.09		0.87
1.1	1.05	1.08	1.12	1.08	2.14	1.04	1.01	1.09		0.91
1.2	1.02	1.04	1.06	1.04	2.29	1.05	1.04	1.08		0.98
1.3	1.00	1.00	1.00	1.00	2.45	1.06	1.07	1.08		1.07
1.4	0.97	0.96	0.95	0.96	2.63	1.08	1.09	1.08		1.14
1.5	0.95	0.92	0.89	0.92	2.82	1.09	1.12	1.07		1.22
1.6	0.93	0.89	0.85	0.89	3.02	1.11	1.16	1.07		1.30
1.7	0.91	0.86	0.80	0.86	3.24	1.12	1.20	1.06		1.40
1.8	0.88	0.83	0.76	0.82	3.48	1.14	1.24	1.05		1.51
1.9	0.86	0.80	0.72	0.78	3.74	1.16	1.27	1.05		1.63
2.0	0.84	0.77	0.68	0.75	4.00	1.18	1.30	1.04		1.74
2.1	0.82	0.74	0.64	0.72	4.27	1.19	1.34	1.04		1.9
2.2	0.79	0.71	0.61	0.69	4.57	1.21	1.38	1.04		2.0
2.3	0.77	0.68	0.58	0.66	4.88	1.22	1.42	1.03		2.2
2.4	0.75	0.65	0.54	0.63	5.20	1.23	1.46	1.03		2.3
2.5	0.74	0.63	0.51	0.61	5.62	1.24	1.49	1.02		2.5
2.6	0.72	0.61	0.49	0.58	6.03	1.26	1.54	1.02		2.7
2.7	0.70	0.59	0.46	0.56	6.50	1.28	1.59	1.01		2.8
2.8	0.69	0.56	0.44	0.54	6.95	1.30	1.64	1.01		3.0
2.9	0.67	0.54	0.41	0.52	7.45	1.32	1.68	1.00		3.2
3.0	0.66	0.52	0.39	0.50	7.98	1.34	1.73	1.00		3.5
3.1	0.65	0.50	0.37	0.48	8.52	1.36	1.78	1.00		3.7
3.2	0.64	0.48	0.35	0.46	9.16	1.38	1.86	0.99		4.0
3.3	0.62	0.46	0.33	0.44	9.84	1.40	1.89	0.99		4.3
3.4	0.60	0.44	0.31	0.42	10.5	1.42	1.93	0.99		4.6
3.5	0.58	0.43	0.30	0.41	11.2	1.45	1.98	0.98		4.9
3.6	0.57	0.41	0.28	0.39	12.1	1.47	2.03	0.98		5.2
3.7	0.56	0.40	0.27	0.37	12.9	1.50	2.08	0.98		5.6
3.8	0.54	0.38	0.26	0.35	13.8	1.53	2.13	0.97		6.1
3.9	0.52	0.37	0.24	0.33	15.0	1.56	2.18	0.97		6.6
4.0	0.51	0.36	0.23	0.32	17.0	1.59	2.23	0.96		7.0

Remarks. The turbidity coefficients β_r, β_g, β_{rg} and β_{rr} are all supposed to be derived from standard tables, erected under the assumption $\alpha = 1.3$. β_r refers to the short wave radiation derived from measurements with the RG2 filter, β_g to the short-wave radiation given from measurements with the filter OG2. β_{rg} and β_{rr} refers to the bands defined as differences: RG2–OG1 and RG8–RG2 respectively. *Example for use of table.* From value of β_{rg}/β_g given in column 2 find corresponding value of α in first column. From the α-value we find in column 4 the factor with which β_{rg} shall be multiplied to give β_0.

length is slightly less than 0.5 and can for air-masses between 1.0 and 4.0 with good approximation be given by: $\lambda = 0.48\ \mu$. We then obtain:

$$\beta = 1.090\ \beta_r \text{ for the case } \alpha = 1.0 \text{ and}$$
$$= 1.045\ \beta_r \text{ for the case } \alpha = 2.0$$

Practically we get a good agreement with the empirically derived relationship (16).

From a practical point of view the result seems important:

The β values derived by means of the standard filter RG 2 under the assumption $\alpha = 1.3$, are related to the turbidity coefficient B of Schüepp by a constant factor, which is practically independent of α, and only slightly differs from unity. This law holds in the case of the β-values obtained by means of the filter RG 2 but fails, when the effective wavelength of the filtered radiation differs appreciably from 0.5. In such cases, with application of the filters OG 1 and RG 8, as an instance, the constant of equation (16) varies considerably with the value of α, as may be seen from columns 7 and 8 of Table 2.

4. Synthesis concerning the turbidity parameters

It is evident that at low altitudes like Potsdam the spreading of the α-values is so great, that it is first through a knowledge of both the parameters α and β, that we are able to form a clear opinion as regards the aerosole extinction within various parts of the spectrum. This is clear already from the fact, that the extinction at 1.0 μ, expressed through β_0, has an almost opposite annual variation, as the extinction at 0.5 μ, expressed through $\varepsilon_{0,5}$ or through the turbidity coefficient B. The reason for this is included in the fact that at Potsdam the α-values are characterized by an annual variation opposite to that of β. As B is related to β_0 by the equation:

$$B = 0.434\ \beta_0 \cdot 2^{\alpha}$$

high values of α tend to produce high values of B relative to β_0. Both the turbidity coefficients, β_0 and B, each considered alone, are thus liable to give misleading conceptions of the extinction at different wavelengths, if they are applied to individual cases, without due regard to the value of the wavelength exponent α.

It is interesting to note that in the frequency curve for Potsdam, there appears a secondary maximum for $\alpha = 1.3$–1.4, viz. for just the α-values, where the head-maximum generally occurs at high-level stations like Davos.

In fact at Potsdam the winter values for the time December to February show a frequency curve very similar to that of Davos with practically a single frequency maximum for $\alpha = 1.3$ to 1.4. (Fig. 3.)

If we disregard the winter values, the frequency curve for Potsdam shows practically one single maximum at about $\alpha = 1.9$ to 2.0; the maximum at 1.3 to 1.4 has then disappeared. This means that the scattering particles are generally larger in winter than in summer at Potsdam, and that they in the winter adopt a size distribution very similar to what is characteristic for high-level stations.

It is evident, especially from the very extensive investigations of JUNGE, (1952*a*, *c*) that the size of the particles within the aerosol, is determined by a number of factors, of which only few are accessible by simple measurements. As a rule the particles are what Junge calls "mixed nuclei": a small central solid nucleus, on which a liquid solution is deposited to give the form and character of a small droplet. In the case of the maritime haze particles, the the solid part of the nucleus is often missing, and the particle is then simply a small droplet, originally probably deposited on a hygroscopic salt particle. It seems probable that within fresh maritime air over the continent the particles are often largely of the lastnamed kind, viz. simply droplets of a liquid salt solution.

The chief parameters influencing the size of these droplets are temperature and relative humidity of the surrounding air, size and character of the original solid nucleus, and, which has been a rather neglected factor, the radiation to which the particles are subjected. It seems rather natural that the spreading within the size spectrum shall be, as in our result, larger at a low altitude station like Potsdam than at a high altitude station like Davos. In the former case various kinds of particles are carried from the ground and also originating from industrial combustion, in much larger extent than in the mountain regions. From the observations here treated, it is suggested that the size of the particles emanating from ground and from combustion, is in general *smaller* than the particles of the more pure atmospheres

above our mountain stations. The results of various authors concerning the difference in particle size between continental and maritime air deserves attention, (VALKO, 1961) but must be taken with precaution, as it is generally derived without a close consideration of the effect of the fundamental parameters: temperature and relative humidity within the air layers involved, and without regard to pronounced annual variations from other causes than given by the origin of the air masses.

The difference in α between high and low level stations has been brought forward also through a statistical investigation by SCHÜEPP (1949) who found values of α equal to 1.37 for high level stations compared with 1.48 for stations at low levels. MÖLLER (1957) has, in a comprehensive survey, mentioned these findings as a meteorological paradox, worth a closer examination. Such an investigation demands, however, a number of factors to be considered.

5. Historical and critical comments

It is evident from photometric and bolometric investigations of the solar radiation, as it appears after having passed the atmosphere, that the acceptance of a single parameter α, valid for the whole spectrum, represents a simplification of conditions which in reality are much more complicated.

Thus, as regards the extinction by the aerosol, VASSY (1939) distinguishes three types according to the dependence on λ: (*a*) roughly neutral extinction, (*b*) extinction maximal at 5000 Å, (*c*) extinction gradually decreasing with increasing wavelength in the interval $\lambda = 4000$–6000 Å. DESSENS (1947, 1949) adds still a type: (*d*) extinction maximal near 4000 Å (KUIPER, 1952).

With due regard to the extensive spectrobolometric registrations of the Astrophysical Observatory of Smithsonian Institution it may be added that evidently in the cases of clear air and low humidity the type (*c*) is strongly prevailing. The interval within which the extinction is continuously decreasing, may under these conditions be extended to at least 4000–7000 Å, and in many cases up above 1 μ (SCHÜEPP, 1949).

The attempts to express the extinction through the equation (4) in our previous treatment are thus apt to meet with success in cases where we are concerned with the types of extinction (*a*), (*c*) and (*d*), if we disregard to ultraviolet spectrum, but leads in the other cases to a rough estimate, which occasionally may be misleading.

We must thus realize *that in general α is no constant of nature but simply a parameter defined by the arbitrary procedure we choose for its determination*. Generally we derive it from the measurements of two distinct wavelength integrals $\sum \lambda_1$, and $\sum \lambda_2$ under the assumption that equation (4) holds. As this assumption in most cases is only approximately true, the derived α-value is more or less dependent upon *which wavelength groups* we choose for its determination. *The justification of a general determination of α, under the simplifying assumptions made, must then be sought in the possibilities which it creates of correlating the α-values thus found with other conditions*. The merits of a determination of both the turbidity parameters α and β can be summarized in the following main points.

1. They enable us to evaluate the total amount of scattering and absorbing particles in the atmosphere above, in a more exact—or perhaps more correctly: less inexact—way than through a determination of β alone. The optical density of the aerosol thus determined is a factor which ought to be considered in connection with the problem concerning the total albedo of the earth, where the albedo of the cloudfree atmosphere is a factor of some importance (VALKO, 1961; MCCORMICK & BAULCH, 1962; ÅNGSTRÖM, 1962).

2. The α-values give an easily attained rough estimate of the predominating particle size characteristic for the aerosol.

3. It seems evident that the solar constant values derived by the Smithsonian Institution are to some extent influenced by changes in the atmosphere, a fact which explains the rather low correlation found between simultaneous solar constant values from different localities. As most other "sources of error" seem to have been effectively eliminated in the "short method" applied by Abbot, there is still a possibility that the particle size has had an influence on the spectral distribution of the light from the aureole around the sun, and thus indirectly on the derived solar constant values.

Abbot claims to have found relationships between his apparent solar constant values and

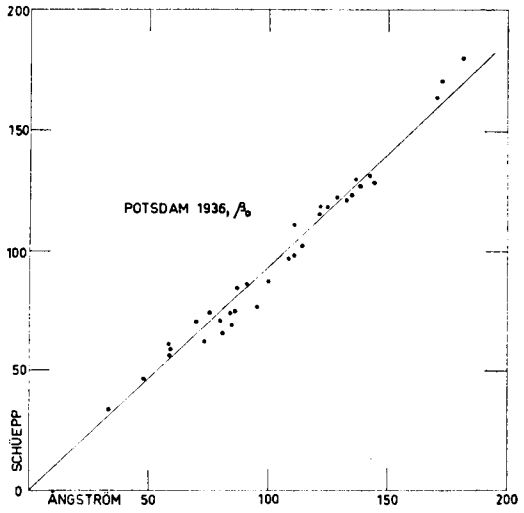

FIG. 6. Comparison of values for β_0 derived according to methods of Schüepp and of the present author.

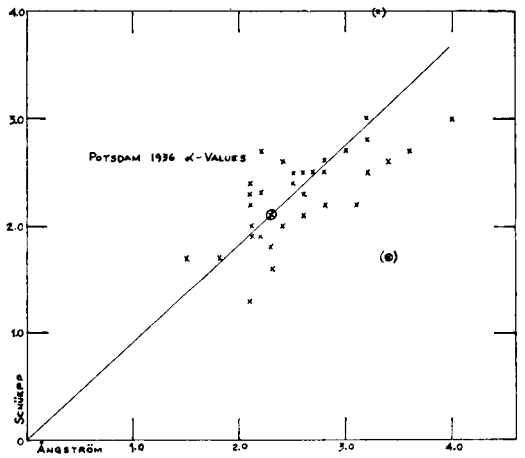

FIG. 7. Comparison of values for α derived by Schüepp and by present author, respectively.

the weather, especially precipitation, over wide regions (ABBOT, 1960). The possibility of a relationship between particle size within the aerosole and the further development within the atmosphere as regards condensation and precipitation, seems to include a field where the observations from the meteorological network of radiation stations, and a derivation from these observations of the turbidity parameters, may offer opportunities for further progress.

4. The determination of the parameters α and β, furnishes a basis for *interpolations* and *extrapolations* within the solar spectrum, for which purpose spectrobolometric records only rarely are at our disposal.

In a previous paper I have emphasized the need of greater accuracy in the radiation measurements, on which all methods for determining the turbidity parameters must be based. The recent works (ÅNGSTRÖM, 1961; ÅNGSTRÖM & DRUMMOND, 1961) on filter absorption have made it evident that the radiation behind the glass filters here concerned can not be corrected to represent standard conditions simply by applying a given filter factor. As the filters differ mainly through a smaller or larger shift from standard conditions as regards the "lower cutoff", the filter factor must vary with airmass and turbidity. In applying a constant filter factor, errors in the measured radiation values of up to 3 per cent may occur, when measurements are made under other conditions than those which prevailed at the determination of the filter factor.

The corresponding errors in the β-values may under extreme conditions reach 5 per cent or slightly more, and as they to some extent are of systematic character, a corresponding error in α of about ± 0.3 may be expected. Most of the observations published by Hoelper, Schüepp and Valko are subject to this criticism. The absolute values derived from these observations must therefore—especially concerning α—be accepted with some reserve. As the α-values here derived are all founded upon the observations published by Hoelper, the same is true concerning our own results, as far as absolute values are concerned.

A limited part of the same observations from Potsdam which have been treated here, have also been used by Schüepp for deriving values of α and β through a method using graphical interpolations. From Figs. 6–8 it may be derived that the two methods—Schüepps and our own—give results, which are in reasonably good agreement. The graphical interpolation procedure seems in general not to invite to a high degree of precision, and some divergences at great airmass values may be due to approximations in the method here applied. A further reason for discrepancies lays in the slightly different functions for the scattering by the dustfree clear atmosphere applied by dif-

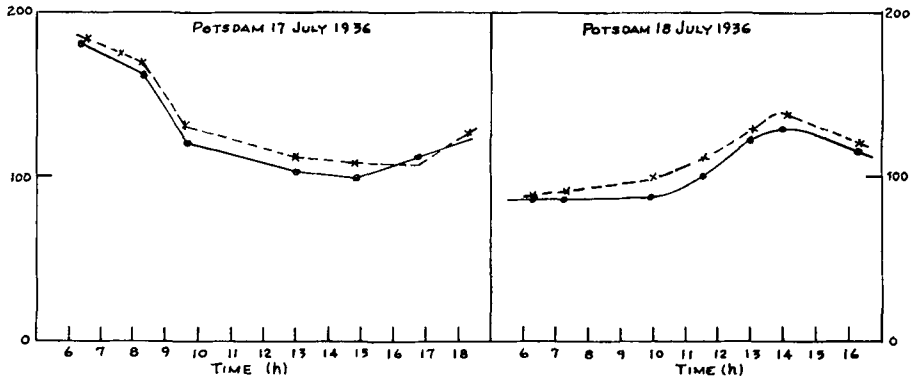

FIG. 8. Turbidity coefficient β_0, computed according to the method of Herovanu–Ångström (× × ×) and according to the method of Schüepp (○○○).

ferent authors. In eliminating the molecular scattering in order to get the extinction by the aerosole alone, the authors thus obtain slightly different values for the lastnamed extinction, and consequently also for the α-values derived from them. The divergency resulting herefrom in the derived β-values is not very large, but considerable differences appear in the α-values. The main cause of such differences seem to lay in the different assumptions made concerning the value of the *depolarization factor*, appearing in the Rayleigh-formula for the molecular scattering. In my recent publications I have used the value 1.061, derived by Penndorf, who has given good reasons for its correctness. Schüepp has, in his thesis, from which a number of values are here taken for comparison, used the value 1.074 of Cabannes, a value also found in a table given in Linkes Meteorologisches Taschenbuch. In the literature we frequently find the value 1.042 of van de Hulst. How the different assumptions concerning the depolarization factor influence the α-values, at the use of the methods, here applied or discussed, is shown in Table 3 below:

TABLE 3.

	Penndorf (1.061)	Cabannes and Götz (1.074)	van de Hulst (1.042)
$\beta = 0.025$	0	− 0.32	+ 0.46
$\beta = 0.05$	0	− 0.17	+ 0.22
$\beta = 0.10$	0	− 0.08	+ 0.17
$\beta = 0.20$	0	− 0.045	+ 0.05

If we accept the values 1.061 of Penndorf, we must thus regard the α-values of Schüepp, and probably also those of Valko, as too low. The correction, which ought to be applied to them is dependent on the β-values, and reaches appreciable amounts first at low turbidity values. The extinction by the pure atmosphere, on which Hoelper bases his computations is the one derived experimentally from high level measurements by Fowle to which Hoelper applies a correction for absorption by ozone. As has been shown by Götz this extinction is slightly lower than what would be obtained assuming a depolarization factor of 1.074. They seem, however, to correspond rather well to a value of the depolarization factor as given by Penndorf.

Without entering at present further into these divergences,—a higher accuracy in the actinometric filter measurements must be reached before a more detailed discussion of this question may be profitable—we may here finish by stating that the present methods—either we use the method of Herovanu and myself, or the one by Schüepp and Valko—both already now give results which from meteorological point of view, appear accurate enough to be of practical value.

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