An experiment in three-dimensional objective analysis[1]

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ABSTRACT

Some experiments in the three-dimensional objective analysis of contour height structure are described. In the scheme adopted, a family of surfaces is fitted to the data available in the vicinity of each grid point and the resulting analysis is in the form of four parameters defining the contour heights above the point at all pressure levels up to 200 mb. The scheme has the merit that many varieties of data can be exploited simultaneously. Verification statistics indicate that the analyses are good except in the lower stratosphere and an additional parameter would be necessary to overcome this shortcoming.

Introduction

Methods for the objective analysis of fields of contour height and other parameters have been developed in many countries and now form an established part of the procedure leading up to the preparation of numerical forecasts. The requirement is normally for the determination of values at a regular array of grid points by interpolation between irregularly distributed observations and the main advantage of the objective analysis systems is the considerable saving of time and labour which is effected. So far, objective analyses have been computed on a two-dimensional basis, that is the process used has essentially been applied independently at the various levels required, as with the subjective analysis of contour charts.

It is true that when constructing charts for several levels the forecaster adopts various procedures to achieve vertical consistency and to help levels which are deficient in data by extrapolation from levels where data is more plentiful; equivalent steps can be taken in the computed analyses by introducing statistical and empirical links between the various levels. Nevertheless, such three-dimensional representations of, say, the contour height structure are basically derived as a series of two-dimensional analyses. The forecaster is limited to this form of analysis because he can only draw contours for one level at a time, but there is in principle no reason why we should not objectively analyse all levels simultaneously in the course of one computation. This we regard as full three-dimensional analysis and in the present paper some preliminary experiments on such a basis are briefly described.

Analysis procedure

In the two-dimensional analysis system used in the Meteorological Office, a value is interpolated at each grid point in turn by fitting a bivariate second degree polynomial to the data in its vicinity. The natural three-dimensional extension of this system is to fit a suitable family of surfaces $z = f(x, y, p)$ to the nearby data at all levels. The family requires sufficient flexibility to provide for variations in pattern from level to level and the choice tentatively adopted has the form:

$$z = Q_1 L^3 + Q_2 L^2 + Q_3 L + Q_4, \qquad (1)$$

where

$$Q_r = a_r x^2 + b_r y^2 + 2h_r xy + 2g_r x + 2f_r y + c_r$$

and $\quad L = \log (p/p_0), \quad p_0 = 1000$ mb.

This family has 24 degrees of freedom and represents a family of surfaces which are quadrics (elliptic or hyperbolic paraboloids); the vertical variation of contour height z above any point (x, y) is cubic in log p. It was realised that four degrees of freedom might prove inadequate to represent accurately the observed vertical structure above a point in all cases but it was

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decided to experiment on this basis to acquire some experience of the general method.

In order to determine the family of surfaces to be used for interpolation at one grid point, we minimize the expression:

$$E = \sum\sum lm(z-z_0)^2 + T^2 \sum\sum lm(\mathbf{V}-\mathbf{V}_0)^2 + \sum\sum l'm(z-z_F)^2. \qquad (2)$$

In eq. (2) z, $\mathbf{V}$ represent the contour height and geostrophic wind implied by eq. (1), z_0 and $\mathbf{V}_0$ are observed values and z_F forecast values.

l is a horizontal distance weighting function introduced to ensure that the analysis for a particular grid point is biased towards those observations which are nearest; l was given the form $1/(1+\mathrm{K}r^4)$, r being the distance between observation and grid point and K a constant determining the range beyond which the effect of an observation fell off rapidly. The numerical values used for K were 0.186 for surface observations and 0.0116 for upper air observations. With r expressed in grid lengths the former value results in a sharp falling off of l between 1 and 2 grid lengths and between 2 and 4 grid lengths for the latter value of K. This discrimination was imposed because of the greater sparsity of upper air observations which meant that the required number generally came from a much larger area around the grid point under consideration than was the case with surface data.

A similar horizontal distance weighting function l' was applied to the forecast values and this was taken as 1/32 of the weight given to an observation at the same distance, i.e. $l' = \frac{1}{32}l$. This greatly reduced weight was used here to ensure that where observations were available any contamination of the analysis from bad forecast values would be negligible.

m is a vertical weighting factor designed to achieve a closer fit at low than at higher levels, in accord with the known falling off of data accuracy with height; m was taken as 1 at 1000 mb. falling off linearly with pressure to 0.4 at 200 mb.

In eq. (2) a dimensional parameter T^2 is needed to bring the dimensions of the first two terms into line; with z in m and $\mathbf{V}$ in msec^{-1} T^2 has the dimensions sec^2. By varying the numerical value assigned to T^2 we can allow the weight given to the wind observations relative to the contour height observations to be varied. On the basis of extensive experience of a similar fitting process in two-dimensional analysis, a value of $T^2 = 16$ sec^2 was used in the present work.

We write double summations in Eq. (2) to indicate that the summations have to be carried out three-dimensionally viz. horizontally over the available data as well as vertically up the soundings. It is to be noted that all kinds of data can be injected into the process simultaneously, e.g. conventional upper air data, surface observations in the form of 1000 mb heights and winds, aircraft winds, D values, etc. These data need not refer to standard pressure levels. In addition to the vertical and horizontal distance weighting, it would be a simple matter to discriminate between different kinds of data on grounds of quality.

Minimization of the expression eq. (2) leads to 24 linear algebraic equations which were solved by pivotal condensation. The resulting analysis is in the form of four parameters A, B, C, D for each grid point defining the vertical structure of contour height as $z = AL^3 + BL^2 + CL + D$, and the contour field for any pressure level is then determined. The four parameters can be identified as the 1000 mb height (D), parameters related to the temperature (C) and lapse rate (B) at 1000 mb, also the curvature (A) of the temperature profile. We note that as

$$T = \frac{p}{R\varrho} = -\frac{g}{R}\frac{\partial z}{\partial(\log p)}$$

the implied vertical temperature structure above any point is quadratic in log p.

Results

Seven experimental three-dimensional analyses were computed for 00 GMT on 22 to 28 November 1961 using all available data both surface and upper air (seven levels) up to 200 mb. A 24×20 grid was used extending from Hudson Bay to the Urals in the North and to the North African coast in the south. For the analysis at any one grid point up to four upper air soundings and up to 20 surface observations within four grid lengths were used as available. The forecast values which enter in the third term of the right-hand side of eq. (1) fulfil the dual purpose of ensuring that the family of surfaces is determinate even in sparse data areas and that in those circumstances the analysis deri-

TABLE 1. *RMS discrepancies of contour height (m) between objective and subjective analyses.*

Pressure level (mb)	22 Nov. 1961	23 Nov. 1961	24 Nov. 1961	25 Nov. 1961	26 Nov. 1961	27 Nov. 1961	28 Nov. 1961	Overall RMS value
1000	18	14	25	17	12	12	20	17
700	17	17	33	19	15	20	19	21
500	22	19	26	19	20	18	21	21
300	34	30	36	35	35	30	29	33
200	75	77	77	89	84	66	66	77

ves appropriate benefit from history. Initially, 27 forecast values were used, nine at each of the levels 1000, 500 and 200 mb, the nine values being those at the grid points surrounding and including that under consideration. However, in areas devoid of data, although a good fit to the forecast values at these pressure levels was achieved, the contour heights implied at other levels were often absurd. This was overcome by introducing additionally forecast values interpolated at two intermediate levels, making 45 values in all. The analysis in its parametric form is readily converted to fields of contour height at any required pressure levels.

The three-dimensional analyses cannot be illustrated fully here for all levels because of the large number of diagrams involved but two examples for particular levels are given later. In the meantime as a general assessment of their quality it may be said that at all levels up to 500 mb the analyses were on the whole very good, indeed at least as good as those we are accustomed to obtain from the two-dimensional process. This may have been partly because each level derived benefit from neighbouring levels. There was some tendency for the intense depressions to be too shallow in the three-dimensional analyses, but this fault initially beset the two dimensional analyses and could be remedied by similar methods (see CORBY, 1961).

At higher levels, however, the analysis errors increased until at 200 mb they were much too large for the analyses to be regarded as acceptable. This is brought out in Table 1 which gives the root mean square discrepancies between the objective and subjective analyses evaluated over the grid for the various levels.

The root mean square discrepancies of geostrophic wind given in Table 2 show a similar trend.

In setting a standard for objective analyses, a stringent but reasonable demand is that they should differ from their subjective counterparts by no more than analyses drawn independently from the same data by different analysts differ from each other. This attitude acknowledges the skill and experience which enters into the hand-drawn analyses and properly regards them as a target to be aimed for in the computed analyses. The work of BERGGREN (1957) in comparing analyses of the same data by different forecasters suggests that at 500 mb the root mean square discrepancy to be expected between two independent hand-drawn analyses is of the order of 20–25 m. On this basis the overall rms discrepancies of the present three-dimen-

TABLE 2. *RMS discrepancies of geostrophic wind (kt) between objective and subjective analyses.*

Pressure level (mb)	22 Nov. 1961	23 Nov. 1961	24 Nov. 1961	25 Nov. 1961	26 Nov. 1961	27 Nov. 1961	28 Nov. 1961	Overall RMS value
1000	11	9	12	9	8	9	12	10
700	11	9	14	10	9	12	11	11
500	13	12	15	12	12	13	12	13
300	21	16	20	16	16	18	16	18
200	23	22	23	23	20	22	23	22

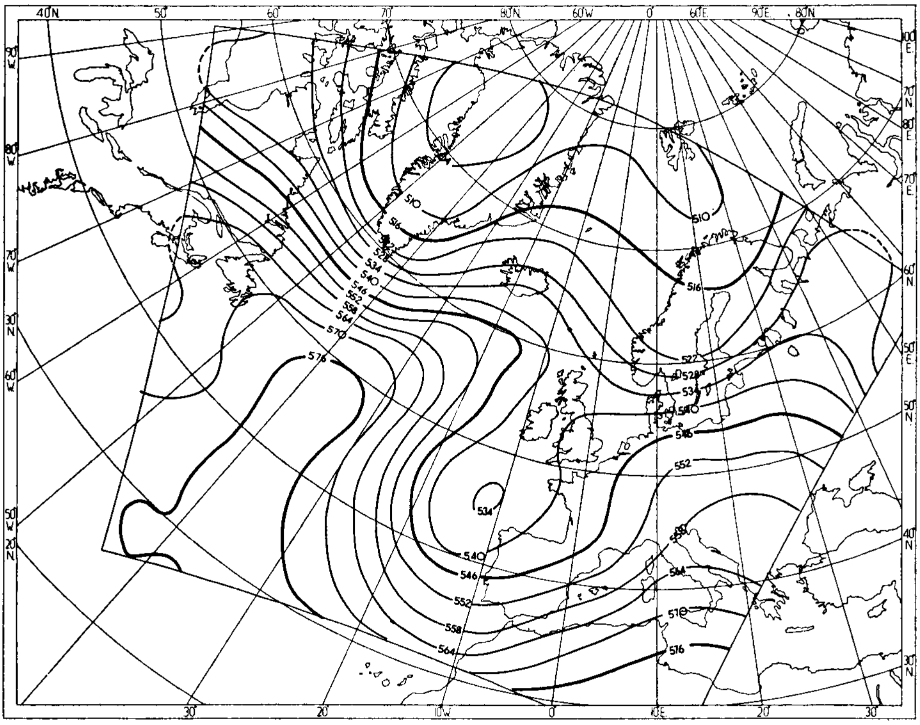

Fig. 1*a*. 500 mb contours for 00 GMT 27 November 1961 as given by the three-dimensional objective analysis process.

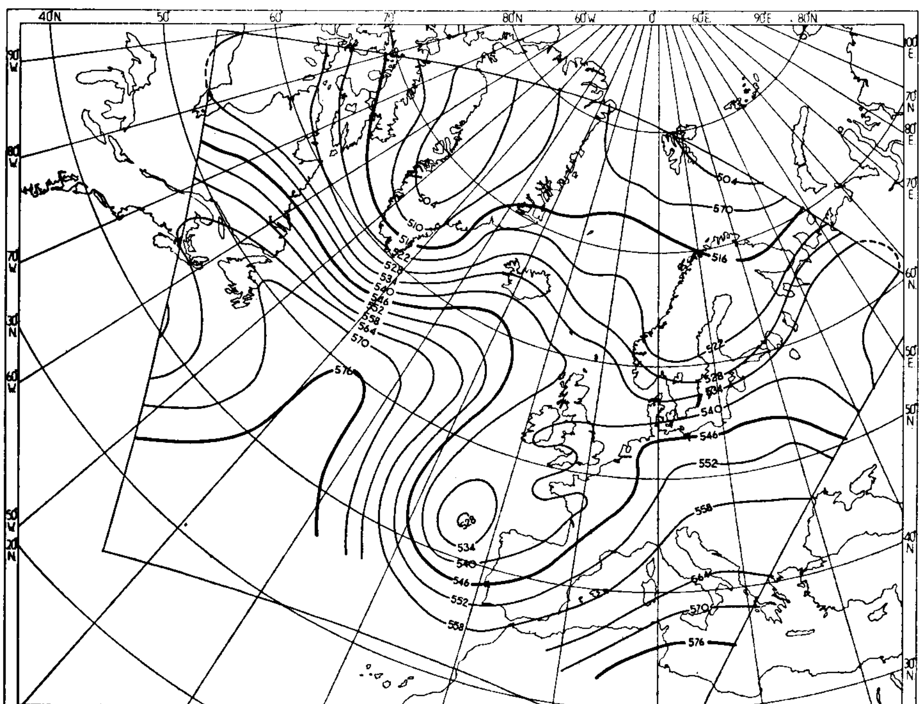

Fig. 1*b*. Corresponding subjective 500 mb analysis.

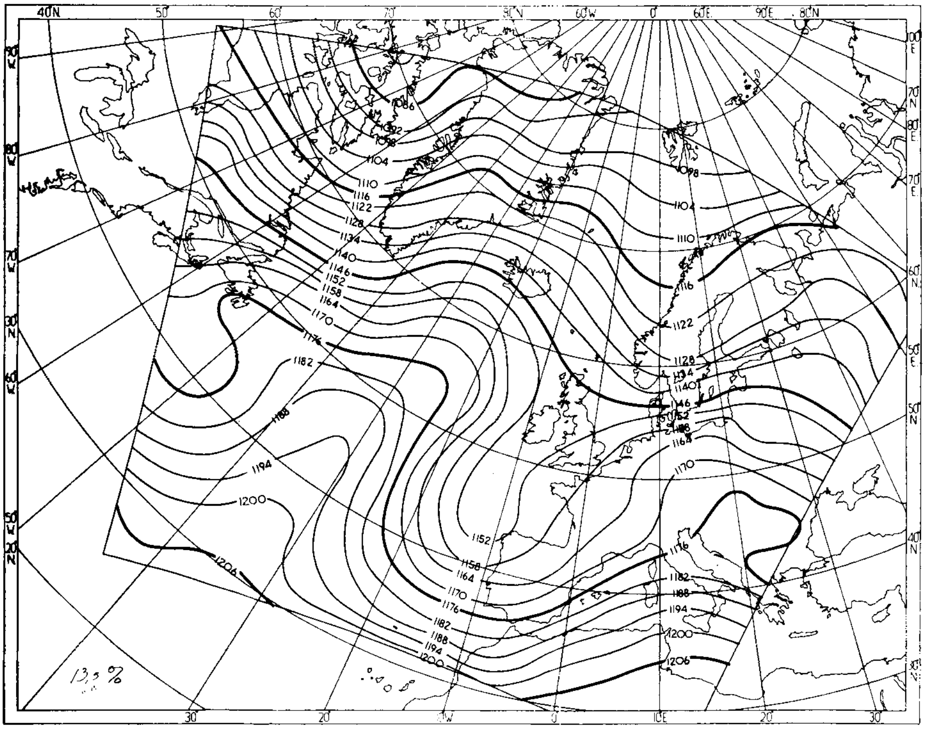

Fig. 2*a*. 200 mb contours for 00 GMT 27 November 1961 as given by the three dimensional objective analysis process.

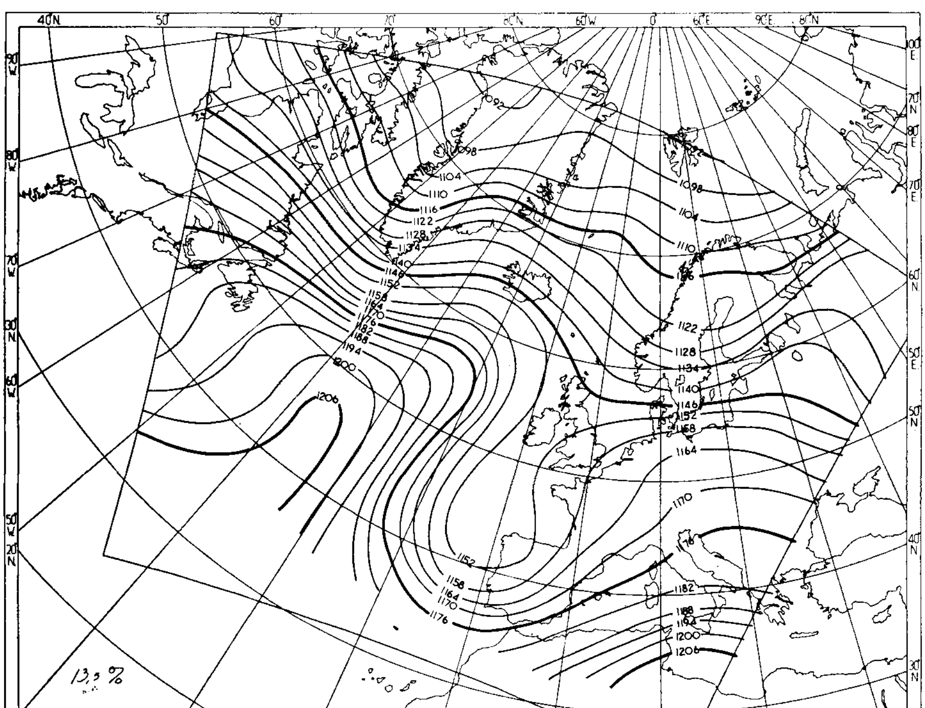

Fig. 2*b*. Corresponding subjective 200 mb analysis. Note the poor quality of the objective analysis especially in the northwest (see text).

sional objective analyses, which range from 17 m at the surface to 21 m at 500 mb can be regarded as highly satisfactory at these levels, in confirmation of the impression given by visual examination of the charts. An example of a typical analysis at 500 mb is given in Fig. 1 together with the corresponding subjective analysis for comparison.

Some increase in the discrepancies at higher levels is to be expected and was deliberately provided for by the form of the vertical weighting function adopted in the fitting procudure. However, the increase to an overall rms discrepancy of 77 m at 200 mb is clearly excessive. An example of one of the analyses at 200 mb is given in Fig. 2*a*. The very poor quality of the analysis, especially in the northwest of the area can be seen by comparison with the corresponding subjective analysis, Fig. 2*b*. An examination of the error fields at 200 mb in the individual cases showed that the discrepancies mostly took the form of large areas in northern latitudes with contour heights at 200 mb systematically much too low. The explanation proved to be that the representation in terms of four parameters for the vertical structure was inadequate in regions where the tropopause lay well within the depth of the atmosphere being analysed three-dimensionally. This is best made clear by reference to the vertical temperature structure implied in the analysis in various circumstances, this being quadratic in log p. Thus in a region where the soundings have a form of the type shown schematically by the full line in Fig. 3a the temperature structure implied in the analysis might be of the form given by the dashed curve. Although the detail of the temperature structure is smoothed out, the corresponding contour heights can be accurately represented, usually to within about 20 m at all levels. However, with a temperature structure of the general form given in Fig. 3*b*, which is typical of the Arctic in winter, the fitted structure implied in the analysis often had a shape dominated by the tropospheric data. The implied temperature, being quadratic in log p, necessarily has a profile with a constant curvature and thus no possibility of a reversal of curvature at the tropopause. It follows that in such cases contour heights in the stratosphere must be badly fitted and this in fact accounted for the errors in the 200 mb analyses in northern latitudes.

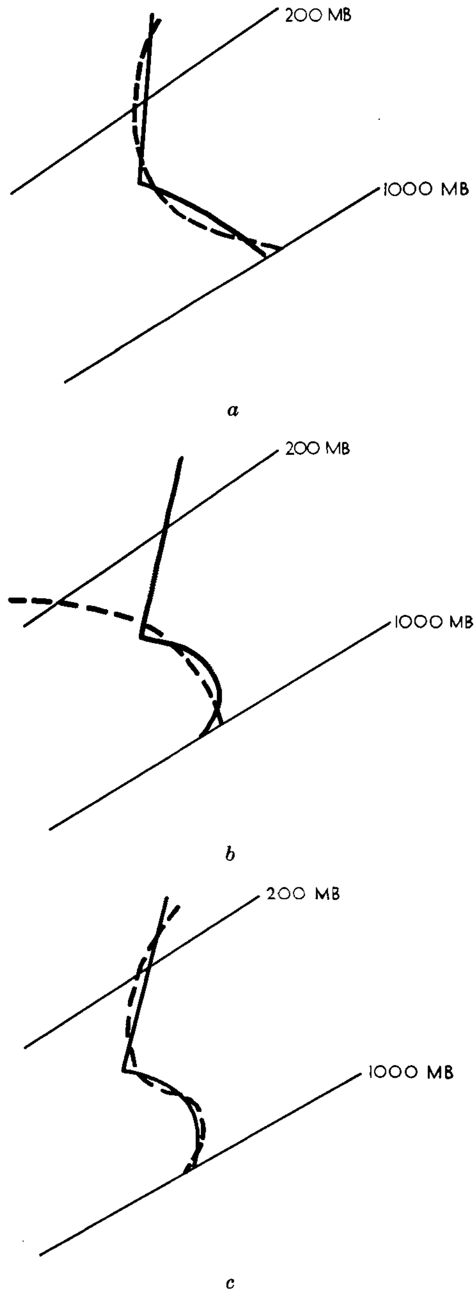

Fig. 3. Schematic temperature soundings (—) and corresponding profiles implied in the analyses (---).

Clearly the number of degrees of freedom provided here is inadequate if the three-dimensional analysis is required to cover simultaneously the lower stratosphere as well as the troposphere. An extra parameter for the vertical resolution, making a quartic in log p for the contour height representation and hence cubic in log p for the implied temperature structure, allows a reversal of the curvature of the temperature profile as in the schematic example of Fig. 3*c*. No full three-dimensional analyses on this extended basis have yet been computed but a number of extreme soundings, both real and artificial, were so fitted with the clear indication that the extra degree of freedom provided all the flexibility needed to fit the contour heights to within a few metres at all levels to 200 mb.

Conclusions

A three-dimensional objective analysis scheme on the lines used for the experiments described above appears practicable. The principal merit of the scheme is that many varieties of data both surface and upper air can be injected and used simultaneously. If the analysis is required to cover the troposphere alone, the fields of four parameters would be sufficient to define the three-dimensional structure of contour height adequately. If the analysis is required to include the lower stratosphere simultaneously, then five parameters are necessary. The formulation of a numerical forecasting model directly in terms of such parameters has not yet been investigated.

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