The operational forecasting model used in the norwegian[1] meteorological service

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ABSTRACT

A two-parameter geostrophic model is described. Primary emphasis is placed on the integration scheme used. This scheme implies computation of trajectories and interpolation of vorticity and a certain function of thickness in the endpoints of the trajectories.

The Norwegian Meteorological Institute runs two different models on their computer, a barotropic and a two-parameter model. The two models are very alike, the only difference being that the so-called development term is absent in the barotropic model.

For this reason only the two-parameter model will be discussed here. And further, since the physical properties of this type of model are well known, we shall mainly be concerned with the method used by the integration.

The model is quasi-geostrophic, and the input data are a stream function and a thickness. The basic assumption is that

$$\Phi(x, y, t, p) = \hat{\Phi}(x, y, t) + F(p)\,\Phi_T(x, y, t), \qquad (1)$$

where Φ is the geopotential, and $\hat{}$ symbolizes a vertical mean with respect to pressure. The function F is supposed to be known, whereas Φ and Φ_T are the fields which are forecasted.

It is further supposed that the stability defined by

$$\frac{\partial T}{\partial \Phi}$$

is independent of height. Using this second assumption we can find the analytic expression for the function F corresponding to a thickness defined by two particular pressure surfaces. This means that the model can handle any thickness as one of the basic parameters, provided certain constants are properly adjusted. Actually there is one version in operation that uses the thickness between 500 and 1000 mb, and another one which uses the thickness between 300 and 700 mb. It is probably best in models of this kind to use a thickness which represents a large part of the troposphere.

The stream function used as input data is derived from 500 mb heights, and is supposed to give the vertical mean wind. Experience has shown that this surface probably is a little too high. A pressure surface somewhere between 500 and 600 mb would be better.

There is no friction, heating, or mountain effect in the model, and the horizontal boundary conditions are simply that ω is zero at 1000 mb and at the top of the atmosphere. However, the model contains a Helmholtz term in order to control the very long waves.

If we introduce the basic model assumption in the vorticity equation, and afterwards form the vertical mean, we get the vertical mean vorticity equation

$$\frac{\partial \hat{\zeta}}{\partial t} + \hat{\mathbf{v}} \cdot \nabla(\hat{\zeta} + f) + c\,\mathbf{v}_T \cdot \nabla \zeta_T = 0, \qquad (2)$$

which is one of our prognostic equations. The last term is the development term.

Further, if we subtract this mean equation from the vorticity equation, then derive an expression for ω by vertical integration, and finally substitute this expression for ω in the thermodynamic equation, we get

$$\frac{\partial}{\partial t}\left(\Phi_T - \frac{a}{f}\zeta_T\right) + \hat{\mathbf{v}} \cdot \nabla\Phi_T + \frac{a}{f}\hat{\mathbf{v}} \cdot \nabla\zeta_T + \frac{a}{f}\mathbf{v}_T \cdot \nabla(\hat{\zeta} + f) + \frac{b}{f}\mathbf{v}_T \cdot \nabla\zeta_T = 0. \qquad (3)$$

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This is our prognostic equation for the thickness. The constants are defined as

$$a = \int_{p_1}^{p_2} \sigma\left(\int_{ps}^{p} F\,dp\right)dp,$$

$$b = \int_{p_1}^{p_2} \sigma\left(\int_{ps}^{p} (F^2 - c)\,dp\right)dp,$$

$$c = \widehat{F^2},$$

$$\sigma = \frac{\partial\Phi}{\partial p}\,\frac{1}{\theta}\,\frac{\partial\theta}{\partial p},$$

where p_s is the surface pressure, 1000 mb, p_1 and p_2 are the top and bottom pressure corresponding to the thickness Φ_T.

We may note that the last term in the thickness equation is similar to the development term in the vorticity equation. This is not astonishing since the term represents the influence on the thickness from the asymmetric part of the vertical velocity, which again is connected with development of vertical mean vorticity. When the coefficients a, b, and c are computed, it turns out that in the thickness equation this term is much smaller than the other terms. This is not the case, however, for the development term in the vorticity equation.

The integration of these equations is performed by means of a method which we have called quasi-Lagrangian. An outline of the method will be given in the following. Some features are described more in detail in connection with an experiment with the barotropic equation (Økland, 1962).

The application of the quasi-Lagrangian method is based upon the fact that the dominating terms in the prognostic equations can be written as the individual derivative of certain quantities, and therefore are immediately integrated with time if the trajectories can be computed.

In this way we get from the vorticity equation

$$[\hat{\zeta} + f]_{t_1}^{t_2} + c\int_{t_1}^{t_2} \mathbf{v}_T \cdot \nabla\zeta_T\,dt = 0, \qquad (4)$$

where the time integral is performed along the trajectory from time t_1 to time t_2, which means that the time variation is relative to a particle moving along the trajectory. In the two-parameter model this integral is approximated by the mean of the values in the end-points of the trajectory

$$\zeta_2 = \zeta_1 + f_1 - f_2 - \frac{c}{2}[(\mathbf{v}_T \cdot \nabla\zeta_T)_1 + (\mathbf{v}_T \cdot \nabla\zeta_T)_2]. \quad (5)$$

The reason why this rather crude approximation gives sufficient accuracy for a long time step is that the patterns of the thickness field from which the term is computed, to some extent is translated in the mean wind field, and so the time variation of the development term along a trajectory tends to be small.

We have performed two more transformations upon the vorticity equation. However, they are not basically necessary for the method, and it is therefore natural at this point to describe some of the main principles followed by the numerical integration.

The calculation of a time step begins with the computation of trajectories, starting in the grid-points and going backwards in the velocity field from time t_2 to time t_1. Actually, what we wish to obtain in this way is the locations of the points, which during the time interval from t_1 to t_2 move up to the grid-points. The reason why these backward trajectories are computed is, of course, that we wish to know the vorticity in a regular square grid at time t_2 because this enables us to derive the stream function from the vorticity by means of a simple Poisson equation.

The next thing to do will then be to calculate the value of the right hand side of our vorticity equation, in the computed end-points of the trajectories. Since the velocity and the thickness are known only in the grid-points as time t_1, it is necessary to interpolate in order to find the value in the end-points of the trajectories.

At this point it should be mentioned that owing to our prognostic equation (5), the thickness also must be known at time t_2. This means that the thickness has to be integrated before the stream function for the time step in question.

So far nothing has been said about the method used by the computation of trajectories. There are many possible ways. However, here will only be given a brief description of how it actually is done in the model.

The basic assumption is that the stream function is stationary during the time step from t_1 to t_2, so that the trajectories actually are computed in steps in a stationary stream field. The stream field used is not the field given at time t_1, however, but a stream field corresponding to a time half way between t_1 and t_2. This is in fact a sort of centered time step, and means that the stream function and also the thickness must be saved for two consecutive time steps.

Obviously the time steps should be made as long as possible in order to save computation time. No stability criteria in the ordinary sense exists for this method of integration. However, the assumption that the stream field can be considered to be stationary during the computation of the trajectories, obviously cannot permit too long time steps. In the present model we use 12 hr, but since we wish to compute the trajectories in a velocity field corresponding to the middle of the time step, the stream function and also the thickness must be computed for every 6 hr.

The reason why the time steps can be made so long is partly that a space mean of the stream function is introduced, in the same way as Fjørtoft did in connection with his graphical method (FJØRTOFT, 1952). Such a mean field obviously removes the shorter and more rapidly varying wave components.

The thickness equation can be treated in a similar way. At first we write it as

$$\left(\frac{\partial}{\partial t}+\mathbf{v}^*\cdot\nabla\right)\left(\Phi_T-\frac{a}{f}\zeta_T\right)=R, \qquad (6)$$

where
$$\mathbf{v}^*=\mathbf{k}\mathbf{x}\cdot\nabla\psi^*$$

$$\psi^*=\psi+\frac{4a}{d^2f^2}(\bar{\psi}-\psi)-\frac{a}{f}.$$

The bar symbolizes the usual four point space mean, and d is the mesh size of the square grid. R consists of comparatively small terms, and this actually is the reason why the field ψ^* is introduced.

If R is neglected, the equation states that

$$\Phi_T-\frac{a}{f^2}\nabla^2\Phi_T,$$

which is a function of the thickness only, is conserved in the velocity field $\mathbf{v}^*$. If, therefore, the trajectories in the field $\mathbf{v}^*$ are computed, this equation gives

$$\left(\Phi_T-\frac{a}{f^2}\nabla^2\Phi_T\right)_2=\left(\Phi_T-\frac{a}{f^2}\nabla^2\Phi_T\right)_1. \qquad (7)$$

It may be noted that the field $\mathbf{v}^*$ also has the smaller wave components removed.

The computation of the vorticity is a crucial point by the method, and much care has been taken to make these computations accurate and consistent. The mesh size of the grid is 600 km on 60° N. This is a great value compared to the smallest wave components which we wish to describe, and therefore special precautions have to be taken in order to avoid misinterpretation. Interpolation of rather high order is used. Further, the vorticity is not computed as point values, but as mean values over surfaces corresponding to the squares formed by the grid.

The last point was described in detail earlier (ØKLAND, 1962). The general concept is as follows. Consider the squares formed by the straight lines connecting the midpoints of the meshes, and note that the common five-point approximation to the vorticity gives a mean value of the vorticity over these squares, or rather the circulation along their boundary.

It was just said that the computed trajectories were those ending in the grid-points. Actually this is not what is done in connection with the integration of the vorticity equation. Instead, we compute the trajectories ending in the midpoints of the meshes. This then gives us the approximate location of the surfaces which during the actual time step move up to, and coincide with the squares used for the vorticity computation. Then the circulation along the boundary of these distorted surfaces is computed, by means of interpolated values of the velocity in the midpoints of the sides.

What advantages has this method of integration?

Owing to the long time steps, the model is fast, and for that reason well suited for computers of medium speed. However, the program will have to be comparatively complicated.

The model has been run operationally for one and a half years both as a barotropic model without the development term, and as a two-parameter model. The two-parameter model has shown considerable skill in forecasting baroclinic developments. However, no comprehensive comparison has been done between the results given by this model and other baroclinic models. The quasi-Lagrangian integration scheme gives truncation errors of another type than the Eulerian, although they will not necessarily be less (ØKLAND, 1958). But we do not know much about how these errors influence the two-parameter prognoses. Also, the optimum values of the coefficients a, b, and c in the prognostic equations are still uncertain. These are some of the problems which we will try to solve in the near future.

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