A dynamical theory of spiral rain band in tropical cyclones

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ABSTRACT

Elastoid waves and elastoid-gravoid waves are treated for a horizontal circular vortex or a simple model of tropical cyclones. The dynamical possibility for a banded or spiral region of upward motion is shown.

1. Introduction

Spiral rain bands in tropical cyclones were revealed by WEXLER (1947) through analysis of radar observations, and many investigators have directed their attention to them. The bands are generally formed near the center and propagate outwards (SENN *et al.*, 1958). As a possible mechanism for their formation HAURWITZ (1947), WEXLER (1947) and KUETTNER (1959) have suggested elongation of convection cells in the tangential direction by vertical wind shear. On the other hand, TEPPER (1958) has proposed a hypothesis of internal gravity waves. Some further studies are needed to establish the suggestion by KUETTNER and others, since we have yet no clear evidence for the existence of Benard's cell in severe weather situations like a tropical cyclone. The gravity wave hypothesis seems to be less comprehensive, since the persistent existence of an inversion layer, which is a necessary condition for the occurrence of gravity waves, is not confirmed in tropical cyclones, except in the eye.

Although the convective characters of the rain bands cannot be disregarded, KESSLER and ATLAS (1956) attribute the band formation to a dynamical (unknown) mechanism. KRISHNAMURTI (1961, 1962), in fact, solved numerically the Navier-Stokes equations for steady symmetric and moving asymmetric cyclones, and obtained a few ring-shaped or spiral-shaped regions of upward motion. However, a physical interpretation of the mechanism cannot be found in his papers. These facts have directed the attention of the present author to a possible dynamical mechanism for the bands.

2. Elastoid waves

Fluid motion between concentric cylinders of different angular velocity has a cellular feature along the rotational axis under certain conditions (TAYLOR, 1923). The existence of a similar feature in the boundary layer of a flow along a concave wall was shown by GÖRTLER (1940). These are well-known phenomena in the boundary layer theory called Taylor-Görtler vortices. BJERKNES *et al.* (1933) showed the possibility for this type of motion in a frictionless rotating fluid and gave the axi-symmetric motion propagating along the rotational axis only, the name *elastoid wave*.

The existence of an elastoid wave in a tropical cyclone does not immediately imply the possibility of a banded structure in radial direction, but in the vertical. Under certain conditions, however, we have a possibility for a periodical feature in the radial direction for elastoid waves in a circular vortex.

We shall in this section treat purely elastoid waves and not *gravoid waves* which propagate in the tangential direction. An approximate treatment of elastoid-gravoid waves will be given in Sect. 3.

We consider a circular vortex with a vertical axis of rotation in an homogeneous incompressible atmosphere on the rotating earth. Small disturbances are superimposed on the basic motion. In a cylindrical coordinate system (r, θ, z) where the z-axis coincides with the rotational axis, the following equations hold in the undisturbed circular vortex, if hydrostatic equilibrium and gradient wind balance are assumed

$$\left.\begin{aligned} \frac{U^2}{r} + fU &= \frac{\partial}{\partial r}(SP) \\ -g &= \frac{\partial}{\partial z}(SP). \end{aligned}\right\} \quad (1)$$

Here U and P are the undisturbed values of the tangential velocity and of the pressure, respectively. S, f and g are constant specific volume, Coriolis parameter and gravitational acceleration, respectively. The assumption of a constant S is made in order to reveal the dynamical features as clearly as possible. Naturally, the effect of baroclinicity cannot be neglected in actual cyclones and a non-vanishing of the vertical shear of the tangential velocity may be inconsistent with such an assumption.

If the disturbances are small and symmetric, the system of equations is

$$\left.\begin{aligned} &\frac{\partial u}{\partial t} + v\left(\frac{\partial U}{\partial r} + \frac{U}{r} + f\right) + w\frac{\partial U}{\partial z} = 0, \\ &\frac{\partial v}{\partial t} - u\left(\frac{2U}{r} + f\right) = -\frac{\partial (Sp)}{\partial r}, \\ &\frac{\partial w}{\partial t} = -\frac{\partial (Sp)}{\partial z}, \\ &\frac{v}{r} + \frac{\partial v}{\partial r} + \frac{\partial w}{\partial z} = 0, \end{aligned}\right\} \quad (2)$$

where u, v, w and p are small perturbations of the tangential, radial and vertical velocities and of the pressure, respectively.

As elastoid waves, the perturbations are assumed to be proportional to $\exp(imz) \cdot \exp(i\sigma t)$. The system (2) is then reduced to

$$\left.\begin{aligned} &i\sigma u + \zeta_a v - \eta w = 0, \\ &i\sigma v - \left(\frac{2U}{r} + f\right) u = -\frac{\partial (Sp)}{\partial r}, \\ &\frac{\sigma}{m} w = -Sp, \\ &\frac{v}{r} + \frac{\partial v}{\partial r} + imw = 0, \end{aligned}\right\} \quad (3)$$

where $\zeta_a = \frac{\partial U}{\partial r} + \frac{U}{r} + f$ and $\eta = -\frac{\partial U}{\partial z}$.

From (3) we obtain

$$\left.\begin{aligned} &\sigma^2 \frac{\partial^2 v}{\partial r^2} + \left[\frac{\sigma^2}{r} - i\eta m\left(f + \frac{2U}{r}\right)\right]\frac{\partial v}{\partial r} \\ &\quad + \left[\zeta_a m^2\left(f + \frac{2U}{r}\right) - \sigma^2 m^2 - \frac{\sigma^2}{r^2} - \frac{i\eta m}{r}\left(f + \frac{2U}{r}\right)\right] v = 0, \\ &w = \frac{i}{m}\left(\frac{\partial v}{\partial r} + \frac{v}{r}\right), \\ &\sigma u = \frac{\eta}{m}\frac{\partial v}{\partial r} + \left(\frac{\eta}{m} + ir\zeta_a\right)\frac{v}{r}, \\ &Sp = -\frac{i\sigma}{m^2}\left(\frac{\partial v}{\partial r} + \frac{v}{r}\right). \end{aligned}\right\} \quad (4)$$

In a condition of steady state, $\sigma = 0$, the system (4) reduces to

$$\left.\begin{aligned} &\frac{\eta}{m}\frac{\partial v}{\partial r} + \left(\frac{\eta}{m} + ir\zeta_a\right)\frac{v}{r} = 0, \\ &w = \frac{i}{m}\left(\frac{\partial v}{\partial r} + \frac{v}{r}\right), \\ &u = Sp = 0, \end{aligned}\right\} \quad (5)$$

where the last equation is derived from the second and third equations of (3). Thus, we have the following solutions:

$$\left.\begin{aligned} &v = -\frac{A}{r}\exp\left(-\frac{im\zeta_a}{\eta} r\right), \\ &w = -\frac{A}{r}\frac{\zeta_a}{\eta}\exp\left(-\frac{im\zeta_a}{\eta} r\right), \\ &u = Sp = 0, \end{aligned}\right\} \quad (6)$$

if ζ_a and η are constant and where A is an arbitrary constant. Taking imaginary parts, the solutions (6) become

$$\left.\begin{aligned} &v = \frac{A}{r}\sin\frac{m\zeta_a}{\eta} r \cos mz, \\ &w = -\frac{A}{r}\frac{\zeta_a}{\eta}\cos\frac{m\zeta_a}{\eta} r \sin mz, \\ &u = Sp = 0, \end{aligned}\right\} \quad (7)$$

under the boundary conditions:

$$\left.\begin{aligned} &v = 0 \text{ at } r = 0, \\ &w = 0 \text{ at } z = 0 \text{ (the earth's surface)}. \end{aligned}\right\} \quad (8)$$

The solutions (7) imply that the radial and vertical velocity fields have a banded feature in the radial direction with an interval nearly equal to L_r:

$$L_r = \frac{2\pi\eta}{m\zeta_a}. \tag{9}$$

The radial wavelength L_r is proportional to the vertical shear of the tangential velocity η and the vertical wavelength L_z ($\equiv 2\pi/m$), and inversely proportional to the vertical component of the absolute vorticity ζ_a. The values of η and ζ_a needed for computation of L_r have been taken from the field of tangential velocity in Hurricane Cleo as given by KRISHNAMURTI (1961). These fields are reproduced in Figs. 1 and 2, converted from p-coordinate to z-coordinate. In spite of fairly large variations it seems possible to select the following average values:

$$\eta = 1-2\times10^{-3}\ \mathrm{sec}^{-1},$$
$$\zeta_a = 3-7\times10^{-4}\ \mathrm{sec}^{-1}.$$

Then, the wavelength L_r takes values between 14 and 132 km, as shown in Table 1. The distances between successive bands observed in aircraft reconnaissance flights are 15–45 km in Hurricane Carrie, September 15, 1957 (NHRP, 1958). In KRISHNAMURTI'S (1961) results for a

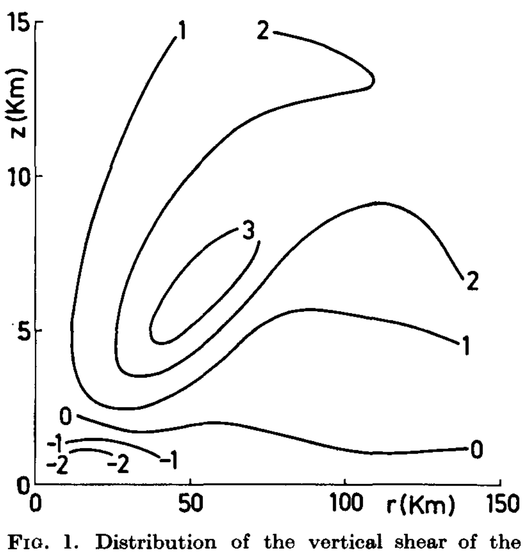

FIG. 1. Distribution of the vertical shear of the tangential wind velocity for Hurricane Cleo, 1958, derived from KRISHNAMURTI'S (1961) paper. Unit: 10^{-3} sec^{-1}.

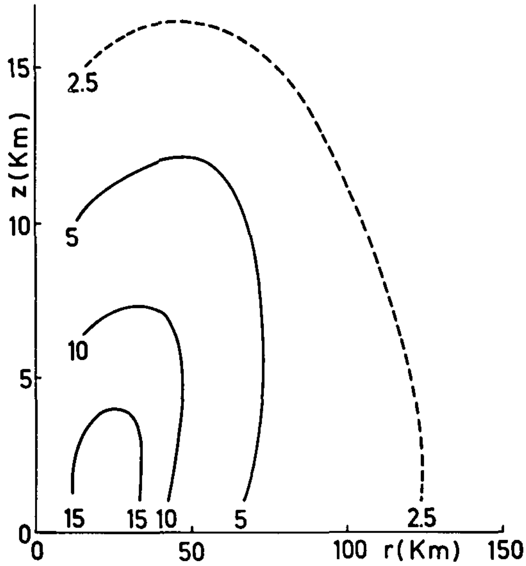

FIG. 2. Distribution of the vertical component of the absolute vorticity for Hurricane Cleo, 1958, reproduced from KRISHNAMURTI'S (1961) paper. Unit: 10^{-4} sec^{-1}.

steady symmetric hurricane derived from the actual data of Hurricane Cleo, the regions of downward motion are found at distance of about 80 and 130 km from the center and at the center, that is, the wavelength is 50–80 km in terms of this paper. These values fall within the range determined on a theoretical basis.

A treatment of the non-steady state will now be carried out under the restriction of $\partial U/\partial z = 0$, which may not be too serious for our discussion of elastoid waves. Then, eqs. (4) are reduced to

$$\left.\begin{aligned}
&\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + v\left(k^2 - \frac{1}{r^2}\right) = 0,\\
&w = \frac{i}{m}\left(\frac{\partial v}{\partial r} + \frac{v}{r}\right),\\
&u = i\frac{\zeta_a}{\sigma}v,\\
&Sp = -i\frac{\sigma}{m}\left(\frac{\partial v}{\partial r} + \frac{v}{r}\right),
\end{aligned}\right\} \tag{10}$$

where
$$k^2 = m^2\left[\frac{\zeta_a([2U/r]+f)}{\sigma^2} - 1\right]. \tag{11}$$

We have, if $k^2 = \text{constant} > 0$, the following solutions:

TABLE 1. *Theoretical wavelength in radial direction,* L_r.

η ($\times 10^{-3}$ sec^{-1})	1		2	
ζ_a ($\times 10^{-4}$ sec^{-1})	3	7	3	7
L_r (km) for $m = 2\pi/10$ km^{-1}	33	14	66	29
L_r (km) for $m = 2\pi/20$ km^{-1}	66	20	132	57

$$\left.\begin{aligned}
v &= AJ_1(kr) + BY_1(kr),\\
w &= \frac{i}{m}\left\{A\left[\frac{k}{2}J_0(kr) + \frac{J_1(kr)}{r} - \frac{k}{2}J_2(kr)\right]\right.\\
&\qquad \left. + B\left[\frac{k}{2}Y_0(kr) + \frac{Y_1(kr)}{r} - \frac{k}{2}Y_2(kr)\right]\right\},\\
u &= \frac{i\zeta_a}{\sigma}[AJ_1(kr) + BY_1(kr)],\\
Sp &= -\frac{i\sigma}{m^2}\left\{A\left[\frac{k}{2}J_0(kr) + \frac{J_1(kr)}{r} - \frac{k}{2}J_2(kr)\right]\right.\\
&\qquad \left. + B\left[\frac{k}{2}Y_0(kr) + \frac{Y_1(kr)}{r} - \frac{k}{2}Y_2(kr)\right]\right\},
\end{aligned}\right\} \quad (12)$$

where A and B are arbitrary constants and J and Y are Bessel functions of the first and second kind. Introducing the boundary conditions $v = 0$ at $r = 0$ and $w = 0$ at $z = 0$ (the earth's surface) we obtain the following imaginary parts of the solutions:

$$\left.\begin{aligned}
v &= AJ_1(kr)\cos mz \sin \sigma t,\\
w &= -\frac{A}{m}\left[\frac{k}{2}J_0(kr) + \frac{J_1(kr)}{r} - \frac{k}{2}J_2(kr)\right]\sin mz \sin \sigma t,\\
u &= \frac{\zeta_a}{\sigma}AJ_1(kr)\cos mz \cos \sigma t,\\
Sp &= \frac{\sigma A}{m^2}\left[\frac{k}{2}J_0(kr) + \frac{J_1(kr)}{r} - \frac{k}{2}J_2(kr)\right]\cos mz \cos \sigma t.
\end{aligned}\right\} \quad (13)$$

If we confine ourselves to a region of great distance from the center, $kr \gg 1$, the solutions (13) are further reduced to

$$\left.\begin{aligned}
v &= \frac{A}{\sqrt{\pi kr/2}}\cos\left(kr - \frac{3\pi}{4}\right)\cos mz \sin \sigma t,\\
w &= -\frac{Ak}{m\sqrt{\pi kr/2}}\cos\left(kr - \frac{\pi}{4}\right)\sin mz \sin \sigma t,\\
u &= \frac{A\zeta_a}{\sigma\sqrt{\pi kr/2}}\cos\left(kr - \frac{3\pi}{4}\right)\cos mz \cos \sigma t,\\
Sp &= \frac{A\sigma k}{m^2\sqrt{\pi kr/2}}\cos\left(kr - \frac{\pi}{4}\right)\cos mz \cos \sigma t,
\end{aligned}\right\} \quad (14)$$

Thus, when $k^2 > 0$, the small disturbances u, v, w and p have roughly periodic distributions in the radial direction, and the wavelength L_r is approximately equal to $2\pi/k$. If $k^2 < 0$, the disturbances have no radially banded feature, but show an exponential distribution. The identity (11) shows that the condition $k^2 > 0$ is satisfied, only if

$$T > T^*,$$

where $T = 2\pi/\sigma$ and $T^* = 2\pi/\sqrt{\zeta_a([2U/r] + f)}$. The value of $(2U/r) + f$ needed for determination of the critical period T^* has been taken from KRISHNAMURTI's data on Hurricane Cleo (Fig. 3). Selecting an average value of 5–8 $\times 10^{-4}$ sec^{-1} and assuming $\zeta_a = 3$–7×10^{-4} sec^{-1} (Table 2), we find for T^* the value 140–270 min. The

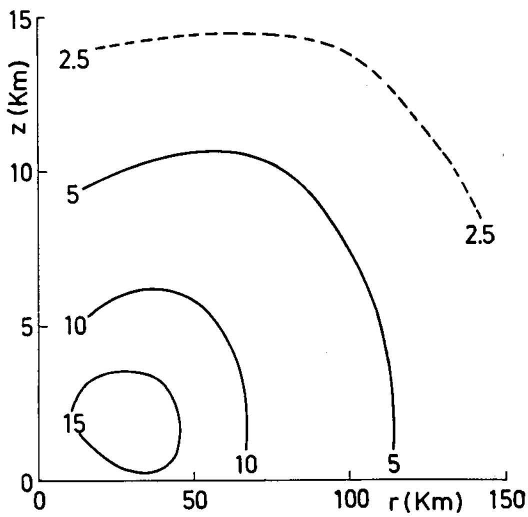

FIG. 3. Distribution of $2U/r$ for Hurricane Cleo, 1958, derived from KRISHNAMURTI's (1961) paper. Unit: 10^{-4} sec^{-1}.

TABLE 2. *Critical value of the period T^*.*

ζ_a ($\times 10^{-4}$ sec^{-1})	3		7	
$(2U/r)+f$ ($\times 10^{-4}$ sec^{-1})	8	5	8	5
T^* (min)	220	270	140	180

wavelength L_r can be calculated for specified values of T ($>T^*$), ζ_a and $(2U/r)+f$. For $\zeta_a = 5\times 10^{-4}$ sec^{-1} and $(2U/r)+f = 7\times 10^{-4}$ sec^{-1}, the value of L_r is 7.4–72 km, corresponding to $T=$ 183–299 min (Table 3).

Radially progressive waves can be obtained by superposing a similar wave of phase difference of $\pi/2$ in time and in radial direction upon the solutions (14), as is commonly done in wave theory. The velocity of radial propagation C is equal to σ/k, and is in this case (Table 3) found to be 0.4–6.6 m/sec. The outward propagation velocity of a rainband, relative to the center of Typhoon Helen, 1958 was estimated from radarscope data by TATEHIRA (1961) to be about 4 *m*/sec.

3. Elastoid-gravoid waves

Purely gravoid waves in a circular vortex or a tropical cyclone have been already treated by several authors (e.g. BJERKNES *et al.*, 1933; MASUDA, 1952). For the present purpose of examining the possibility of a spiral-shaped region of upward motion in a circular vortex, an approximate treatment of elastoid-gravoid waves will be attempted.

The equations for axi-asymmetric small perturbations superimposed upon a circular vortex described in Sect. 2 are

$$\left.\begin{aligned}
&\frac{\partial u}{\partial t} + U\frac{1}{r}\frac{\partial u}{\partial \theta} + \zeta_a v - \eta w = -\frac{1}{r}\frac{\partial (Sp)}{\partial \theta},\\
&\frac{\partial v}{\partial t} + U\frac{1}{r}\frac{\partial v}{\partial \theta} - u\left(\frac{2U}{r}+f\right) = -\frac{\partial (Sp)}{\partial r},\\
&\frac{\partial w}{\partial t} + U\frac{1}{r}\frac{\partial w}{\partial \theta} = -\frac{\partial (Sp)}{\partial z},\\
&\frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} + \frac{v}{r} + \frac{\partial w}{\partial z} = 0.
\end{aligned}\right\} \quad (15)$$

Assuming that u, v, w and p are proportional to $\exp(imz)\cdot\exp(il\theta)\cdot\exp(i\sigma t)$, eqs. (15) become

$$\left.\begin{aligned}
&\frac{\partial v}{\partial r}\left(\sigma+\frac{l}{r}U\right) + \frac{v}{r}\left(\sigma+\frac{l}{r}U - l\zeta_a\right) = \\
&\quad - w\left[\frac{l\eta}{r} + im\left(\sigma+\frac{l}{r}U\right)\left(1+\frac{l^2}{r^2m^2}\right)\right],\\
&\frac{\partial v}{\partial r}\left(\frac{2U}{r}+f\right) + \frac{v}{r}\left(f - \frac{l^2U}{r} - l\sigma + \frac{2U}{r}\right)\\
&\quad = \frac{il}{mr}\left(\sigma+\frac{l}{r}U\right)\frac{\partial w}{\partial r}\\
&\quad + iw\left[\frac{l^2}{m^2r^2}\left(\frac{\partial U}{\partial r}-\frac{U}{r}\right) - m\left(\frac{2U}{r}+f\right)\right],\\
&\frac{l}{r}u = -mw + i\frac{\partial v}{\partial r} + i\frac{v}{r},\\
&Sp = -\frac{w}{m}\left(\sigma+\frac{l}{r}U\right).
\end{aligned}\right\} \quad (16)$$

This system (16) can be simplified if we confine ourselves to the case of steady motion ($\sigma=0$) and of no vertical shear of the tangential velocity ($\eta=0$), and furthermore to a region at large distance from the center ($l^2/r^2m^2 < 1$). We obtain

$$\left.\begin{aligned}
&\frac{\partial^2 v}{\partial r^2} + m^2n^2 v = 0,\\
&w = \frac{i}{m}\frac{\partial v}{\partial r} + iv\frac{1-(r\zeta_a/U)}{rm},\\
&u = -\frac{mr}{l}w + \frac{ir}{l}\frac{\partial v}{\partial r} + \frac{i}{l}v,
\end{aligned}\right\} \quad (17)$$

TABLE 3. *Radial wavelength L_r and radial velocity of propagation C for $\zeta_a = 5\times 10^{-4}$ sec^{-1} and $(2U/r)+f = 7\times 10^{-4}$ sec^{-1}.*

T (min)		183	190	209	233	262	299
L_r (km)	for $m = 2\pi/10$ km^{-1}	36	26	16	12	9.2	7.4
C (m/sec)	for $m = 2\pi/10$ km^{-1}	3.3	2.3	1.3	0.9	0.6	0.4
L_r (km)	for $m = 2\pi/20$ km^{-1}	72	51	32	24	18	15
C (m/sec)	for $m = 2\pi/20$ km^{-1}	6.6	4.5	2.5	1.7	1.1	0.8

TABLE 4. *Radial distance r of a point of the azimuthal angle $\theta=\pi/2$ with the same phase as that at the point $\theta=0$ and $r=50$ km.*

ζ_a ($\times 10^{-4}$ sec^{-1})	3		7		
$(2U/r)+f$ ($\times 10^{-4}$ sec^{-1})	8	5	8	5	
$(U/r)^2$ ($\times 10^{-8}$ sec^{-2})	16	5	16	5	
Possible integral value of l	1	1	1	1	2
n	0.7	1.4	1.6	2.5	0.87
mnr for $\theta=0$, $r=50$ km, $m=2\pi/10$ km^{-1}	22	44	50	79	27
r (km) for $\theta=\pi/2$, $m=2\pi/10$ km^{-1}	45	48	48	49	44
mnr for $\theta=0$, $r=50$ km, $m=2\pi/20$ km^{-1}	11	22	25	40	14
r (km) for $\theta=\pi/2$, $m=2\pi/20$ km^{-1}	42	45	45	48	40

$$Sp=-\frac{lU}{m}\frac{w}{r},$$

where $$n^2+1=\frac{\zeta_a([2U/r]+f)}{l^2(U/r)^2}.$$

If n^2 is a positive constant, the system (17) gives the following solutions:

$$\left.\begin{aligned} v&=Ae^{imnr},\\ w&=A\left[\frac{i}{rm}\left(1-\frac{r\zeta_a}{U}\right)-n\right]e^{imnr},\\ u&=A\frac{r\zeta_a}{lU}e^{imnr},\\ Sp&=A\left[n-\frac{i}{rm}\left(1-\frac{r\zeta_a}{U}\right)\right]\frac{lU}{mr}e^{imnr},\end{aligned}\right\}\quad(18)$$

where A is an arbitrary constant. Remembering the periodic factor exp $(il\theta)$, and superimposing a similar wave with a phase difference of $\pi/2$ in the radial and tangential directions upon the wave (18), the following system is obtained:

$$\left.\begin{aligned} v&=Ae^{i(mnr+l\theta)},\\ w&=A\left[\frac{i}{rm}\left(1-\frac{r\zeta_a}{U}\right)-n\right]e^{i(mnr+l\theta)},\\ u&=A\frac{r\zeta_a}{lU}e^{i(mnr+l\theta)},\\ Sp&=A\frac{lU}{mr}\left[n-\frac{i}{rm}\left(1-\frac{r\zeta_a}{U}\right)\right]e^{i(mnr+l\theta)}.\end{aligned}\right\}\quad(19)$$

It is easily seen that these solutions (19) generally give a shape of the same phase-line that is not circular. The equation for phase isolines is

$$mnr+l\theta=\text{constant}\qquad(20)$$

and a calculation shows that the radial distance of the point at $\theta=\pi/2$ where the phase is the same as at $\theta=0$ and $r=50$ km is 40–49 km for the given values of ζ_a, $(2U/r)+f$, $(U/r)^2$ and m, (see Table 4). This result shows the possibility for a spiral shape of regions of equal phase, e.g., of regions of upward motion or a rainband.

4. Concluding remarks

Dynamical possibilities for a banded structure and for a spiral shape of equal phase regions of upward motion have been shown to exist in a case of a simple circular vortex in an incompressible homogeneous atmosphere, introducing elastoid wave and elastoid-gravoid wave. The treatment in a future study will be extended to more realistic models of tropical cyclones and also a more detailed verification of the theoretical results will be made by comparison with observational data.

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