On the dissipation of kinetic energy in the atmosphere[1]

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ABSTRACT

After a discussion of some general aspects of the atmospheric energy cycle results from computations of the frictional changes of kinetic energy are given. Large losses of kinetic energy are found in the boundary layer and in the region near the tropopause below the level of maximum wind. The total rate of dissipation of kinetic energy in the troposphere amounts to 10 watts m^{-2}, of which 4 watts m^{-2} are dissipated below 900 mb. The mean kinetic energy in the troposphere was 24×10^5 joules m^{-2}. The rate of dissipation of the kinetic energy in the first kilometer above the ground and its dependence upon the geostrophic wind speed are studied.

1. The atmospheric energy cycle

Due to differential heating, potential and internal energy are continuously created in the atmosphere and converted into kinetic energy, which finally through action of friction dissipates into heat. The investigation of this fundamental energy cycle has turned out to be difficult. We know only the order of magnitude of the mean rate of the energy transformations and very little is for instance known about the dependence of friction on the parameters of the large-scale flow. A knowledge of the generation and dissipation rates of kinetic energy is, however, of paramount importance in attempts to predict the future state of the atmosphere by numerical means.

The energy equations integrated over the whole mass M of the atmosphere are WHITE and SALTZMAN (1956)

$$\frac{\partial}{\partial t}\int_M K dm = -\int_M \alpha\omega dm + \int_M \mathbf{v}\cdot\mathbf{F} dm, \qquad (1)$$

$$\frac{\partial}{\partial t}\int_M (\Phi + I) dm = \int_M \alpha\omega dm + \int_M Q dm. \qquad (2)$$

K, Φ and I refer to kinetic energy, potential energy and internal energy per unit mass, respectively, $\mathbf{v}$ is the horizontal velocity, α specific volume, $\mathbf{F}$ the friction force and Q the rate of non-adiabatic heating per unit mass.

The term $-\int_M \alpha\omega dm$ of equation (1) appears with opposite sign in equation (2) and describes conversion of potential plus internal energy (the total potential energy) into kinetic energy. In order for the mean kinetic energy of the atmosphere to be maintained in spite of friction this term must be positive. It follows that upward motion must in general be associated with warmer air than downward motion.

In long time averages of equations (1) and (2), terms on the left hand side become negligible. The mean input of heat into the atmosphere must then be equal to the average work done by frictional forces.

A relation between the driving mechanism and the atmospheric energy cycle is obtained through the introduction of the concept of available potential energy (LORENZ, 1955). It is defined as the difference between the total potential energy and the minimum total potential energy which could result from adiabatic redistribution of mass. It is measured, approximately, by a weighted vertical average of the horizontal variance of temperature

$$A = \frac{1}{2}\int_M \gamma c_p \overline{T'^2} dm. \qquad (3)$$

Here A is the available potential energy, $\bar{T}$ denotes the horizontal area average over the whole earth of the temperature measured along a pressure surface, $T' = T - \bar{T}$ and $\gamma = (\bar{T} - p c_p R^{-1} \partial\bar{T}/\partial p)^{-1}$ is a measure of the mean static stability.

An equation describing the rate of change of the available potential energy can be derived

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from the thermodynamic equation. In approximate form this is

$$\frac{\partial A}{\partial t} = \int_M \alpha\omega dm + \int_M \gamma \overline{T'Q'} dm. \tag{4}$$

The second term on the right-hand side of (4) represents the rate of generation of available potential energy due to differential heating and thus describes the primary energy transformation for atmospheric motions.

Comparing (4) with (1) it is seen that the intensity of the atmospheric energy cycle can in principle be determined from any of the integrals $G = \int_M \gamma \overline{T'Q'} dm$, $C = -\int_M \alpha\omega dm = -\int_M \mathbf{v} \cdot \nabla_p \phi dm$ or $D = \int_M \mathbf{v} \cdot \mathbf{F} dm$, which averaged over long periods, must be equal. However, none of these expressions is easy to evaluate. The nonadiabatic heating Q and the frictional force F are the basic, partly external parameters determining the long-term behaviour of the atmosphere. Unfortunately their dependence on the motion itself which is required for an evaluation of the G- and D-terms, is to a great extent unknown. On the other hand, computation of the C-term presupposes a knowledge of the spatial distribution of either ω or the ageostrophic part of the wind, and these are difficult to determine for the global scales of motions.

Concerning estimates of the generation of available potential energy, a paper by CLAPP (1961) may be mentioned. He has estimated that part of the energy generation which is associated with the stationary component of the motion as revealed by monthly normal maps.

Recently quite extensive attempts to estimate the conversion term C have been made by WIIN-NIELSEN (1959), SALTZMAN and FLEISHER (1960) and JENSEN (1960). In the first two papers the equations for a two-layer, frictionless, adiabatic and quasigeostrophic model of the atmosphere were used for computation of ω, while Jensen for this purpose used the so-called adiabatic method. In principle this consists of solving the thermodynamic equation for ω after assuming zero non-adiabatic heating. In many cases the adiabatic method gives a good approximation to the actual vertical velocities. It can, however, not be used in studies of energy conversions. This was implicitly shown by Lorenz in the derivation of (4). The conversion integrand $\alpha\omega$ will, if ω is computed by the adiabatic method, reduce at each pressure level into a triple correlation term which probably is negligibly small and in fact has been disregarded in obtaining (4).

In his classical paper on the energy of the earth's atmosphere, BRUNT (1926) gave an estimate of D, the dissipation rate of kinetic energy, which is often cited in the literature as a value of the mean intensity of the atmospheric energy cycle. For the lowest kilometer Brunt assumed a distribution of wind according to the well-known Taylor theory. Taking the most probable global mean values for the parameters used, he obtained a value of 3 watts m^{-2}. Between 1 km and 10 km levels the wind profile was taken from sample cases. Assuming the coefficient of eddy viscosity to be here the same as in the first kilometer he found the dissipation of kinetic energy between 1 km and 10 km to be of the order 2 watts m^{-2}. Hence the total mean dissipation of kinetic energy in the troposphere would be of the order 5 watts m^{-2}. More recent estimates (BALL, 1961) agree in order of magnitude with this value.

Other estimates of the global changes of kinetic energy have been given by PALMÉN (1959) and by PISHAROTY (1954).

The lack of aerological observations from large parts of the earth is one of the difficulties encountered in attempts to estimate the energy transformations on a global scale. However, important information may be obtained by studying energy changes in those regions where the network is dense.

2. Method

The following expression is obtained when the kinetic energy equation is solved for the frictional term and integrated over a mass M, enclosed within a volume of horizontal area S and the pressure levels p_1 and p_2,

$$\int_M \mathbf{v} \cdot \boldsymbol{F} dm = \frac{\partial}{\partial t} \int_M K dm + \frac{1}{g} \oint_L \int_{p_1}^{p_2} K V_N dp dL + \frac{1}{g} \int_S [K\omega]_{p_1}^{p_2} dS + \int_M \mathbf{v} \cdot \nabla_p \phi dm. \tag{5}$$

Here L = length of the periphery of S and V_N = component of $\mathbf{v}$ normal to L, positive outward.

The first term on the right side of (5) describes the local rate of change of kinetic energy in the volume considered. The second term represents the net horizontal outflow of kinetic energy, while the third integral stands for the net flow of kinetic energy through the upper and lower boundaries. The fourth term, finally, represents the rate of generation of kinetic energy inside the volume due to the work done by the horizontal pressure forces.

For a region with a sufficiently dense network of aerological stations the terms on the right hand side of (5) can be evaluated and the rate of the frictional loss of kinetic energy in the volume obtained as a residual.

In the present study, data from British stations for the periods from January 2nd to 13th and January 16th to 21st, 1954 have been used. In Fig. 1 the locations of the stations are shown and also the grid system used in the computations. Sixteen pressure levels from 950 mb to 200 mb level were used. Below the 950 mb level surface observations were used and, when available, supplemented by 1000 mb data.

In order to evaluate the area, line, and pressure integrals any given property P was assumed to be a linear function of the horizontal coordinates. If A, B, and C denote the corners of a triangle, the integral of P over the area S of the triangle is given by the formula:

$$\int_S P ds = S\bar{P} = 1/3(P_a + P_b + P_c) \cdot S,$$

where P_a, P_b and P_c are the values of P at the corners. Because of the linearity only the mean gradient of the property P could be determined in each triangle.

In a triangular grid system the line integrals can be computed in the following way:

$$\oint_L P V_N dL = L\widetilde{PV}_N = \tfrac{1}{2}[L_{AB}(P_A V_{NA}^{AB} + P_B V_{NB}^{AB}) + L_{BC}(P_B V_{NB}^{BC} + P_C V_{NC}^{BC}) + L_{CA}(P_C V_{NC}^{CA} + P_A V_{NA}^{CA})].$$

Here L_{AB}, L_{BC} and L_{AC} are the sides of a triangle ABC, and $L = L_{AB} + L_{BC} + L_{CA}$.

In the notation used for the wind components V_{NA}^{AB}, for instance, represent the component of the wind at station A outward from the triangle and normal to the side L_{AB}.

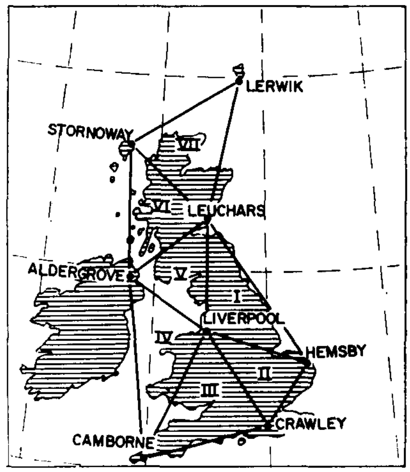


FIG 1.

The mean ω in each triangle can be computed with the aid of the continuity equation from which one gets by integration

$$\bar{\omega}(p) = \bar{\omega}(p_s) + \frac{1}{S}\int_S \int_p^{p_s} \nabla_p \cdot \mathbf{v}\, dp dS = \overline{\frac{\partial p_s}{\partial t}} + \frac{1}{S}\int_S \nabla_p \cdot \int_p^{p_s} \mathbf{v}\, dp ds = \overline{\frac{\partial p_s}{\partial t}} + \frac{L}{S}\int_p^{\tilde{p}_s} V_N dp.$$

The pressure tendency $\partial p_s/\partial t$ at the earth's surface, which is an observed quantity, was throughout the period chosen for this study found to be so small that it could be entirely neglected.

The pressure integration was made first for the layer from the ground up to 950 mb level and then for each 50 mb layer up to 200 mb level. Finally, eq. (5) was averaged in time with data for every twelve hours. Most of the computations were made on the electronic computer BESK.

3. General results

In Fig. 2, $W_F = \mathbf{v} \cdot \mathbf{F}(\Delta p/g)$, the rate of loss of kinetic energy due to the work done by frictional forces, is given for different vertical layers, averaged over the region represented by the triangles I–V and over the two periods considered. The variation with pressure of the mean wind speed V and the mean static stability $\partial\theta/\partial z$ (θ is the potential temperature) is also shown.

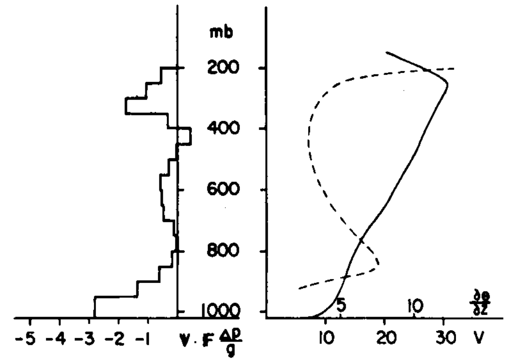


FIG. 2. Left side: the mean work done by frictional forces in different layers (unit: watts m^{-2}). Right side: the mean wind speed (heavy line) in m sec^{-1} and the mean static stability (dashed line) in °C km^{-1}.

The total value of W_F summed over all layers below 200 mb level amounts to 10.4 watts m^{-2}. The mean kinetic energy in this layer was 23.6 × 10^5 joules m^{-2}. If the kinetic energy and the rate of dissipation were constant the kinetic energy would be renewed in 2.6 days. This implies that a numerical prediction of the tropospheric motion field for more than one or two days cannot be very successful unless a proper mechanism for frictional dissipation of kinetic energy is incorporated in the model.

Considering the vertical variation W_F is found to have large negative values below approximately 900 mb, as well as near the tropopause just below the level of the maximum wind. In the middle troposphere W_F is generally negative but its magnitude is smaller than in the lower and upper troposphere. It is interesting to notice the great similarity between the vertical distribution shown in Fig. 2 with the yearly mean rate of generation of kinetic energy over the British Isles, computed by SMITH (1955). His results show two maxima of the energy generation, one in the boundary layer and the other in the upper troposphere somewhat below the wind maximum. The mean rate of generation, 8.8 watts m^{-2}, obtained by Smith may be compared with the total value of W_F, 10.4 watts m^{-2}, as given above.

It will be shown in the next section that the magnitude of W_F in the first kilometer above the ground depends primarily upon the stress between the atmosphere and the underlying earth's surface. For a qualitative discussion of the W_F-minimum in the upper troposphere it is assumed that the frictional force is due to change with height of the vertical component of the eddy stress:

$$\mathbf{F} = \frac{1}{\varrho}\frac{\partial \boldsymbol{\tau}_z}{\partial z}.^{(1)}$$

It is furthermore assumed that

$$\boldsymbol{\tau}_z = \varrho K \frac{\partial \mathbf{v}}{\partial z},$$

where K is the coefficient of eddy viscosity. Qualitatively, K must decrease with increasing static stability. Disregarding the change of wind direction with height one can, considering the vertical variation of the mean wind speed and of the mean static stability (Fig. 2), expect the largest vertical shear of the stress $\partial \boldsymbol{\tau}_z/\partial z$) to occur below the level of maximum wind. In this regard the location of the upper troposphere minimum of W_F seems quite reasonable. From the values of W_F and V an approximate mean value of 4 × 10^5 cm^{2}sec^{-1} for K between 400 mb and 200 mb levels can be deduced.

In Table 1 the contributions from the different terms in (5) are given for the entire layer from the earth's surface to the 200 mb level and also separately for the two layers surface–900 mb and 900 mb–200 mb.

Below 900 mb the work of friction amounts to 4.2 watts m^{-2}. This loss of kinetic energy is almost entirely compensated for through the work done by the horizontal pressure forces. This means physically that there is in the first kilometer above the earth's surface an approximate balance between pressure, Coriolis and frictional forces. This is not only true for the time averages given in Table 1 but is also, according to the data, approximately valid at each instant.

TABLE 1. *Values of different terms in the kinetic energy equation (5) and of the work of frictional forces (W_F). Unit: watts m^{-2}.*

Layer	Local change	Horiz. adv.	Vert. adv.	Gener.	W_F
Surface–900 mb	−0.1	0.0	0.0	−4.1	−4.2
900 mb–200 mb	−3.0	−0.6	−3.3	+0.7	−6.2
Surface–200 mb	−3.1	−0.6	−3.3	−3.4	−10.4

[1] In the rest of the article $\boldsymbol{\tau}_z$ is called stress.

Between 900 mb and 200 mb W_F has a total value of 6.2 watts m^{-2}. Here the contributions from horizontal advection and from generation are small but this is only so in the time averages. These quantities show in the daily values large and to considerable extent compensating fluctuations. With regard to the instantaneous changes, the two terms representing local changes of kinetic energy and the net vertical flux are usually of less importance.

In order to determine the accurracy of the numerical values given in Fig. 2 and Table 1, a second computation of W_F has been made taking occasions with large values of ω at 200 mb as unrealistic and replacing these data in the upper troposphere by interpolated values. The general form of W_F as a function of pressure was only affected to a relatively small extent. However, the value of W_F for the layer 900 mb–200 mb, 6.2 watts m^{-2}, is probably accurate only within a factor of two.

For the purposes of the next section, W_F $(=\mathbf{v}\cdot\mathbf{F}[\Delta p/g])$ is given in a different form. Introducing z as a vertical coordinate one has, assuming

$$\mathbf{F} = \frac{1}{\varrho}\frac{\partial \tau_z}{\partial z},$$

$$W_F = \int_{p_1}^{p_2} \mathbf{v}\cdot\mathbf{F}\frac{dp}{g} = \int_{z_2}^{z_1} \mathbf{v}\cdot\frac{\partial \tau_z}{\partial z}\,dz.$$

An integration by parts yields

$$W_F = [\mathbf{v}\cdot\tau_z]_{z_2}^{z_1} - \int_{z_2}^{z_1} \tau_z\cdot\frac{\partial \mathbf{v}}{\partial z}\,dz \qquad (6)$$

(6) is an approximate expression for the physical fact that the change of kinetic energy due to friction (W_F) in a certain volume of the atmosphere is the result of the net transport of kinetic energy by frictional eddies through the boundaries and the net transformation of the kinetic energy into eddy energy inside the volume. The latter process is called the dissipation of kinetic energy and is denoted by D.

Due to the large static stability the stress at the 200 mb level, as well as the eddy transport of kinetic energy through this level, must be small. Therefore, the total value of W_F for the layer surface–200 mb, 10.4 watts m^{-2}, gives approximately also the rate of dissipation of kinetic energy in this layer.

4. Dissipation of the kinetic energy in the first kilometer above the ground

The purpose of this section is to study the dissipation within the first kilometer as dependent on the magnitude of the geostrophic wind which here represents the driving force of the motion. For two regions with obviously different surface roughness the value of the "geostrophic drag coefficient" as well as the mean value of the eddy viscosity in the first kilometer will be computed.

An x,y,z-coordinate system will be used in this section. The x-axis is taken along an isobar and the perpendicular y-axis is directed towards the lower pressure.

On the basis of the quasi-balance between pressure, Coriolis and frictional forces, one can express the work of friction forces in terms of a cross isobaric mass transport, which in turn is related to the difference in stress between the bottom and the top of the layer considered. One first obtains, taking the upper boundary to be 1000:

$$W_F(0\text{–}1000\,\mathrm{m}) = \int_0^{1000} \mathbf{v}\cdot\nabla p\,dz = fV_g M_y, \qquad (7)$$

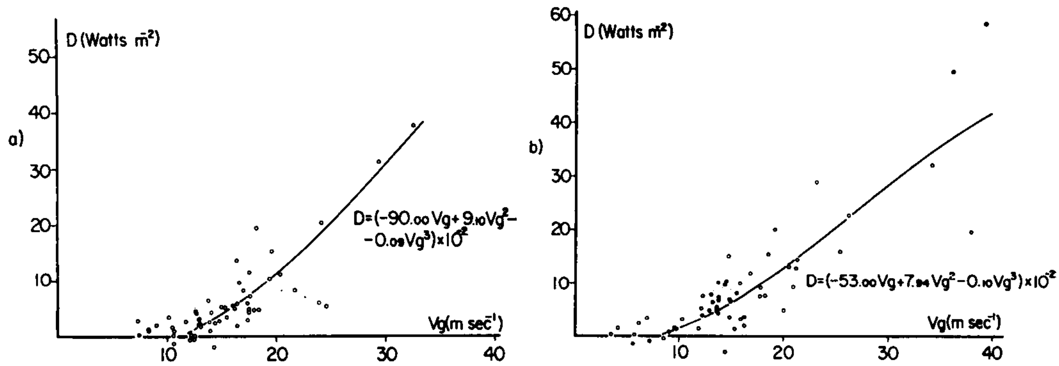
where V_g is the speed of the geostrophic wind and M_y is the integrated cross isobaric mass transport. The vertical variation of the geostrophic wind was found from the data to be relatively unimportant and has been neglected in (7). From the equation of motion one obtains: $M_y = (\tau_{x,0} - \tau_{x,1000})/f$, where the index x indicates a component along the x-axis. Hence we have

$$W_F = V_g(\tau_{x,0} - \tau_{x,1000}). \qquad (8)$$

Assuming the value of the eddy viscosity at 1000 meters height to be 1.0×10^5 cm^{2}sec^{-1} the contribution from the term $V_g\tau_{x,1000}$ was estimated to be on the order of 0.3 watts m^{-2}. The left-hand side of (8) is on the order of 4 watts m^{-2} (Table 1). With an error of about 10% we can thus neglect $\tau_{x,1000}$ in (8). To the same degree of approximation we can in eq. (6), when applied to the first kilometer above the ground, also neglect the vertical eddy flux of kinetic energy. We can then write (8) as

$$D(0\text{–}1000) = \tau_0\cos\alpha_0\,V_g, \qquad (9)$$

where α_0 is the angle between the drag at the ground and the geostrophic wind vector (or



a. *b.*

FIG. 3. The rate of dissipation of kinetic energy in the first kilometer above the ground as a function of the geostrophic wind speed at the surface, (*a*) for the region of the triangles II and III; (*b*) for the region of the triangles V and VI.

TABLE 2. *Values of mean cross-isobaric angle α_0 with standard deviation, the ratio of the mean speed V_a of the surface wind (measured at the anemometer level) to the speed V_g of the geostrophic wind at the surface, the square of the geostrophic drag coefficient and the eddy viscosity K (cm^2sec^{-1}) for the first kilometer for different regions.*

Triangle	I	II	III	IV	V	VI	VII
α_0	$28° \pm 12°$	$25° \pm 11°$	$20° \pm 11°$	$26° \pm 11°$	$31° \pm 12°$	$34° \pm 16°$	$24° \pm 16°$
V_a/V_g	0.39	0.42	0.45	0.46	0.36	0.38	0.46
C^2		$1.0 \cdot 10^{-3}$			$1.4 \cdot 10^{-3}$		
K		$1.0 \cdot 10^5$			$1.2 \cdot 10^5$		

the angle between the wind in the surface layer and the isobars). Inserting for τ_0 the expression which LETTAU (1959) has introduced,

$$\tau_0 = \varrho_0 C^2 V_{g,0}^2 (^1) \tag{10}$$

we obtain

$$D(0\text{–}1000) = \varrho_0 C^2 V_{g,0}^3 \cos \alpha_0. \tag{11}$$

C is called geostrophic drag coefficient.

In Fig. 3 the rate of dissipation, as given by the generation term, is plotted against the speed of the geostrophic wind at the surface. The left part of the figure refers to the combined region of the triangles II and III, while the right part represents the region of the triangles V and VI.

Certain similarities are seen between these two cases. The rate of dissipation is almost constant for geostrophic wind speeds less than 12 m sec^{-1}. Above this value the dissipation increases with the wind speed. In the interval where most of the data occur, 10 m $sec^{-1} < V_g <$ 25 m sec^{-1}, Fig. 3 gives larger dissipation in the mountainous region (part b) than in the flat region (part a). It must be remembered that the rate of dissipation in both regions has been determined using observations only from the corner stations. The wind profiles at these stations need not reflect much of the effect of the topography inside the area. On the other hand, the values of the mean cross isobaric angle in different triangles and the ratio of the mean surface wind (measured at the anemometer level) to the mean geostrophic wind indicate that the effect of varying surface roughness is at least qualitatively depicted in the results (see Table 2). The cross isobaric angle increases, and

[1] Theoretical support to the proportionality between surface stress and the square of the geostrophic wind can be found in the paper by ROSSBY and MONTGOMERY (1935).

the ratio V_a/V_g decreases with increasing roughness.

C^2 has been computed from eq. (11) using the values given for α_0. K has been determined by using the data of Fig. 3 in the expression for the rate of dissipation given in Taylor's classical theory for the atmospheric boundary layer (see e.g. BRUNT, (1941)).

$$D = \sqrt{\frac{fK}{2}}\, \varrho_0 \sin 2\alpha_0\, V_g^2. \qquad (12)$$

Both the drag coefficient as well as the coefficient of eddy viscosity are larger for the rougher region. The magnitudes of these coefficients seems to be somewhat low when compared with some earlier estimates. LETTAU (1950) deduced for instance from the "Leipzig wind profile" a value $C^2 = 1.4\ 10^{-3}$ and for a relatively smooth area in the southern U.S. the author HOLOPAINEN (1961) found $C^2 = 1.5\ 10^{-3}$. The values of C^2 given in Table 2 are probably underestimated by about 20% due to two effects. First, the mean value of the eddy stress at 1000 m which was neglected in (8), must be positive in order to be consistent with the vertical variation of wind at this level. In the second place there is probably an underestimation of the cross-isobaric mass transport due to the very few readings that usually are made in the lowest 500 m of a wind sounding.

The mean value of K, 1.1 10^5 cm^{2}sec^{-1}, obtained from Table 2, is the same as from the Leipzig wind profile.

Naturally C^2 as well as K depend on the thermal stratification of the air. It seems, however, that such a dependence cannot be investigated with the aid only of standard daily observations.

It is finally of interest to know the importance of the first few decameters above the ground in relation to the total dissipation in the first kilometer. This can be roughly estimated noticing first from eq. (6) that the rate of dissipation for this lowest layer is approximately given by the vertical eddy flux of kinetic energy from layers above. Thus,

$$D(O - H_a) \simeq \tau_0 V_a \quad (\tau_a \simeq \tau_0), \qquad (13)$$

where H_a is the height of the anemometer. Comparing (13) with (9) it is seen that

$$D(O - H_a)/D(O - 1000) \simeq V_a/V_g \cos \alpha_0.$$

From Table 2 a mean value for this ratio of 0.47 is obtained. Hence, almost half of the total dissipation of kinetic energy in the first kilometer above the ground occurs already below the anemometer level.

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