Further aspects on the diurnal temperature variation at the surface of the earth

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(Manuscript received January 26, 1962, revised version February 12, 1963)

ABSTRACT

From a number of independent clear-sky observations of global radiation and temperature at the surface of the earth, typical values are derived for the diurnal variation of these elements for certain specified lengths of the day. An attempt is made to relate the typical variation of radiation with that of surface temperature by the aid of a formula recently suggested by the author. The agreement is good and the constants used in the formula resemble those previously used by the author.

Depending on the actual conditions (type of soil, humidity gradient in the air, etc.) various combinations of the two constants should be used in the formula. It is demonstrated how the choice of constants affects the amplitude and phase lag of the temperature curve. Also discussed is the possibility of incorporating turbulence effects in the formula.

1. Introduction

According to a theory presented by the present author in a recent paper (LÖNNQVIST, 1962), there exists under certain assumptions a simple connection between the diurnal variation of the temperature at the surface of the earth and that of the global radiation.

The assumptions can be summarized as follows. The energy contribution from outside, i.e. the global radiation S, is compensated at the surface by conduction, on one hand, and by evaporation and long-wave net radiation on the other. The conduction is treated in accordance with standard methods, assuming constant conductivity conditions in the soil and neglecting molecular conduction in the air. The long-wave net radiation is put proportional to the temperature of the surface which is an acceptable assumption as long as the temperature variation is moderate. The evaporation (and condensation) is treated in the same way as the net radiation. Empirical data were given which seem to justify such an assumption.

Provided the diurnal variation of the global radiation is represented by

$$S = S_m + \sum_{j=1}^{n} [b_j \cos j\omega t + c_j \sin j\omega t], \qquad (1)$$

the temperature variation at the surface can be written as follows

$$T_o = T_{om} + \sum_{j=1}^{n} [p_j \cos j\omega t + q_j \sin j\omega t], \qquad (2)$$

where

$$\begin{cases} p_j = \dfrac{(\beta_j + h)\, b_j - \beta_j c_j}{(\beta_j + h)^2 + \beta_j^2}, & (3) \\[2ex] q_j = \dfrac{\beta_j b_j + (\beta_j + h)\, c_j}{(\beta_j + h)^2 + \beta_j^2}. & (4) \end{cases}$$

Here
$$\beta_j = \beta_d \sqrt{j} \qquad (5)$$

and
$$\beta_d = \varrho c \sqrt{\varkappa} \sqrt{\frac{\omega_d}{2}}, \qquad (6)$$

where ϱc is the heat capacity and $\varkappa$ the thermometric conductivity of the soil. ω_d is the angular frequency of the diurnal variation, i.e. 0.727×10^{-1} sec^{-1}. Obviously the solution of the problem depends on the two constants, β_d and h. In the numerical example discussed in the previous paper the following figures were used:

$$\beta_d = 2.3 \times 10^{-4} \text{ cal/cm}^2 \text{ sec deg},$$
$$h = 2.7 \times 10^{-4} \text{ cal/cm}^2 \text{ sec deg}.$$

How the choice of constants affects the amplitude and phase lag of the temperature curve will be discussed in section 5 of the present paper.

2. Normal diurnal variation of global radiation from clear skies for various lengths of the day

In the numerical example given in the previous paper, (LÖNNQVIST, 1962), the duration of daylight, d, was equal to 16.6 hours. To make sure that the fairly good agreement did not depend on the particular length of the day, the formula should be verified for other values of d as well. Since there are very few data available in the literature giving simultaneous measurements of the insolation from a cloudless sky and the temperature of the surface of the earth, we have chosen to base our investigation on normal values computed from a large number of independent observations of global radiation, on the one hand, and surface temperature, on the other.

Our first task is then to find typical values for the global short-wave radiation at clear skies. The values should be reduced for the albedo of the surface. In order to standardize the amplitude of the diurnal variation, the data should be given for a certain latitude. Data are wanted for certain specified lengths of the day, say, $d = 8$, 12, 16 and 24 hours. Our investigation can be briefly summarized as follows.

Disregarding first the effects of the atmosphere and the albedo, it is a purely geometrical problem to determine the theoretical maximum radiation at any time of the day for a given latitude and for various lengths of the day. The geometrical values should be reduced for the combined effect of absorption, backward scattering and albedo. In order to establish normal values for this reduction and by that the typical diurnal variation of clear-sky insolation, global radiation data for Stockholm (59.4° N) and for Östersund (63.2° N), representing a number of cloudless cases, were used together with the same kind of data for Pälkäne, Finland (61.2° N), published by FRANSSILA (1936). Though applicable to quite another latitude, data for the Nebraska project, published by LETTAU and DAVIDSON (1957), were used for comparison. The values finally chosen as typical

TABLE 1. *Normal values for the clear-sky global radiation in cal/cm² min at about 60 degrees latitude for specified values for the length of the day, d.*

Local time	$d = 8$	$d = 12$	$d = 16$	$d = 20$ hours
00	0.00	0.00	0.00	0.00
02	0.00	0.00	0.00	0.02
04	0.00	0.00	0.04	0.14
06	0.00	0.03	0.19	0.40
08	0.02	0.19	0.46	0.66
10	0.16	0.41	0.66	0.84
12	0.25	0.48	0.71	0.91
14	0.16	0.41	0.66	0.84
16	0.02	0.19	0.46	0.66
18	0.00	0.03	0.19	0.40
20	0.00	0.00	0.04	0.14
22	0.00	0.00	0.00	0.02

at about 60 degrees latitude are given in Table 1. The figures indicate that the effective insolation at any time of the day and of the year amounts to about 50 per cent of the theoretical maximum radiation. The data given in Table 1 are represented graphically in the upper part of Fig. 1.

3. Normal diurnal temperature variation at the surface of the earth for various lengths of the day at a Swedish land station

A similar study was made concerning the typical temperature variation at the earth's surface. The values were chosen so as to apply to clear and calm weather at a land station at about 60 degrees latitude. A brief summary of this comprehensive investigation is given here.

Data published by FRANSSILA (1936) provide four cases for which d varies between 16.6 and 19.3 hours. Data by LETTAU and DAVIDSON (1957) provide three cases for which $d = 13$ hours. These series of observations contain simultaneous measurements at the surface and at the screen level. Using the discrepancy between the temperature curves at the two levels for establishing certain corrections, it was found possible to utilize in addition screen temperatures observed at stations where surface temperatures had not been observed. Owing to this possibility, 17 cases with d varying between

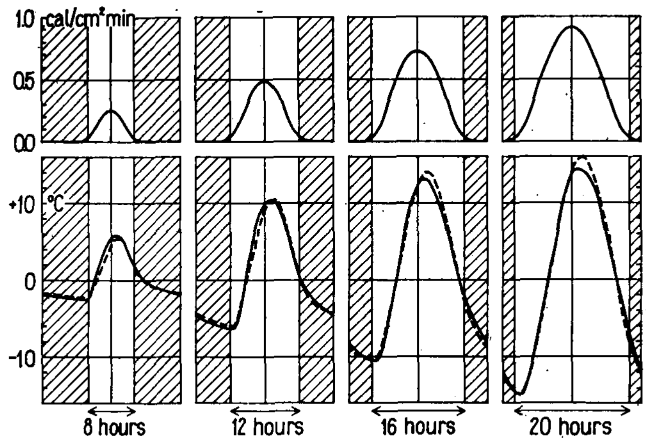

FIG. 1. Typical data for the diurnal variation of clear-sky insolation (upper part) and surface temperature (lower part) at about 60 degrees latitude, given for specific durations of daylight. Broken lines indicate computed temperature variations, assuming $\beta_d = 2.4 \times 10^{-4}$ and $h = 2.2 \times 10^{-4}$ cal/cm²sec deg. Night hours are shaded.

5.6 and 18.4 hours were taken from Uppsala (59.9° N); three cases with $d = 24.0$ hours (midnight sun touching the horizon) were taken from observations made at Pajala (67.2° N).

In order to find the connection between the shape of the surface-temperature curve and the length of the day, the following technique was used. Each individual temperature curve was carefully drawn. The accurate time was determined for the maximum, the minimum, and three other characteristic points on each curve. The times found were plotted in a diagram against the length of the day, d. Five curves were drawn in the diagram—one for each characteristic point—to fit the corresponding plots as closely as possible. Another graph gave the amplitude as a function of the length of the day. Obviously, with the aid of such diagrams the typical temperature curve can be reconstructed for any value of d. Values for the surface-temperature deviation from the mean at various hours of the day, evaluated in this way, are given in Table 2. The same values are represented graphically in the lower part of Fig. 1. They are shown by a full line.

TABLE 2. *Normal values for the temperature at the surface of the earth, given as deviations from the mean in degrees Celsius, for specified lengths of the day, d.*

Local time	$d = 8$	$d = 12$	$d = 16$	$d = 20$ hours
00	−2.0	−4.7	− 8.2	−12.1
02	−2.2	−5.4	− 9.7	−14.5
04	−2.4	−6.0	−10.5	−13.8
06	−2.6	−6.3	− 8.7	− 7.1
08	−2.5	−3.1	− 1.1	+ 0.7
10	+2.1	+5.5	+ 7.5	+ 8.5
12	+5.4	+9.9	+12.4	+13.8
14	+5.0	+9.8	+12.5	+14.1
16	+2.0	+6.0	+ 9.4	+11.9
18	−0.2	+0.6	+ 3.3	+ 5.7
20	−1.1	−2.2	− 1.9	− 0.7
22	−1.6	−3.7	− 5.6	− 7.1

4. Verification of the formula for various lengths of the day

After having carried out the climatological investigation summarized in Tables 1 and 2 and represented by full lines in Fig. 1, it is possible to test the formula (2) for data appertaining to various lengths of the day. The broken lines in Fig. 1 show the results of computations based on the radiation curves given in the upper part of the same figure. The same combination of β_d and h was used in all the four cases. The constants were chosen so as to give the best possible fit in all cases taken together, viz.

$$\beta_d = 2.4 \times 10^{-4} \text{ cal/cm}^2 \text{ sec deg},$$

$$h = 2.2 \times 10^{-4} \text{ cal/cm}^2 \text{ sec deg}.$$

Obviously, both values are fairly close to those used by the author in the previous paper. They can be considered as typical for land stations in Sweden and Finland. However, the conditions might differ considerably from one day to the other.

It might be worth while to mention in this connection what has been stressed by various authors studying the Brunt formula for the fall of surface temperature during a clear night, viz. that the dependence of soil constants seriously limits the practical application of the theory. Empirical frost prediction formulas are usually more successful. The same limitation applies to our formula, and even more so since evaporation has been included in the theory. However, irrespective of its usefulness for practical purposes, the formula adds to our understanding of the connection between energy exchange and temperature at the surface of the earth.

5. The effect of various combinations of β_d and h

Depending on actual conditions (type of soil, humidity gradient in the air etc.), different combinations of β_d and h should be used in the formula. Consequently, amplitude and phase lag of the surface temperature variation depends in a certain way on soil and air conditions. In order to study this question various combinations of the two constants are inserted in the formula without changing the insolation conditions.

A somewhat similar problem has been studied by LETTAU (1951). The differences in the way of approach are so large, however, that no direct comparison will be made in this paper, although some results obtained by Lettau will be discussed in section 6 of the present paper.

As already shown in the previous paper, the ratio of the diurnal amplitudes of surface temperature and global radiation can be written as follows

$$\left(\frac{A_T}{A_S}\right)_j = \frac{1}{\beta_j \sqrt{1+(\cot\vartheta_j)^2}}, \tag{7}$$

where $$\cot\vartheta_j = \frac{h+\beta_j}{\beta_j}. \tag{8}$$

TABLE 3. *Ratio of temperature and radiation amplitudes (in deg Celsius per cal/cm² min) for certain combinations of β_d and h.*

	Rate of cooling by conduction		
Rate of cooling by evaporation and heat radiation	Small $\beta_d = 0.3 \times 10^{-4}$	Normal $\beta_d = 2.3 \times 10^{-4}$	Very large $\beta_d = 60 \times 10^{-4}$
Abs. min. $h = 0.0$	393	51	2.0
Small $h = 1.0 \times 10^{-4}$	125	42	2.0
Normal $h = 3.0 \times 10^{-4}$	50	29	1.9
Large $h = 10 \times 10^{-4}$	16	13	1.8

Here ϑ_j is the phase lag of the jth harmonic of the surface temperature against the corresponding harmonic of the radiation.

Let us assume that the radiation curve is symmetric relative to noon, which is in fact the normal case. Then the first harmonic, $j = 1$, of the surface temperature describes fairly well the total amplitude and exceedingly well the time for the temperature maximum, whereas the time for the temperature minimum at the surface is highly influenced by the other harmonics. To simplify the computations the following summary is based on the first harmonic only. Table 3 gives the ratio between the amplitude of the temperature variation (in degrees Celsius) and that of the variation of global radiation (in cal/cm² min) for certain combinations of β_d and h. The three values chosen for β_d represent freshly fallen snow, wet clay and turbulent ocean water, respectively. Likewise, the values chosen for h are intended to cover the whole range of possible values.

TABLE 4. *Phase lag (in hours) of surface temperature maximum against insolation maximum for various combinations of β_d and h.*

	Rate of cooling by conduction		
Rate of cooling by evaporation and heat radiation	Small $\beta_d = 0.3 \times 10^{-4}$	Normal $\beta_d = 2.3 \times 10^{-4}$	Very large $\beta_d = 60 \times 10^{-4}$
Abs. min. $h = 0.0$	3.0	3.0	3.0
Small $h = 1.0 \times 10^{-4}$	0.9	2.3	3.0
Normal $h = 3.0 \times 10^{-4}$	0.4	1.6	2.9
Large $h = 10 \times 10^{-4}$	0.1	0.7	2.7

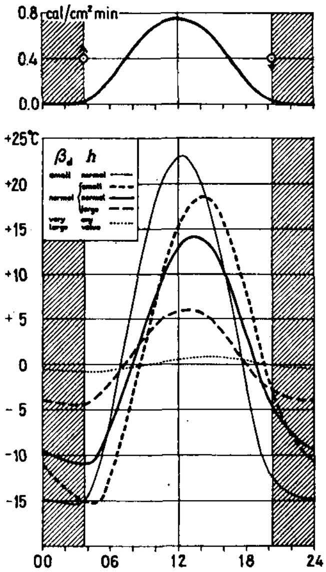

FIG. 2. Diurnal variation of surface temperature for five combinations of β_d and h. Computations are based on the insolation curve given in the upper part. Symbols indicate sunrise and sunset.

For the same combinations of the two constants Table 4 indicates the phase lag of surface temperature against insolation, i.e. the number of hours counting from noon that will elapse until temperature at the surface reaches its maximum. It is evident from the table that the phase lag never exceeds 3 hours. This can also be seen directly from (8), since obviously cot $\vartheta_j \geqslant 1$. The limiting value, 3 hours, is approached for $\beta_d \gg h$, including the case $h \to 0$.

The curves for the temperature at the surface of the earth drawn in Fig. 2 are all based on the same insolation curve but correspond to five different combinations of the constants, chosen among those presented in Tables 3 and 4. The duration of daylight is assumed to be 16.6 hours. We see from the tables and from Fig. 2 the striking difference between the temperature amplitude at a normal ground surface and at a water surface. For turbulent ocean water LETTAU (1951) indicates a range of values for $\varkappa$, viz. 1–100 cm²/sec. The figure for β_d used in Tables 3 and 4 corresponds to the lower one of these limits. Using instead the upper limit the temperature amplitude is reduced to 0.2°C per cal/cm² min.

6. Incorporation of turbulence effects in the formula

In the previous paper the author disregarded the transport of energy to and from the surface of the earth by turbulence in the air. This simplification of the problem was justified by the fact that the case studied was one characterized by light winds. It is worth while to try to introduce turbulence effects in the theory, although from the viewpoint of verification the introduction of winds makes it difficult to avoid cases affected by advection of energy which cannot be accounted for in the theory.

As mentioned above, LETTAU (1951) has presented a theory which in some respects leads to results comparable with those presented by the present author. The differences in the way to tackle the problem can be summarized as follows. Lettau did not disregard turbulence. He did disregard, however, the diurnal variation of cooling by evaporation and heating by condensation. Finally, he studied the first harmonic only. Thanks to Lettau's investigation it is possible to introduce effects of turbulence in our formula. Lettau found that there is a fairly small phase lag of the temperature wave against the wave of turbulent energy transport. He gives the value 25 minutes for for diurnal oscillations. Thus, though not explicitly mentioned by Lettau, the cooling by turbulence is approximately proportional to the temperature at the surface, i.e.

$$L = h_L(T_0 - T_L). \tag{9}$$

In Fig. 3 the cooling L, as given by Lettau, is plotted as a function of surface temperature

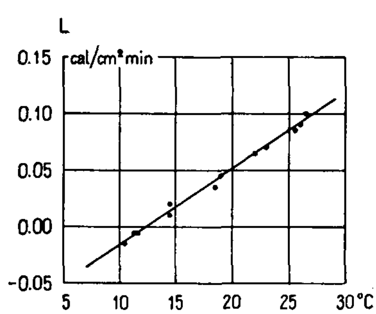

FIG. 3. Energy transport from the surface by turbulence, according to Lettau's computations, presented here as a function of surface temperature. $\mathring{w}_a = 20$ cm/sec. $z_0 = 0.1$ cm.

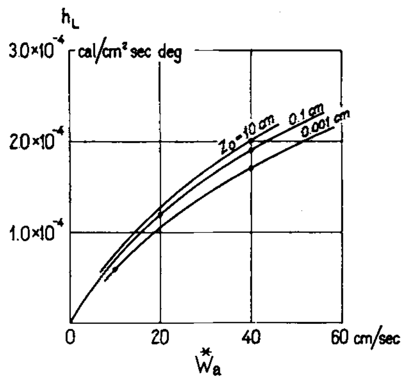

FIG. 4. Turbulent cooling coefficient h_L as a function of $\overset{*}{w}_a$ for different values of z_0. Curves are based on Lettau's computations.

for a special case, viz. adiabatic mixing velocity $\overset{*}{w}_a = 20$ cm/sec and roughness parameter $z_0 = 0.1$ cm. The constant h_L in (9) is determined by the slope of the line in the figure. We find $h_L = 1.2 \times 10^{-4}$ cal/cm²sec deg. Fig. 4 shows values for h_L determined in the same way for various combinations of $\overset{*}{w}_a$ and z_0. According to Lettau $\overset{*}{w}_a$ ranges from 10 to 60 cm/sec for winds ranging from Beaufort force 1 to 5, approximately.

Obviously, we can insert in equations (3) and (4)

$$h = h_F + h_W + h_L, \tag{10}$$

where h_F indicates the rate of cooling by longwave net radiation. For a relative net radiation equal to 25 per cent, h_F amounts to 0.3×10^{-4} cal/cm² sec deg;

h_W indicates the rate of cooling by evaporation (and heating by condensation). The numerical value seems to vary considerably from one situation to another. In the particular case studied in detail by LÖNNQVIST (1962), h_W was equal to 2.5×10^{-4} cal/cm²sec deg;

h_L indicates the rate of energy transport by air turbulence. A numerical estimate can be obtained from Fig. 4. For a wind of about 2 Bf, $h_L = 1.2 \times 10^{-4}$ cal/cm²sec deg.

It is evident from these values that in low-wind conditions the diurnal variation of non-conductional cooling is predominantly an effect of evaporation and condensation.

7. Certain limitations in the application of the theory

Some limitations have already been mentioned, viz. the dependence of soil constants and the assumption of simple laws for the energy transport at the surface by net radiation, evaporation and turbulence. These limitations, however, are not too serious. We have shown that for a reasonable choice of constants the computed temperatures agree fairly well with those observed.

From the formula we obtain values for the deviation from the diurnal mean temperature and not for the temperature itself. The mean value may change considerably from one day to the other. This fact is demonstrated in Fig. 5. It shows the temperature recorded in Uppsala during the clear-sky period January 22–25, 1953. Lacking measurements at the surface of the earth, we have used screen temperatures instead. The duration of daylight was equal to 7 hours. Winds were WNW 4 m/sec throughout the period. The full line shows the observed temperature variation with all its irregularities. The dotted line shows the running mean computed over 24 hours. The temperature variation shown in the left-hand bottom is the four-day average of the deviations from the running

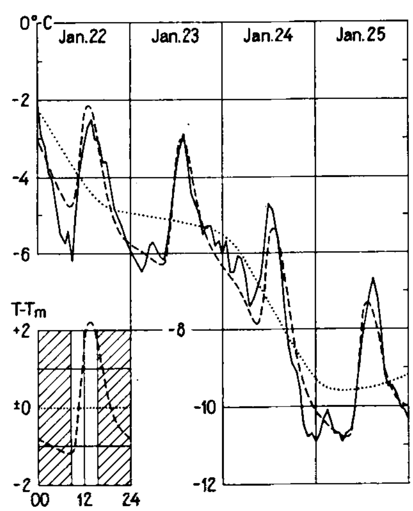

FIG. 5. Screen-temperature variation at Uppsala, Jan. 22–25, 1953. Curves indicate observed temperature (full line), running 24-hour mean (dotted line), and average diurnal variation (broken line), shown both separately and superimposed on the running mean.

mean. The broken line in the figure is obtained by superimposing the average diurnal variation on the running mean. Comparing the full and broken lines, the standard deviation is equal to 0.5°C.

From this example it is evident that the changes in the diurnal mean from one day to the other may occasionally be of the same magnitude as the periodic variation itself.

The fundamental assumption for our theory is that the diurnal variation of radiation is periodic, in which case we obtain a periodic solution for the variation of surface temperature. It would seem that this assumption makes it impossible to handle cases in which radiation is far from periodic, e.g. a cloudless day followed by an overcast day. In order to show how this can be done, nevertheless, we have chosen the case represented in Fig. 6. The observations were made by FRANSSILA (1936) at Pälkäne. Data are available from 08 local time on August 10 to 07 local time on August 11, 1934. In the upper part of the figure the full line shows the variation of the net short-wave plus long-wave radiation during the period of observations. Also shown are the changes in cloudiness that took place. The clouding over after midnight is well reflected in the curve for short-wave and long-wave radiation.

The way to handle this case is to introduce fictitious radiation data before and after the observed data in such a way that a 48-hour curve is obtained which can be treated as periodic, the period being 48 hours. In this case 8 harmonics are used instead of 4. Even harmonics describe together what is common to the two days, whereas odd harmonics describe the differences between the two days. According to equations (5) and (6), we shall use

$$\beta_{j=1} = \beta_{2d} = \beta_d \sqrt{1/2}.$$

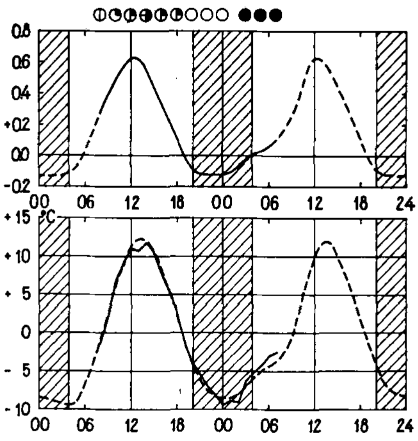

FIG. 6. Cloudiness, radiation and surface temperature at Pälkäne, Aug. 10–11, 1934. In the upper part full line indicates observed long-wave and short-wave radiation balance, broken line represents fictitious data. In the lower part broken line indicates computed surface temperatures, full line represents observed data.

In the lower part of Fig. 6. the full line shows the temperature actually observed at the surface of the earth. The broken curve represents the computed temperature variation, using optimum values for the constants, viz.

$$\beta_d = 1.8 \times 10^{-4} \text{ cal/cm}^2 \text{sec deg}$$
$$h = 3.8 \times 10^{-4} \text{ cal/cm}^2 \text{sec deg}.$$

In this case the standard deviation of the temperature differences is equal to 0.7°C. Winds were somewhat stronger in this case than in the case studied in the previous paper. Advection might to some degree be responsible for the less good agreement in this case.

It should be mentioned that the shape of the fictitious radiation curve does not affect the result very much except for that part of the curve which describes the conditions a few hours prior to the start of observations.

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