A theoretical analysis of lee wave clouds observed by Tiros I[1]

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ABSTRACT

A Tiros I cloud photograph taken April 18, 1960 over the southern part of South America is examined. The clouds were very regularly orientated in parallel bands on the lee side of the Andes. The cloud pattern is assumed to be quasi-stationary. It seems to be quite clear that the clouds are formed by the waves which are set up in the air flow over the mountain range. The wind direction varied little with height and was almost perpendicular to the mountain.

The observed wave length is compared with a theoretically computed value obtained by using a method previously presented by the author (Döös, 1961). The agreement between these values is good. The amplitude of the disturbances produced by the mountain has also been evaluated. For comparison the wave length has also been evaluated by means of the parcel method.

1. Introduction

The relatively new opportunity to obtain observations of clouds with the aid of satellites is indeed very promising. One important advantage of this technique is that it offers the possibility of getting pictures of clouds over vast areas where density of ordinary observations are low. This is certainly a very important complement to routine analysis of weather maps. Another advantage is that it is possible to reveal certain features of the horizontal distribution of the clouds, which in many cases is practically impossible to detect from observations made at the earth's surface even when the station density is relatively good. Several reports have already illustrated this fact quite clearly. One example is the interesting satellite photographs of cellular cloud patterns presented by KRUEGER & FRITZ (1961).

In this report we shall study a satellite picture of orographically produced clouds which occurred over a large area to the east of the southern part of the Andes. We shall also compare the observed distances between the clouds with the wave length computed with the aid of available meteorological observations using a method previously presented by the author (Döös, 1961).[2]

2. The satellite picture and the synoptic situation

In Fig. 1a part of the satellite picture, which was taken from Tiros I at 1738 GMT, April 18, 1960 in its 250th orbit, is shown. We notice that the photograph was taken when the satellite passed over the southern part of South America. In order to identify the location of the clouds a 2° latitude–longitude grid and the coastline have been superimposed. In the picture the 2000 m contour line of the mountains has also been superimposed. A more complete map of the topography has been obtained from the very detailed map Mapa Hipsometrico de la Republica Argentina y Regiones Limitrofes (1930) and is shown in Fig. 2.

Before we turn to the discussion of the clouds shown in this picture we shall study the synoptic situation.

The network of radiosonde stations in South

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[2] This article will hereafter be denoted by the symbol I.

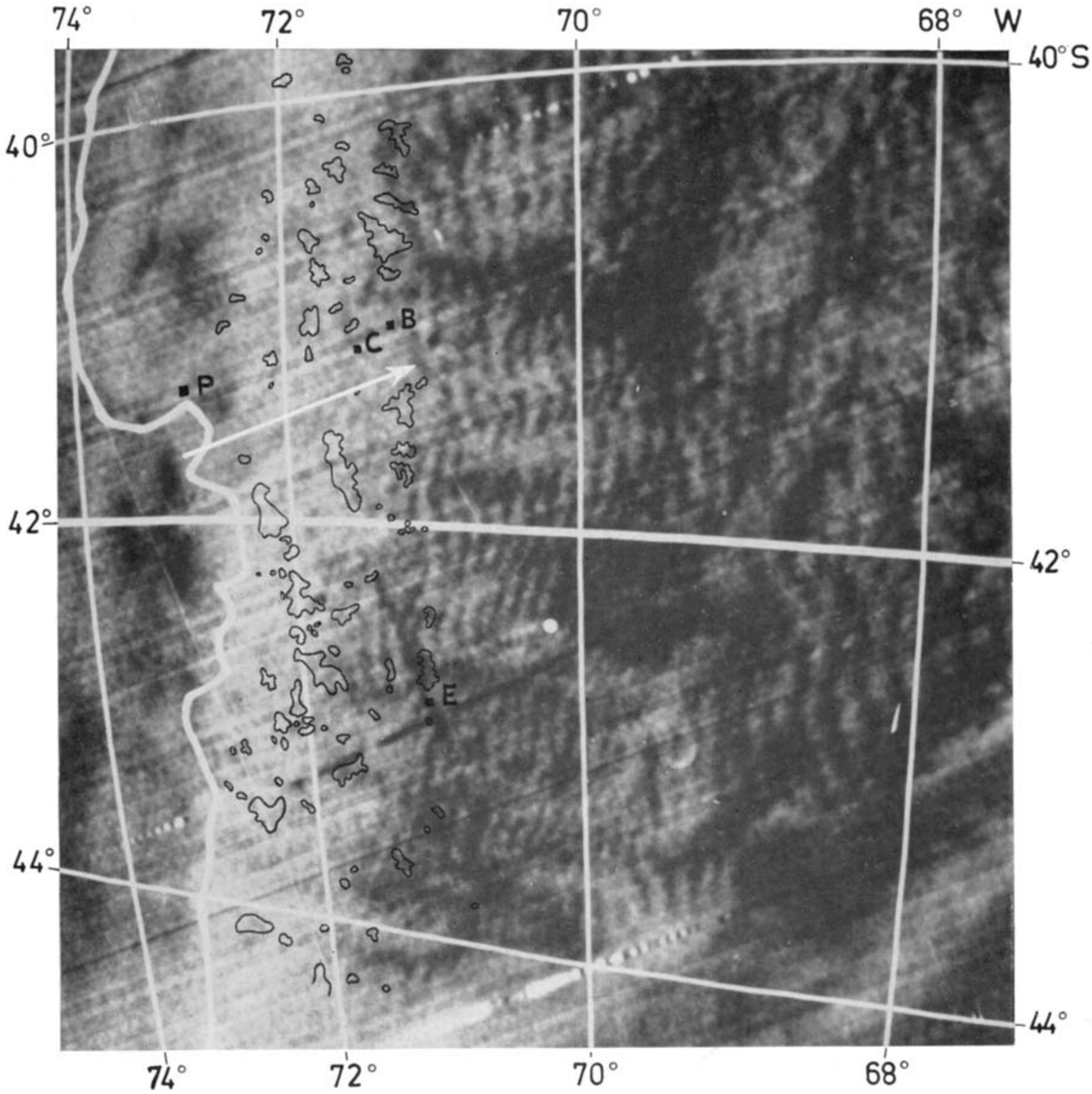

FIG. 1. Tiros I wide angle cloud photograph with superimposed 2° latitude-longitude grid. The picture was taken at 1738 GMT, April 18, 1960. The black lines show the 2000 m level of the mountains. The white arrow indicates the average wind direction (250°) in the troposphere. The positions of the radiosonde station at Puerto Montt and the stations which make cloud observations, Cerro Catedral, Bariloche and Esquel, are also given here.

America is indeed very sparse. It is therefore very fortunate that in the particular area we are concerned with there is one station, Puerto Montt (85–801, rawinsonde). The synoptic situation is given by the surface maps at 1200 and 2300 GMT (cf. Fig. 3). These analyses are obtained from the daily bulletin (Carta del tiempo) published by the national weather service in the Argentine. It is evident that the accuracy of the analyses is relatively poor due to the paucity of data. However, it seems to be quite clear that this day there were no radical

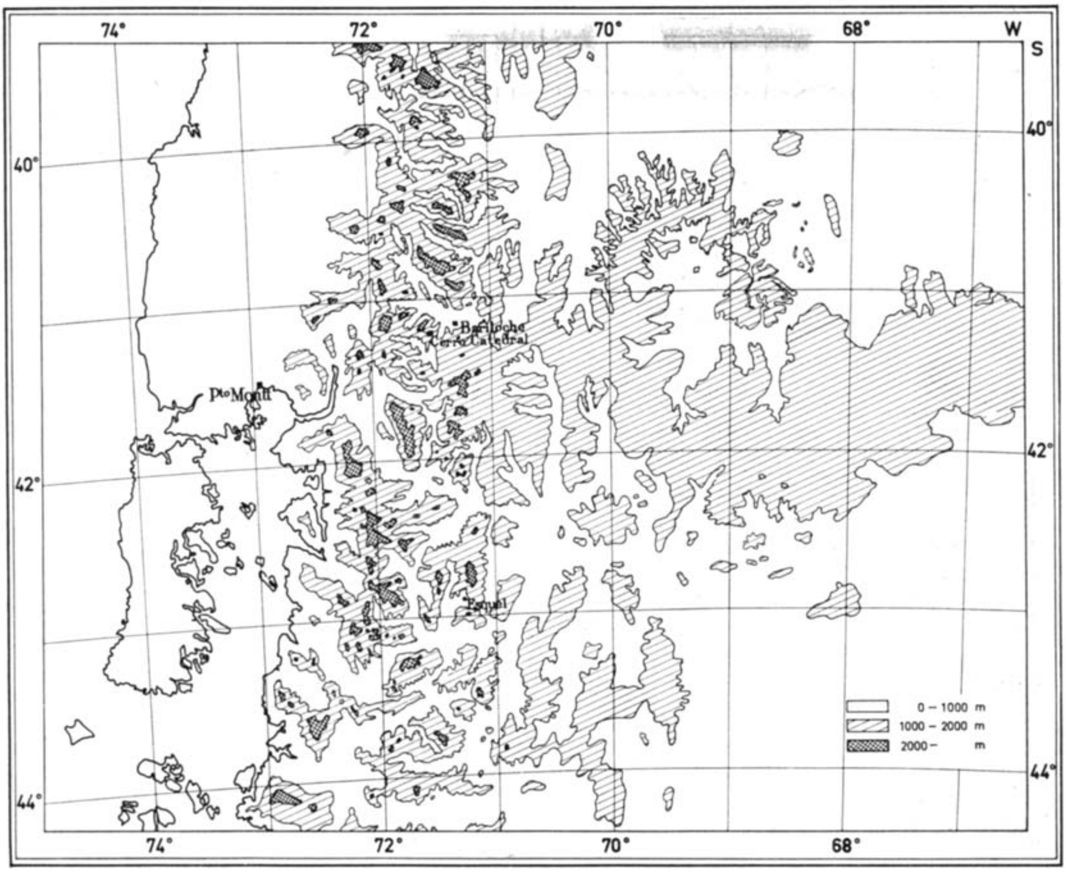

FIG. 2. Topographic map for the region indicated by dashed lines in Fig. 3. This region is approximately the same as the satellite photograph in Fig. 1.

changes of the synoptic situation in the region we consider.

The satellite picture was taken almost exactly between the soundings released at Puerto Montt at 1130 GMT and 2330 GMT. Unfortunately the first one of these two soundings did not reach as high as the 500 mb surface due to icing. In the following theoretical treatment we are therefore forced to make use of the latter sounding only and assume that this is representative for the time when the satellite picture was taken. Since the weather situation was relatively stable in the region we are interested in, it is not likely that this is a serious assumption. The vertical variation of the wind speed, wind direction and temperature obtained from this sounding are shown in Fig. 4 (the dashed lines). We notice that the wind direction varies comparatively little with height and that it is approximately perpendicular to the mountain range.

If we now return to the satellite picture (cf. Figs. 1 and 2) we observe that on the windward side of the mountain range the cloud cover is fairly light, while the mountain range seems to be completely covered by clouds. On the lee side of the mountain, the clouds are very regularly orientated in bands over a large area. Judging from the photograph it is most likely that the type of clouds to the east of the mountain are stratocumulus or altocumulus with the special form undulatus, perlucidus. This is also in close agreement with the cloud observations made at the stations Puerto Montt, Cerro Catedral, Bariloche and Esquel, which are given in Table 1. This table shows that there were very little changes with time, both regarding the type of clouds and the

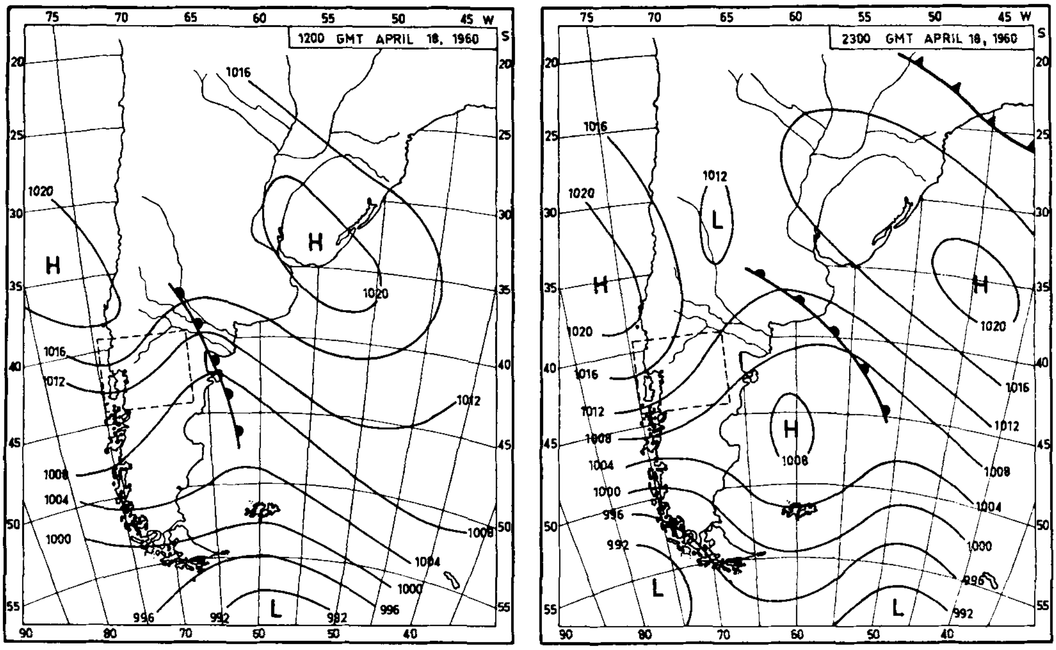

FIG. 3. Surface weather maps for 1200 and 2300 GMT, April 18, 1960. The dashed rectangle indicates the region for the topographic map shown in Fig. 2.

amount of sky covered. We notice that the stations at Bariloche and Esquel, which are situated to the east of the mountain range,

TABLE 1.

Station	Time, GMT	N	N_h	C_l	h	C_m	C_h
Puerto Montt, 6 m	1200	8	8	7	4	×	×
	1500	8	8	7	3	×	×
	1800	8	8	7	3	×	×
	2100	–	–	–	–	–	–
	2300	8	8	7	3	×	×
Cerro Catedral, 1955 m	12	7	7	7	0	2	0
	15	7	4	7	1	2	0
	18	8	8	0	2	2	×
	21	7	4	7	1	2	0
	23	8	8	7	1	×	×
Bariloche, 836 m	12	7	4	5	5	4	0
	15	7	4	5	5	4	6
	18	7	7	5	5	0	0
	21	7	7	5	5	0	0
	23	7	5	5	5	0	0
Esquel, 810 m	12	7	7	8	6	0	0
	15	6	6	8	6	0	0
	18	5	5	8	6	0	0
	21	4	4	8	6	0	0
	23	6	6	8	6	0	0

report stratocumulus the whole day. The code figures indicate that these clouds are of the type which are not formed by flattening of cumulus. We also see that at Bariloche altocumulus translucidus (probably lenticularis) are reported at 1500 GMT.

With the observations which are available there is no possibility to show that the clouds are quasistationary; however, it is very likely that they are.

With regard to the position of the radiosonde station and the possibility of obtaining a fairly accurate estimate of the distances between the clouds in the wind direction we shall now study the conditions in the region between the latitudes 41° and 42°S. A cross-section of the topography has been constructed by averaging the height values along the mountain range between these two latitudes and is shown in Fig. 5*a*. We notice that the crest of the mountain range is about 1800 m above the sea level. However, if we measure the height of the crest above the surrounding ground it is only approximately 800 m.

The wave length has been estimated by measuring the distance over seven cloud bands which are by and large situated in the region

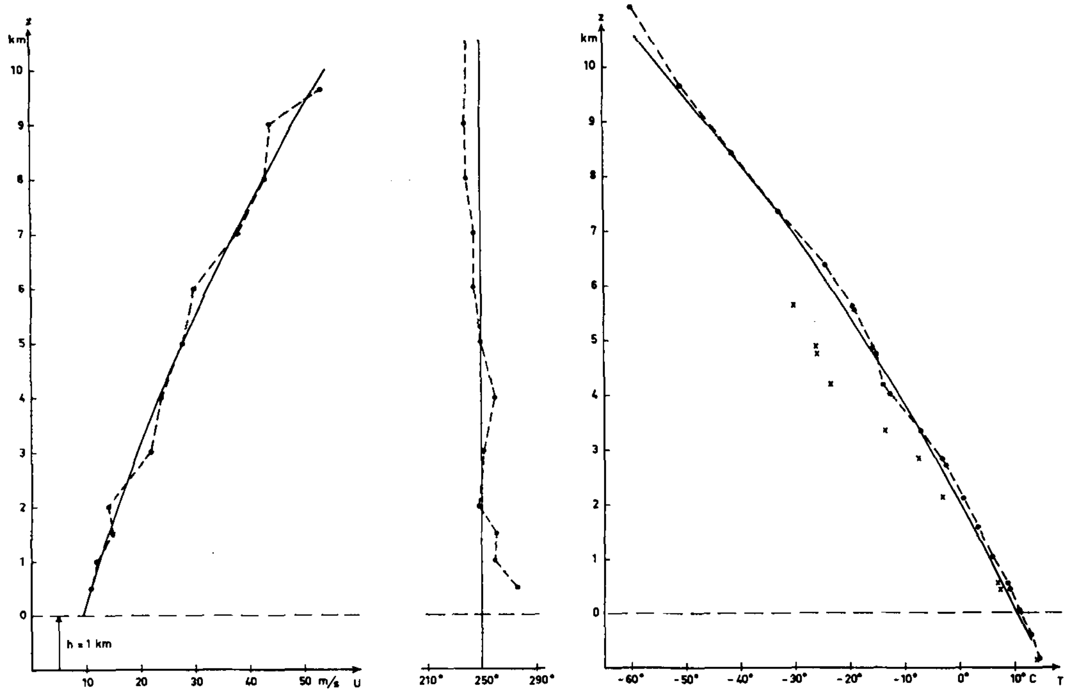

Fig. 4. The vertical distribution of the wind speed, wind direction and the temperature for 2330 GMT, April 18, 1960. The dashed lines and the crosses (the dew point temperature) show the observed conditions as obtained from the radiosonde station at Puerto Montt. The solid lines represent the idealized distributions with height.

bounded by the latitudes 40° and 41°S and the longitudes 70° and 71°W. The average value for the wave length we obtain in this way is 11.3 km. The uncertainty of this value is approximately 1 km.

We shall compare this value for the wave length with a theoretically computed value. The method we shall use is described in I and permits us to compute the stream lines over an ideal mountain profile provided we know the vertical variation of the wind and temperature.

3. Theoretical treatment

The basic assumptions in this theory (I) are that the motion is non-viscous, laminar, isentropic and stationary. It is furthermore assumed that the undisturbed current blows perpendicular to the mountain ridge which has an infinite extension. The characteristic scale of the motion is so small that the effect of the earth's rotation can be neglected. Making use of these assumptions the following perturbation equation for the vertical velocity w' is derived in I

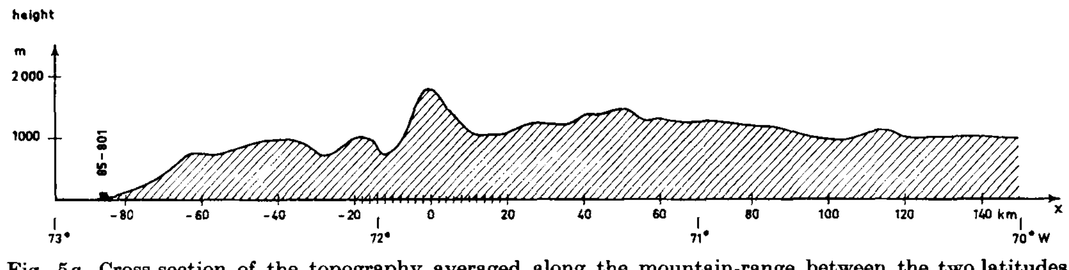

Fig. 5a. Cross-section of the topography averaged along the mountain-range between the two latitudes 41° and 42°S. The position of the radiosonde station at Puerto Montt (85–801) is given by the dot to the west of the mountain ridge.

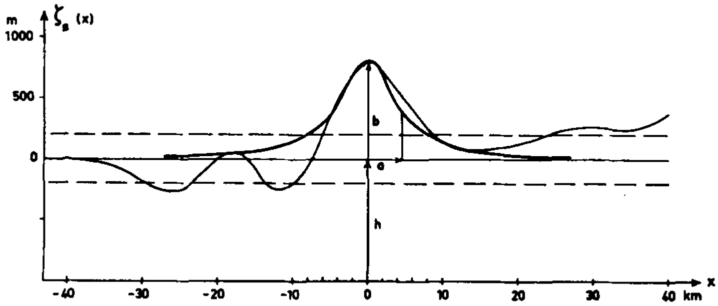

Fig. 5*b*. The heavy line shows the idealized mountain profile (5). *a* denotes the half width of the mountain and *b* the height of the mountain measured from the mean level of the ground. The height of the mean level above the sea level is denoted by *h*. The numerical values of these parameters are in the figure: $a = 4.5$ km, $b = 0.8$ km and $h = 1$ km. The thin line is a part of the actual mountain profile shown in Fig. 5*a*.

$$\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial z^2} + \alpha(z) w_1 = 0, \tag{1}$$

where $$w_1 = w' \exp\left\{-\frac{g - R\gamma}{2RT_0} z\right\}, \tag{2}$$

$$\alpha(z) = \frac{g(\gamma_d - \gamma)}{U^2 T_0} - \frac{1}{U}\frac{d^2U}{dz^2} + \left(\frac{\gamma_d - \gamma}{T_0} - \frac{g}{\varkappa R T_0}\right)\frac{1}{U}\frac{dU}{dz} - \frac{2}{\varkappa R T_0}\left(\frac{dU}{dz}\right)^2 - \left(\frac{g - R\gamma}{2RT_0}\right)^2. \tag{3}$$

The velocity of the undisturbed current is denoted by U while γ and T_0 represent its lapse rate and the temperature respectively. g is the acceleration of gravity, γ_d the dry adiabatic lapse rate, $\varkappa$ the ratio of the specific heats at constant pressure and constant volume and R the gas constant for unit mass.

The lower boundary condition requires the flow to be tangential to the surface, while the upper boundary condition states that the energy of the waves remains finite at great heights. Using these boundary conditions equation (1) is solved in I for the case when the coefficient (3) can be represented by the function

$$\alpha^*(z) = \alpha_0 e^{-\lambda z} + \alpha_1, \tag{4}$$

where α_0, α_1 and λ are positive parameters.

If the profile of the mountain range has the following form

$$\zeta_s(x) = \frac{b}{1 + \left(\frac{x}{a}\right)^2}, \tag{5}$$

where a and b are constants, the expression for the displacements of the streamlines becomes

$$\zeta(x, z) = \begin{cases} -2\pi a b \dfrac{U(0)}{U(z)} \exp\left\{\dfrac{g - R\gamma}{2RT_0} z\right\} \cdot \displaystyle\sum_{n=1}^{N} \left[e^{-ak} \sin kx \frac{J_\nu\left(\beta e^{-\frac{\lambda z}{2}}\right)}{\frac{d}{dk} J_\nu(\beta)} \right]_{k=k_n} + I_1 & \text{for } x > 0 \\ I_2 & \text{for } x < 0, \end{cases} \tag{6}$$

where $$\left.\begin{aligned} \nu &= \frac{2}{\lambda}\sqrt{k^2 - \alpha_1}, \\ \beta &= 2\frac{\sqrt{\alpha_0}}{\lambda}. \end{aligned}\right\} \tag{7}$$

k is the wave number. By making use of (7) the wave numbers $k_1, k_2, \ldots, k_N$ are obtained from the positive roots $\nu_1, \nu_2, \ldots, \nu_N$ of the equation

$$J_\nu(\beta) = 0. \tag{8}$$

The symbols I_1 and I_2 in (6) represent terms which are important only in the close vicinity of the mountain.

The number of roots of equation (8) depends on the value of the parameter β. This can be seen in Fig. 6, where curves for $J_\nu(\beta) = \text{const}$ are shown in a ν, β-plane. (From JAHNKE & EMDE, 1945).

4. Results

The necessary information we require in order to compute the wave length is the observed vertical distributions of the wind and temperature which are represented by the dashed lines in Fig. 4. With these data the coefficient $\alpha(z)$ (3) can be evaluated. It is

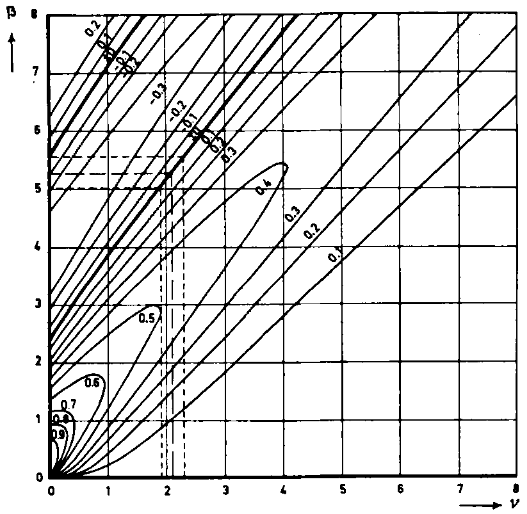

FIG. 6. Curves $J_\nu(\beta) = \text{const}$ in the ν,β-plane. The dashed lines represent the three cases for different values of the parameter β.

evident that $\alpha(z)$ cannot be represented exactly by the function $\alpha^*(z)$ (4). However, below the tropopause a relatively close agreement can be obtained. The solid lines in Fig. 4 show the idealized distributions of wind and temperature which can be represented by $\alpha^*(z)$. The values of the parameters which occur in this function are given in Table 2.

TABLE 2.

h (km)	α_0 (km^{-2})	α_1 (km^{-3})	λ (km^{-1})	β	ν	L (km)
0.8	1.821	0.008	0.488	5.53	2.31	11.0
					0.009	70.3
1.0	1.652	0.008	0.488	5.27	2.11	12.0
1.2	1.499	0.008	0.488	5.02	1.91	13.2

The value of the parameter α_0 depends on the choice of the height h of the mean level of the ground above the sea. Studying the profile of the mountain range (Fig. 5a) we notice that to the west of the crest the surface is somewhat below 1000 m and to the east it is slightly higher. In order to obtain an estimate of how much the result depends on this parameter we have chosen three different values of h (0.8, 1.0 and 1.2 km). In Table 2 the roots of equation (8) are also given. We notice that for the value of β we have in the case $h = 0.8$ km, the equation has two roots.

Concerning the parameters which determine the shape of the mountain we have evaluated the solution for three different values of the parameter a (4, 4.5 and 5 km). The parameter which gives the height of the mountain is set to 0.8 km. In Fig. 5b is shown the idealized profile for $a = 4.5$ km. It is to be pointed out here that the parameters a and b have no influence on the wave length of the produced disturbances. It is only the amplitude of the waves which are dependent on a and b. By studying the solution we also notice that the amplitude is directly proportional to b.

The vertical variations of the amplitudes for the different values of the height of the surface h and the half width of the mountain a are shown in Fig. 7.

5. Discussion

Before we turn to a discussion of the results obtained, it should be pointed out that we cannot consider this case as a perfect case for testing a theoretical method for computing the form of the streamlines in an air flow passing over a mountain ridge. In the first place we only know the form of the actual streamlines indirectly through the clouds. In this case we have furthermore assumed that the observed cloud pattern is quasi-stationary. This seems to be quite plausible, but it has not been possible to confirm it.

We see thus that our possibility of verifying the theory is limited. This is particularly so for the amplitude. Regarding the wave length we are somewhat more fortunate. By comparing the observed wave length obtained by measuring the distances between the cloud bands on the satellite picture with the theoretically computed values we find a good agreement.

The uncertainty of the computed values for the wave length depends on a number of factors. The choice of the height of the mean level above the sea seems to be one of the more important ones. Judging from Figs. 2 and 5 we see that the variation of the ground level in the x-direction is not so great as to make it possible to obtain an estimate of the uncertainity of the wave length by changing h by 0.2 km. This leads to a change in the wave length by approximately 1 km (cf. Table 2). In

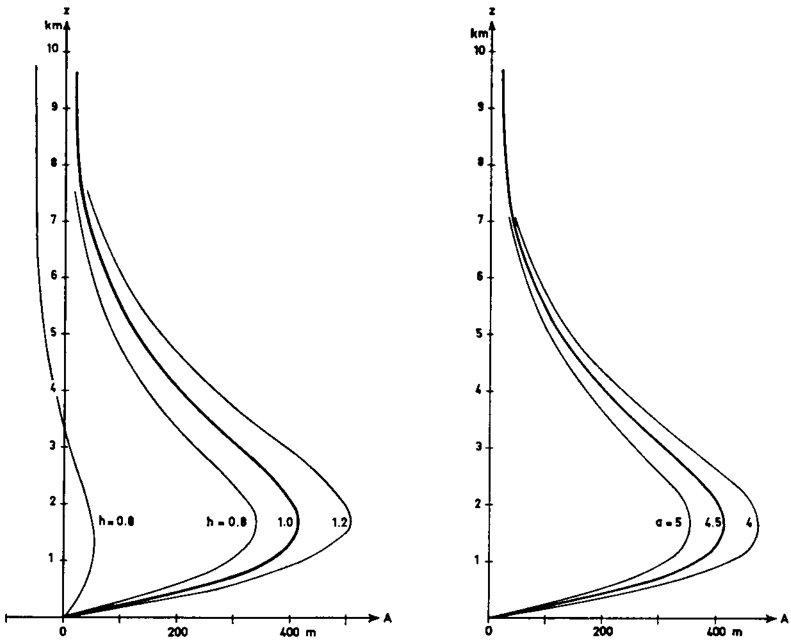

FIG. 7. The vertical variations of the amplitudes for different values of the height of the mean level (to the left) and the half width of the mountain (to the right).

the case $h = 0.8$ km there are two roots of equation (8). Consequently there are also two wave solutions. In Fig. 7 we see that the solution which has the longest wave length has a relatively small amplitude. It is therefore of little interest in this connection. The fact that for certain values of the parameter β, two or more wave solutions can occur is sometimes very clearly illustrated by cloud observations.

Another cause of the uncertainty is that we have idealized the observed wind and temperature distribution with height in order to make it possible to replace the coefficient $\alpha(z)$ (3) by the simple function $\alpha^*(z)$ (4). In addition to this we have been forced to assume that the sounding at 2330 GMT is representative for the conditions at the time when the picture was taken. How severe these assumptions are is difficult to estimate. As can be seen in Fig. 4 the idealized wind and temperature distribution does not deviate too much from the observed ones in the troposphere. Concerning the large deviations above the tropopause, it should be mentioned that PALM & FOLDVIK (1960) have shown that the conditions in the stratosphere have little influence on the motion in the middle and lower troposphere. The dependence of the wind on the computed value of the wave length can be estimated with the aid of the various solutions obtained for the different values of h. A change in h can, in fact, also be interpreted as a vertical displacement of the curves for the wind and temperature relative to the mean level. For example, a decrease in the mean level is equivalent to a displacement upwards of the curves for U and T_0 by the same amount. The effect of this is that the wind decreases and the temperature increases. However, the relative change in the temperature will be rather small and the stability is approximately the same. Consequently, a change in h corresponds to a change of the wind.

This discussion, together with the corresponding estimate in I, suggests that the inaccuracy due to the idealization of the wind and temperature distribution with height is approximately 1 km.

In conclusion it seems reasonable to state that, in spite of the inaccuracies in both the observed and computed value for the wave length, the case supports the validity of the theoretical method.

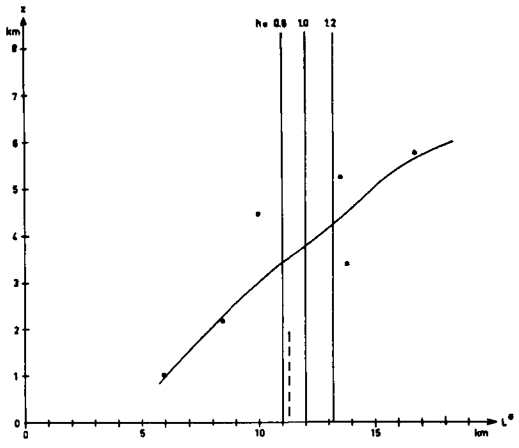

FIG. 8. The slanting solid line shows the vertical variation of the wave length computed with the aid of formula (9), using the idealized wind and temperature distributions with height. The dots give the corresponding wave lengths for the actual wind and temperatures. The vertical solid lines give the wave lengths obtained with the present method for $h =$ 0.8, 1.0 and 1.2 km. The dashed line indicates the observed wave length.

6. The "parcel method"

For comparison the wave length has also been evaluated using the very simple so called "parcel method" which is based on the principle that if an air parcel is subjected to a vertical displacement it attempts to return to its equilibrium level provided the stratification is stable. This method gives the following expression for the wave length.

$$L^* = 2\pi U \sqrt{\frac{T}{g(\gamma_d - \gamma)}}, \quad (9)$$

where the symbols have the same meaning as before. The result of this comparison is shown in Fig. 8. The slanting line represents the variation of L^* with height when the idealized distribution of the wind and temperature is used. The dots show the values which are obtained when the actual wind and temperature observations are used. The vertical lines show the wave length obtained with the present method for different values of the parameter.

We notice that the parcel method gives an increasing wave length with height. This is usually not the case in reality, although we cannot verify this in the present case. However, it is quite clear that the parcel method gives a relatively poor agreement with the observed wave length around the level where the clouds occur, approximately at 1 km.

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