A two-dimensional model of mother-of-pearl clouds

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ABSTRACT

A two-dimensional model of a mother-of-pearl cloud is computed for a specified structure of the atmosphere at the levels of the inversion at about 25 km in the winter polar stratosphere: temperature at the base of the inversion = −80°C, dewpoint depression = 4C°, wind speed = 50 m/sec and a wave motion with amplitude 550 m and wavelength 40 km. Characteristic features of observed clouds such as the asymmetry and the distribution of particle sizes (observed as distribution of colors) are explained. "Ice wings", in earlier works incorrectly interpreted as "ice tails", are explained in the light of three-dimensional mountain waves as ice clouds formed by sublimation when saturation with respect to ice, but not with respect to water, is reached.

1. Introduction

In a previous paper (HESSTVEDT, 1959) the present author investigated the characteristic features of the synoptic situations during occurrences of mother-of-pearl clouds in Norway. It was found that very low temperatures in the stratosphere were always observed, thus increasing the possibility of saturation. Furthermore, winds in the sector W to N, with no change of direction with height, were necessary for mountain waves to reach the considerable heights of 20–30 km. On the crests of these waves the clouds will form due to adiabatic cooling. In another paper (HESSTVEDT, 1960) the growth of the cloud particles was studied. Simple cloud models were constructed and some observational peculiarities such as the uniformity in particle size and the asymmetrical shape of the clouds were explained.

During the last few years our knowledge of the conditions in the stratosphere have increased considerably. In particular, humidity measurements up to about 30 km have shown that the levels where mother-of-pearl clouds form are probably not as dry as was previously believed (see for instance GUTNICK, 1961). Furthermore, improved routine upper wind observations from Norwegian radiosonde stations have made it possible to construct wind profiles up to the level of mother-of-pearl clouds in situations when such clouds occurred (GOTAAS, 1961). It is therefore desirable to revise the cloud model in accordance with the new and better observations. This will be done in this paper. Furthermore, a two-dimensional cloud model has been developed, showing a vertical cross-section through a mother-of-pearl cloud. The procedure is similar to that used by the author in noctilucent cloud studies (HESSTVEDT, 1962).

2. The growth of cloud particles

The rate of growth of water droplets in a cloud may be determined from

$$(r+s)\,\frac{dr}{dt} = \frac{D}{\varrho_w R_v T}\,[e - e_w'(r,T)] \qquad (1)$$

(ROOTH, 1957), where r = droplet radius, t = time, D = diffusivity of water vapor in air, ϱ_w = density of water, R_v = gas constant of water vapor, T = absolute temperature, e = ambient vapor pressure, $e_w'(r,T)$ = equilibrium vapor pressure at the surface of the droplet, $s = (D/\alpha)\sqrt{2\pi/R_v T}$ and α = condensation coefficient. The rate of growth of ice spheres is given by an equation similar to (1); we have only to replace ϱ_w, $e_w'(r,T)$ and α by the analogous expressions ϱ_i, $e_i'(r,T)$ and α'.[1] The equilibrium vapor

[1] In the computations below both α and α' are taken as 0.5.

pressure, $e'_w(r, T)$, depends upon temperature, droplet radius and concentration of dissolved salts. It is, however, easy to show that for our particular problem the effects of the curvature and of the salt concentration are of importance only during the very first stages of the particle growth. For the major part of the particle growth the effects are negligibly small, and for the sake of simplicity we shall neglect them throughout.

If an air parcel is displaced vertically, its temperature will change according to

$$T = T_0 - \gamma z, \qquad (2)$$

where γ = dry adiabatic lapse rate and z = vertical displacement, measured from a reference level where $T = T_0$. Since the released heat of vaporization is negligibly small at such low temperatures, (2) is assumed to cover also moist adiabatic processes. Furthermore, the vapor pressure will vary as

$$e = e_0 \left(\frac{T}{T_0}\right)^{\frac{7}{2}} - NR_v T \varrho_w \frac{4\pi}{3} (r^3 - r_n^3), \qquad (3)$$

where e_0 = vapor pressure at the reference level, N = concentration and r_n = radius of the condensation nuclei. Since, in the case of mother-of-pearl clouds, r is much smaller than s, it is convenient to write (1) in the form

$$\frac{dr}{dt} = \frac{\alpha e_0}{\varrho_w \sqrt{2\pi R_v T_0}[1 + (r/s)]} \left[\left(\frac{T}{T_0}\right)^3 - \left(\frac{T_0}{T}\right)^{\frac{1}{2}} \frac{e_w(T)}{e_0} - NR_v (T_0 T)^{\frac{1}{2}} \varrho_w \frac{4\pi}{3e_0} (r^3 - r_n^3) \right], \qquad (4)$$

where $e_w(T)$ = saturation vapor pressure over a plane water surface.

The last term in the brackets represents the reduction of vapor pressure due to condensation. The concentration of condensation nuclei, N, has been found to be of the order 1 cm^{-3} or even smaller (JUNGE, CHAGNON & MANSON, 1961). It turns out that the liquid (or solid) water content is small compared with the difference between the first two terms in the brackets during the major part of the particle growth. For $N \leq 0.1$ cm^{-3} the liquid water content may be neglected in the computations, whereas for $N = 1$ cm^{-3} it will introduce an adjustment which is of importance only for particle sizes greater than about 2 μ.

3. The cloud model

In order to construct a two-dimensional model of a mother-of-pearl cloud we shall assume a temperature profile in the vicinity of the inversion at about 25 km in the arctic winter stratosphere. For the present problem only the levels within a vertical distance of ± 1.5 km from the inversion are of interest. The selected profile is shown in Fig. 1.

Humidity measurements up to 30 km show that the variation of the frost-point (and consequently also the vapor pressure) is not very great for the levels of interest to us (see for instance GUTNICK, 1961). In our model atmosphere we shall assume the air to be saturated with respect to ice at the base of the inversion, where the temperature is assumed to be -80°C. The frost-point is further assumed to be the same (-80°C in the undisturbed atmosphere) at all levels in question. A frost-point of -80°C corresponds to a dew-point of -84°C and a mixing ratio of 0.0137 g/kg at 24 km (which is the mean height of mother-of-pearl clouds). Air parcels at the base of the inversion will then become saturated after a dry-adiabatic cooling of 4.5C°.

In our model atmosphere we shall assume a wave motion

$$Z = A \sin \frac{2\pi v}{\lambda} t, \qquad (5)$$

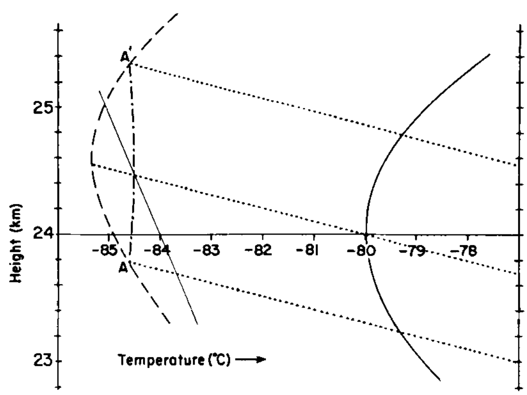

Fig. 1. Thick solid curve: temperature profile in the undisturbed atmosphere. Dashed curve: temperature conditions at the wave-crests. Dotted lines: dry-adiabates. Dash-dotted curve (AA'): temperatures at the lifting condensation levels. Thin, solid line: change of dew-point due to adiabatic lifting.

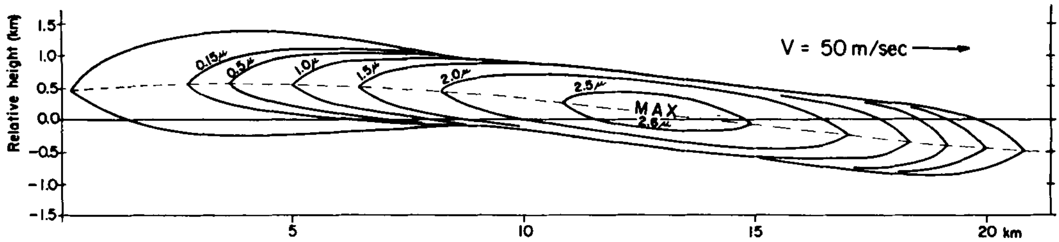

Fig. 2. Two-dimensional model of a mother-of-pearl cloud. The dotted curve indicates the wave motion at the base of the inversion.

which is here assumed to be the same at all levels in question. As appropriate values for the constants we shall assume an amplitude $A = 550$ m, a wind speed $v = 50$ m/sec and a wavelength $\lambda = 40$ km (HESSTVEDT, 1957; GOTAAS, 1961).

It is seen from Fig. 1 that saturation with respect to water is reached within a layer of thickness AA'. The length of AA', consequently, represents the maximum thickness of the cloud, which is clearly highly dependent on the temperature profile near the inversion. The position of the upwind edge of the cloud is determined by the dew point depression at the base of the inversion (and, of course, also by the wave amplitude). The observed fluctuations in the length of mother of pearl clouds are easily explained on the basis of even small variations in these parameters.

In our model cloud the particles are assumed to form upon nuclei of radius $r_n = 0.05$ μ and to grow as water droplets above the condensation level until a droplet radius of 0.15 μ is reached. Then spontaneous freezing of the droplets is assumed to take place. The rate of particle growth will therefore be considerably increased because of the difference in vapor pressures over ice and over water.

The result of the calculations is shown in Fig. 2. The model cloud shows a double asymmetry: The downwind edge of the cloud is situated considerably lower (950 m) than the upwind edge. This is partly due to the transformation from water cloud to ice cloud, partly due to the slow rate of evaporation, which starts when the convection sublimation level has been reached. (For this reason, a pure water cloud or a pure ice cloud would also show the same kind of asymmetry although less marked.) Furthermore, the cloud is seen to be somewhat thicker in the upwind part than in the downwind part. This asymmetry is caused by our assumption that the particles will freeze when they have grown to a certain size (in our case 0.15 μ). Some of the water droplets will never grow to this critical size and will therefore evaporate soon after they have passed the condensation level on the downwind side of the wave crest. The thickness of the cloud in the downwind part is therefore determined by the depth of the layer where the cloud particles are "trapped" by freezing.

The similarity between the model cloud in Fig. 2 and cloud "B" in GOTAAS' paper (1961) is striking. But the variety of cloud forms is, of course, unlimited and so also is the possibility of constructing model clouds by varying the parameters, in particular the temperature profile and dew point depression.

The distribution of colors in mother-of-pearl clouds is a function of the particle size and of the angular distance to the sun. Very often, especially for small clouds, this angular distance varies only little from point to point in the cloud, and the distribution of colors will primarily depend upon variation in particle size. Bearing this in mind, it is of interest to note the similarity between the distribution of particle sizes in Fig. 2 and observed distributions of colors in mother-of-pearl clouds (see for instance STÖRMER, 1932).

4. Concluding remarks

Although some details have yet to be explained we may say that the formation of mother-of-pearl clouds is no longer the great puzzle it was only some ten years ago. The temperature and wind conditions necessary for the clouds to form are fairly well known, and the mean humidity conditions in the stratosphere are such that condensation may very well occur under the synoptic conditions typical for mother-of-pearl cloud situations. Among the details which have still to be explained we may mention: (*a*) whether the bulk of the particles are liquid or solid, (*b*) the faint veils often observed and the "wings" seen on some pictures (GOTAAS, 1961).

With regard to (*a*) some writers conclude from the remarkable iridescence connected with the clouds that the particles are liquid, and they even go so far as to conclude that the temperature must therefore be above −40°C. The latter conclusion is, of course, not justified. When the clouds occur, the temperature has always been found to be somewhere around −80°C, and at this temperature the possibility of supercooling water is probably very small (HESSTVEDT, 1960). Furthermore, it is not impossible that the life-time of an ice particle in a mother-of-pearl cloud is too short for the development of a crystalline shape. Finally, one may question whether the repuirement for spherical particles in order to get iridescence is really necessary for such small particle sizes.

With regard to (*b*) it is first of all important to notice that the "wings" connected with some clouds form an angle of about 60° with the wind direction. Earlier, these "wings" were explained as "ice tails", proving that the bulk of cloud particles were liquid while a smaller fraction was converted into ice particles, the evaporation of which was retarded owing to the difference in equilibrium vapor pressures. This explanation is obviously not correct.

Here a more reasonable explanation will be given: From the shape of mother-of-pearl clouds we may deduce that they are formed by mountain waves set up by individual peaks rather than mountain ranges. WURTELE (1957) studied three-dimensional mountain waves in an atmosphere with constant stability and found that the horizontal projections of the wave crests are hyperbolas, concave downwind. If we assume that the amplitude of the wave is decreasing with increasing cross-wind distance, mother-of-pearl clouds will only form within a limited cross-wind distance, where saturation with respect to water occurs. At greater cross-wind distances, saturation with respect to ice but not with respect to water is reached. In this case only a few particles will form upon the relatively few available sublimation nuclei. This explanation is also supported by the sharp cross-wind edges of the clouds.

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