On the possibility of ice cloud formation at the mesopause

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ABSTRACT

As a contribution to the discussion about the nature of noctilucent clouds some aspects of the ice particle hypothesis are considered. A model of an ice cloud is computed, assuming a temperature profile and wave motions in agreement with observations. Some of the observed characteristics of noctilucent clouds are well reproduced by the model. Similar computations for stratiform clouds show that a diffusion coefficient of the order 10^7 cm^2/sec seems to be required to keep particle sizes within the observed range. It is concluded that the ice particle theory and the cosmic dust theory should be considered as equally important in future studies of noctilucent clouds.

1. Introduction

The nature of noctilucent clouds has been the subject of study among meteorologists and physicists for many years. At present two possibilities are seriously considered: (*a*) the clouds may consist of solid particles of cosmic origin, the concentration of which has a maximum at or just above the mesopause, or (*b*) they are ordinary ice clouds, formed by sublimation in the low temperature region at the mesopause.

The latter possibility was suggested by HUMPHREYS (1933). He estimated the water vapor content in the stratosphere and mesosphere to be 0.25 g/kg at all levels up to the level of the noctilucent clouds, and from the resulting frost point, 160°K (which, from recent stratospheric density data, may be revised to 166°K), he concluded that supersaturation and sublimation may occasionally occur. Humphreys' ideas were later taken up along the same lines by KHVOSTIKOV (1952). However, a brief review of the literature on noctilucent clouds makes it clear that most writers are in favor of the meteoric dust theory.

The steadily increasing amount of observational data from the upper mesosphere makes it possible to draw further the consequences of the sublimation hypothesis. The present author (HESSTVEDT, 1961) showed that the rate of ice particle growth under the conditions which are believed to exist at the noctilucent cloud level (80–85 km) is such that the particles may very well attain the observed sizes (WITT, 1960) within a reasonable time. It was further pointed out that the seasonal and latitudinal variation in the frequency of noctilucent cloud occurrences is easily explained if the clouds really are ice clouds.

In the present paper an attempt is made to construct an ice cloud model and study its development under conditions typical of the high-latitude mesopause.

2. The rate of growth

It has been shown (HESSTVEDT, 1961) that for low pressure the growth of small particles is determined by

$$\frac{dr}{dt} = \frac{\alpha}{\varrho_i \sqrt{2\pi R_v T}} [e - e_i(r)], \tag{1}$$

where r = particle radius, t = time, α = sublimation coefficient, ϱ_i = density of ice, R_v = gas constant of water vapor, T = absolute temperature, e = ambient water vapor pressure, and $e_i(r)$ = equilibrium vapor pressure over an ice sphere of radius r.

If an air parcel in the vicinity of the mesopause is displaced vertically, its temperature will change almost dry-adiabatically, since the released heat of fusion is exceedingly small and may be neglected. Choosing the convection sublimation level as a reference level we may write

$$T = T_0 - \gamma z, \tag{2}$$

where T_0 = frost point, γ = dry adiabatic temperature lapse rate (= 0.96°C per 100 m) and z = vertical displacement measured from the convection sublimation level.

If no sublimation took place we would have

$$e = e_0 \left(\frac{T}{T_0}\right)^{7/2}, \tag{3}$$

where e_0 = ambient vapor pressure at the level $z = 0$ and c_{pv} = specific heat of water vapor at constant pressure. But due to the particle growth some of the water vapor is removed and the vapor pressure is reduced by an amount e^* given by

$$\frac{e^*}{R_v T} = N \varrho_i \frac{4\pi}{3}(r^3 - r_n^3), \tag{4}$$

where N = number of cloud particles per unit volume and r_n = radius of the nucleus. (For the sake of simplicity, the nuclei are assumed to be of uniform size and nucleating efficiency.) Consequently

$$e = e_0 \left(\frac{T}{T_0}\right)^{7/2} - N R_v T \varrho_i \frac{4\pi}{3}(r^3 - r_n^3). \tag{5}$$

From the Clausius-Clapeyron equation

$$\frac{de_i}{dT} = \frac{L_s e_i}{R_v T^2}, \tag{6}$$

where e_i = equilibrium vapor pressure over a plane ice surface and L_s = latent heat of fusion, we get

$$e_i = e_0 \exp\left[-\frac{L_s}{R_v}\left(\frac{1}{T} - \frac{1}{T_0}\right)\right] \tag{7}$$

which, in combination with Kelvin's equation

$$e_i(r) = e_i \exp \frac{2\sigma_{13}}{\varrho_i R_v T r}, \tag{8}$$

where σ_{13} = specific free surface energy at an ice–vapor interface, gives

$$e_i(r) = e_0 \exp\left[\frac{2\sigma_{13}}{\varrho_i R_v T r} - \frac{L_s}{R_v}\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]. \tag{9}$$

Substitution of (5) and (9) in (1) gives

$$\frac{dr}{dt} = \frac{\alpha e_0}{\varrho_i \sqrt{2\pi R_v T}} \left\{ \left(\frac{T}{T_0}\right)^{7/2} - \frac{4\pi N \varrho_i R_v T}{3 e_0}(r^3 - r_n^3) - \exp\left[\frac{2\sigma_{13}}{\varrho_i R_v T r} - \frac{L_s}{R_v}\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]\right\}. \tag{10}$$

The integration of (10) can now be done upon substitution of realistic values of α, e_0 (or T_0), N and r_n, and for a particle trajectory selected in agreement with observation. It is, however, possible to simplify (10) considerably. In the first place, it turns out that the second term in the parenthesis is negligibly small in comparison with the difference between the two other terms, unless N should be $> 10^2$ cm^{-3} which seems to be an unreasonably high concentration. According to LUDLAM (1957) and WITT (private communication), concentrations higher than about 1 cm^{-3} would hardly occur. Physically, this means that the supersaturation is only slightly reduced by the removal of water vapor due to sublimation. Furthermore, the influence of the curvature is also very small, and for all practical purposes it is sufficient to put $T = T_0$ and $r = \bar{r}$ = constant = 10^{-5} cm in the curvature term. (10) can then be written

$$\frac{dr}{dt} = \frac{\alpha e_0}{\varrho_i \sqrt{2\pi R_v T_0}} \left\{ \left(\frac{T}{T_0}\right)^3 - \left(\frac{T_0}{T}\right)^{1/2} \exp\left[\frac{2\sigma_{13}}{\varrho_i R_v T_0 \bar{r}} - \frac{L_s}{R_v}\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]\right\} = \frac{\alpha e_0}{\varrho_i \sqrt{2\pi R_v T_0}} \cdot I. \tag{11}$$

The variation of I with $T - T_0$ is shown in Fig. 1 for T_0 = 162°K, 172°K and 182°K. It is seen that the values of I do not depend significantly upon the assumed value of T_0. Furthermore, when $T - T_0 < -4°$, i.e. for a lifting of more than about 0.4 km above the sublimation level the rates of growth will vary only within narrow limits. For positive values of $T - T_0$ the evaporation is seen to proceed relatively rapidly.

3. Computation of the particle growth

Observations show that noctilucent clouds often form a diffuse veil, while on other occasions very distinct wave motions occur (WITT, 1962). First we shall compute the growth of ice particles which follow trajectories in ac-

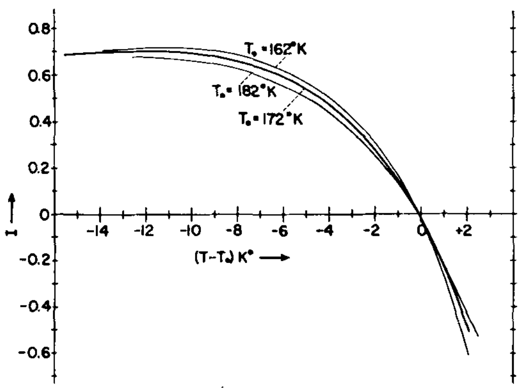

FIG. 1. Variation of I with $T - T_0$ for three different values of T_0 (162°K, 172°K, 182°K).

cordance with Witt's observations. Let us consider a wave defined by

$$z = A \sin \frac{2\pi v}{\lambda} t, \tag{12}$$

where A = amplitude, λ = wavelength and v = particle velocity relative to the wave. Witt refers to two types of waves and in agreement with his data the following values will be used for A, v and λ:

	A	v	λ	
Long waves	1 km	80 m/sec	35 km	(13a)
Short waves	0.4 km	15 m/sec	7.5 km	(13b)

Integration of (11) for these two waves gives for the maximum particle size, which is attained at $z = 0$ at a distance $\lambda/2$:

$$r_{\max} = r_n + 1.8 \times 10^{-3} \alpha e_0 \quad \text{for the long waves,} \tag{14a}$$

$$r_{\max} = r_n + 1.2 \times 10^{-3} \alpha e_0 \quad \text{for the short waves.} \tag{14b}$$

Since r_n, α and e_0 are unknown, precise values for the maximum particle size cannot be given. It is, however, possible to estimate the value of e_0 which would lead to a reasonable growth. WITT (1960) found that $r = 0.13\ \mu$ is a probable mean value in the brightest parts of the clouds. Accordingly, we shall assume that a reasonable particle growth is represented by $r_{\max} - r_n = 0.1\ \mu$, leading to

$$\alpha e_0 = 0.55 \times 10^{-2}\ \text{dynes/cm}^2 \quad \text{for the long waves,} \tag{15a}$$

$$\alpha e_0 = 0.83 \times 10^{-2}\ \text{dynes/cm}^2 \text{ for the short waves.} \tag{15b}$$

The value of α is still unknown. For tropospheric conditions $\alpha = 0.5$ has been suggested. This would lead to a frost point of 172°K for the long waves and 174°K for the short waves. In a typical high latitude summer atmosphere, the air pressure at 82 km is found to be 0.0077 mb (NORDBERG & STROUD, 1961). Accordingly, a vapor pressure of 1.1×10^{-2} dynes/cm² is equivalent to a mixing ratio of 0.9 g/kg.

It is interesting to evaluate the results obtained above in the light of the most recent observations of temperature and humidity in the stratosphere and mesosphere.

Several series of humidity measurements have been made up to a height of 30 km. (For references see MURCRAY, MURCRAY & WILLIAMS, 1962.) The general feature of the height variation of the mixing ratio seems to be a steady increase from a value of the order 0.01 g/kg at 15–20 km to a value of 0.1 g/kg or more at 30 km. The trend of the curves seems to indicate that it is not unreasonable to assume a mixing ratio of the order 1 g/kg at 40 km.

Since no observational data above 30 km are available, any estimate of the water vapor content in the mesosphere is, of course, subject to large uncertainties. As a first approximation we shall assume that the mixing ratio does not vary significantly with height. It is then possible to compute frost points at different levels, based upon fixed mixing ratios. In Fig. 2 such frost points are shown for the mixing ratios 0.1 g/kg (curve A) and 1 g/kg (curve B). The value of the frost point at 82 km, as determined from the calculations in this section (172°K), is marked by a cross. This value seems to fit reasonably well with the values obtained from the observations at 30 km, especially in view of the trend of the mixing ratio from 20 km to 30 km.

The validity of the assumption of a constant mixing ratio in the layer 40–85 km may, of course, be questioned. Unless new methods are developed, there seems to be very little hope that direct humidity measurements can be made at these levels. On the other hand, useful information may be expected from theoretical

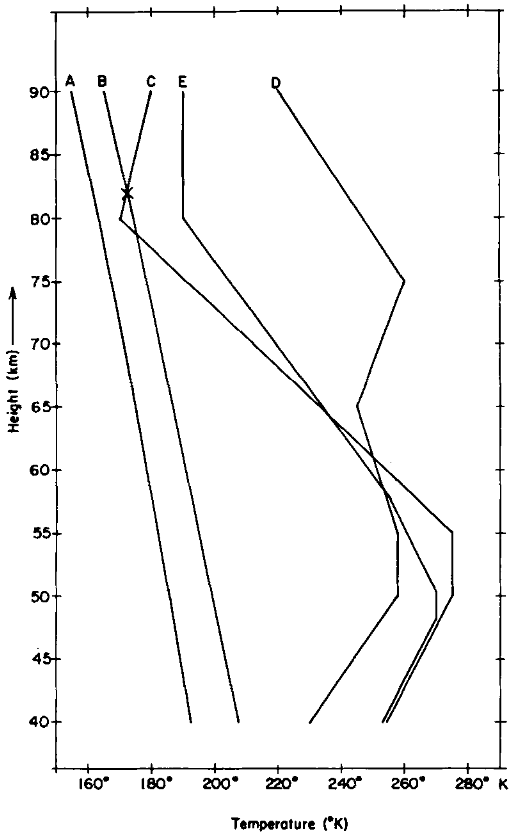

FIG. 2. Curve A: variation of frost point with height (mixing ratio = 0.1 g/kg). Curve B: same as A, but for mixing ratio = 1 g/kg. Curve C: mean temperatures, high latitudes (summer). Curve D: mean temperatures, high latitudes (winter). Curve E: mean temperatures, low latitudes (summer and winter).

considerations. Attempts in this direction have been made by BATES & NICOLET (1950), HORIUCHI (1961) and WALLACE (1962). They have defined a model atmosphere, without diffusion, where the concentrations of the various compounds are determined by a specified set of photochemical reactions. For the sake of simplicity, photochemical equilibrium had to be assumed for the oxygen–hydrogen atmosphere. The concentrations of water vapor obtained for this crude model atmosphere are such that saturation would never occur at the mesopause. However, with the simplifications used, one can hardly expect more than a rough picture of the composition of the atmosphere at the levels of interest to us, in particular with regard to the hydrogen compounds. The life-time of water vapor has been estimated at 1–5 weeks. This means that turbulent diffusion from below may increase the water vapor content at the mesopause considerably. Furthermore, as the water vapor content will increase under night-time conditions, photochemical equilibrium will possibly never occur. Finally, the total amount of hydrogen may undergo time variations by orders of magnitude due to variations in the proton influx. It is also possible that very high concentrations of water vapor may be confined to specific regions, as for instance the auroral zones, where noctilucent clouds are most frequently observed. Thus, no conclusions can so far be drawn regarding the concentrations of water vapor near the mesopause.

In contrast to the humidity, the temperature and its variation with height, season and latitude is fairly well known up to about 90 km. Mean temperature curves are shown in Fig. 2 for both high and low latitudes (winter and summer) (NORDBERG & STROUD, 1961). It is seen that a temperature of 172°K, which, under suitable humidity conditions, was found to provide possibilities for ice cloud formation at about 80 km, should occur relatively frequently in high latitudes during the summer. On individual occasions, temperatures as low as 160°K have been observed. The winter temperatures are considerably higher, and ice clouds can never form at the heights in question. Furthermore, the possibility of ice cloud formation decreases with decreasing latitude.

4. Wave cloud model

Let us assume an atmosphere where the conditions some kilometers above and below the mesopause are defined as follows:

(1) Temperature at the mesopause: 172°K

(2) $dT/dz = -3.5$°K/km below the mesopause
$dT/dz = 3.5$°K/km above the mesopause

(3) constant mixing ratio. Frost point at the mesopause 172°K.

We shall further assume a wave motion in accordance with the data (13a). For the sake of simplicity we shall assume that we have no wind shear, and that the trajectories are parallel at all points in the same vertical. The growth of the particles at the mesopause may then be determined from (11) and (12). For particles

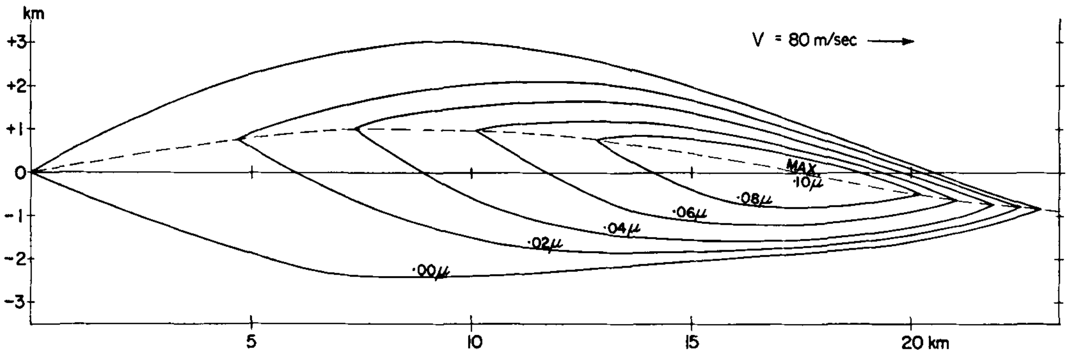

FIG. 3. Cross-section through a model of a wave cloud at the mesopause. Solid lines indicate the distribution of thickness of the ice coating $(r - r_n)$ in microns. Dotted line indicates the particle motion. (Wavelength 35 km, amplitude 1 km, wind speed 80 m/sec.)

above or below the mesopause, (11) must be replaced by a modified equation where the sublimation starts when saturation is reached due to adiabatic cooling of the air.

The results of the calculations are shown in Fig. 3. For reasons already mentioned, absolute particle sizes can not be found. Fig. 3 only gives the value of $r - r_n$, the difference between the radii of the cloud particles and the sublimation nuclei, in relative units. (In order to give an impression of the thickness of the ice coating at the various parts of the cloud, $0.10\ \mu$ is taken as the maximum value for this thickness.) As might be expected, the largest particles are found at the mesopause at a distance $\lambda/2$, where $z = 0$. It may further be concluded that if noctilucent clouds are ice clouds, which behave according to the model, the lower boundary of the clouds, as determined from photogrammetry, cannot be interpreted as a trajectory.

Similar cloud models could be obtained by varying the following conditions. Qualitatively, the effects would be as follows:

(1) the use of a curved temperature profile instead of the two-layer model would tend to increase the total thickness of the cloud,

(2) the use of greater wave amplitudes than 1 km would also tend to increase the total thickness of the cloud,

(3) a damping of the waves on both sides of the mesopause would tend to decrease the total thickness of the cloud whereas the effect of a tilt would probably be very small,

(4) an initial supersaturation at the mesopause would have a tendency to stretch out the cloud between wave-peaks and make the whole cloud more or less continuous.

5. Stratiform cloud model

We shall next consider ice particles growing at a constant temperature $T < T_0$. In this case we have

$$\frac{dr}{dt} = \frac{\alpha}{\varrho_i \sqrt{2\pi R_v T}}\,[e - e_i(r)] = \frac{\alpha e_0}{\varrho_i \sqrt{2\pi R_v T}} \cdot \left\{1 - \frac{4\pi N \varrho_i R_v T}{3 e_0} - \exp\left[\frac{2\sigma_{13}}{\varrho_i R_v T r} - \frac{L_s}{R_v}\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]\right\}, \tag{16}$$

which, with the same simplifications as above, reduces to

$$\frac{dr}{dt} = \frac{\alpha e_0}{\varrho_i \sqrt{2\pi R_0 T}} \cdot \left\{1 - \exp\left[\frac{2\sigma_{13}}{\varrho_i R_v T_0 \bar{r}} - \frac{L_s}{R_v}\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]\right\}. \tag{17}$$

The results of the integration of (17) are shown in Fig. 4 for $T_0 - T = 1, 2, \ldots, 5$ degrees. It is seen that the particles will grow to the assumed full size $(r = r_n + 0.10\ \mu)$ in about 5 min or less. Since noctilucent clouds in general are observed to persist for a much longer time, the acceptability of the ice particle hypothesis requires the existence of a process which effectively prevents the particles from growing to unreasonable sizes. Since the concentration of nuclei is most likely too small to reduce the sublimation rate significantly, only two processes are, *a priori*, likely to be sufficiently effective: turbulent diffusion and precipitation.

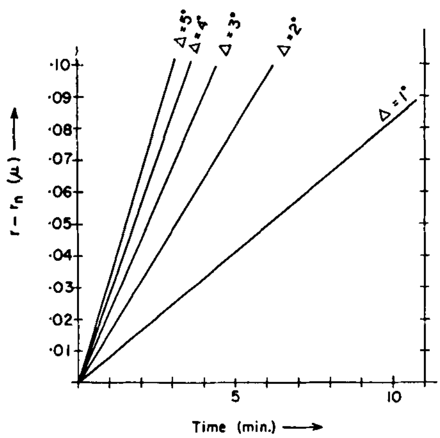

FIG. 4. Growth of ice particles for various degrees of supersaturation ($\Delta = T_0 - T = 1, 2, \ldots, 5°K$).

These two possibilities will now be discussed in more detail.

The problem of transportation of cloud particles from the interior of a cloud layer to the cloud-free air above or below has been studied by MASON (1960). If the relative fall speed of the particles is neglected we may, following Mason, calculate the time t^* it will take to remove for instance 50% of the particles from the central part of a cloud layer of thickness $2z_0$ into the cloudfree air. The relation is

$$\frac{z_0}{\sqrt{4Dt^*}} = 0.81, \tag{18}$$

where D = turbulent diffusion coefficient. For a cloud layer of thickness 2 km ($z_0 = 1$ km) a calculation of t^* for different values of D gives

$t^* \approx 11$ hours for $D = 10^5$ cm²/sec

$t^* \approx 1$ hour for $D = 10^6$ cm²/sec

$t^* \approx 6$ min for $D = 10^7$ cm²/sec

Thus, if we assume $D \approx 10^7$ cm²/sec, 50% of the cloud particles will be removed from the cloud before they grow to unreasonable sizes. (It can further be shown that after 12 min only 19% of the particles are still in the cloud, and after 18 min only 7% of them are left.)

During the latest years several attempts have been made to determine the diffusion coefficients from the rate of widening of meteor trails and from the development of artificial clouds at high altitudes. Assuming isotropic turbulence, values ranging from about 3×10^6 to about 10^8 cm²/sec have been deduced for a height of 82 km (see for instance ZIMMERMAN & CHAMPION, 1961).

For the terminal velocity v_t of small spheres falling in a gas at low pressure we shall apply Stokes' law with Millikan's correction (see for instance KENNARD, 1938). If the mean free path L is much larger than the particle size the formula may be written

$$v_t = \frac{2\varrho g r L \cdot 1.02}{9\eta} \tag{19}$$

where ϱ = density of the particle and η = dynamic viscosity. η may be determined from the formula (SMITHSONIAN, 1951)

$$\eta = \eta_0 \left(\frac{393}{T+120}\right)\left(\frac{T}{273}\right)^{3/2}. \tag{20}$$

Taking $\varrho = 1.2$ g/cm³, $g = 958$ cm/sec², $r = 0.13\ \mu$, $T = 172°K$, $L = 0.573$ cm (according to the ARDC model atmosphere) and $\eta_0 = 1.718$ g/cm sec, we obtain

$$v_t = 17 \text{ cm/sec}. \tag{21}$$

Although the selection of the values of the parameters in (19) involve considerable uncertainties, the fall velocities of ice particles at about 82 km are probably too small to cause an effective removal of cloud particles within approximately 10 minutes, which was estimated as the maximum time the ice particles are allowed to remain in the cloud. The effect of sedimentation upon the size distribution of the cloud particles is therefore believed to be rather small.

Concluding remarks

Our present knowledge of the physical conditions in the vicinity of the mesopause does not permit us to draw definite conclusions as to the nature of noctilucent clouds. The results obtained above show, however, that the possibility of ice cloud formation exists, and that many of the observations may be explained on the basis of cloud models as described above.

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