Statistical studies of the performance of cosmic ray recording instruments

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ABSTRACT

A method is presented to determine the variance of statistical fluctuations in cosmic ray data derived from the neutron monitor, the duplex cubical counter telescope and the directional telescopes. The standard deviation of the error distribution from instruments at the cosmic ray stations at Uppsala, Murchison Bay and Mt. Wellington is estimated by the method. The results are compared with the Poisson distribution and it is stressed that the use of the Poisson distribution will give an underestimate. The results offer standard error calculations of cosmic ray data.

LEGEND OF STATISTICAL SYMBOLS

Population characteristics:
- $E(x)$ = the mean of the variable x;
- $D(x)$ = the standard deviation of the variable x;
- $D^2(x)$ = the variance of the variable x;
- $C(xy)$ = the covariance of the variables x and y.

Calculated sample characteristics:
- $\bar{x}$ = the mean of the variable x;
- $s(x)$ = the standard deviation of the variable x;
- $s^2(x)$ = the variance of the variable x.

Introduction

In dealing with investigations where counting tubes are used, it is usually assumed that each observation is drawn from a Poisson distribution. It is well known that a series of random events will give the probability that n events will occur in the interval t (CURRAN and CRAGGS, 1949):

$$p(x=n)=\frac{(\lambda t)^n e^{-\lambda t}}{n!},$$

where λ is the average number of events per second. As long as the events will occur at random, which holds for most radioactive processes and for cosmic radiation, the Poisson distribution will be theoretically exact. This problem has been discussed by several authors (BATEMAN, 1910; FRY, 1928). For the counting rate N the Poisson distribution gives

$$D^2(N) = N. \qquad (1)$$

The discrete Poisson distribution will rapidly approximate the continuous normal distribution with an increasing n. The Poisson distribution is then of basic importance in error calculations of data from instruments equipped with counting tubes. One must be aware that this estimate will cover only the true random variations in the arrival of particles. No other types of errors are involved.

Statistical treatment

Calculating the variance of a variable which is a function of time will of course give a result including the statistical as well as the time fluctuations. MCCRACKEN (1958) has chosen a very calm cosmic ray period to estimate the statistical fluctuations for a neutron monitor. However, the existence of a daily and a 27-day variation may cause an over-estimation. Thus this method is not suitable for our purpose.

The IGY recommendations to divide the cosmic ray instruments into equal sections to get unbroken continuous registrations, offer the possibility of estimating the statistical fluctuations. Each section has identical sets of electronics and will be working to a great extent as independent instruments measuring the same phenomena. Only the power line is common.

For statistical treatment it is convenient to write the cosmic ray registration:

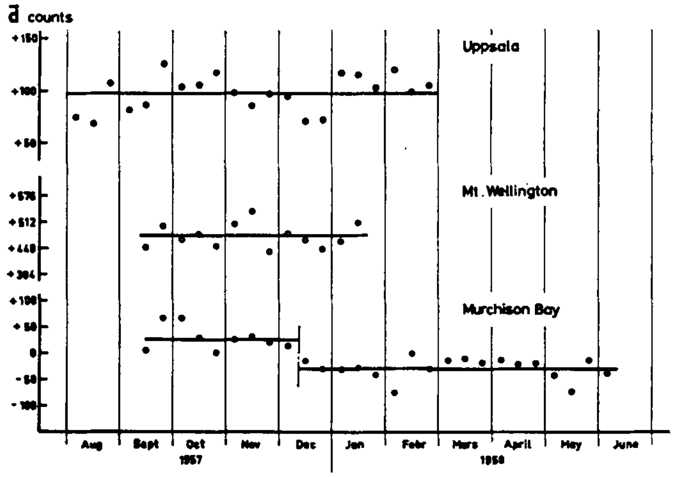

Fig. 1. 10-day means of d from the neutron monitors of Uppsala, Mt. Wellington and Murchison Bay for the periods used in the investigation.

$$N = E(I) + f_1(P,T) + f_2(\Delta I) + \varepsilon. \qquad (2)$$

I = the registered intensity by the instrument at constant atmospheric parameters. It must be observed that not all recorded particles are independent of each others due to multiplicity (McCracken, 1958).

$E(I)$ = the true mean of the intensity I during the period of observation.

$f_1(P,T)$ = the atmospheric influence of I where the pressure P and the temperature T are functions of time. In the following treatment it is sufficient to use the linear form of Duperier's formula (1949) to correct for atmospheric effects:

$$\frac{\Delta N}{N} = \alpha\,\Delta P + \beta\,\Delta T + \gamma\,\Delta H. \qquad (3)$$

ΔN is the difference between the observed and the corrected value; ΔP, ΔT and ΔH are the differences between the mean over the period and the observed value of pressure, temperature and height to a certain pressure level; α, β and γ are constants.

$f_2(\Delta I)$ = the time variation of the particle flux for constant atmospheric parameters. Our knowledge of the time variations is limited as yet.

ε = random variable due to statistical fluctuations. It includes all type of errors in the registration. Assumed approximately normal distributed:

$$E(\varepsilon) = 0$$

ΔP, ΔT, ΔH and ΔI are time variables where ΔH is a function of ΔP and ΔT, ΔP and ΔI are supposed independent. They are not true random variables. However, for longer periods they tend to form symmetrical distributions. In the following their covariances are assumed negligible. This will not be too gross an assumption especially when using the method mentioned below, which will reduce the influence of the existing covariance terms.

Introducing:

$$k = \frac{E(I_x)}{E(I_y)}, \qquad (4)$$

the efficiency ratio between the two sections. z = the number of counts recorded in one section that due to multiplicity in the instrument will be detected simultaneously in the adjacent section, where z is a function of I.

Thus the registered value from a section of a cosmic ray instrument at a certain time can be written (2), (3) and (4):

$$x_1 = kI_{y1} + z_1 + (kI_{y1} + z_1)(\alpha\,\Delta P_1 + \beta\,\Delta T_1 + \gamma\,\Delta H_1) + \varepsilon_x$$

$$y_1 = I_{y1} + z_1 + (I_{y1} + z_1)(\alpha\,\Delta P_1 + \beta\,\Delta T_1 + \gamma\,\Delta H_1) + \varepsilon_y$$

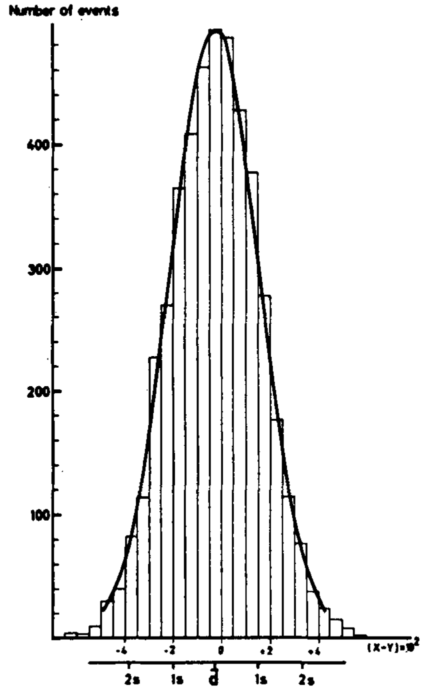

Fig. 2. The distribution of d from the neutron monitor of Murchison Bay 11.12.1957–20.6.1958 with normal distribution fitted.

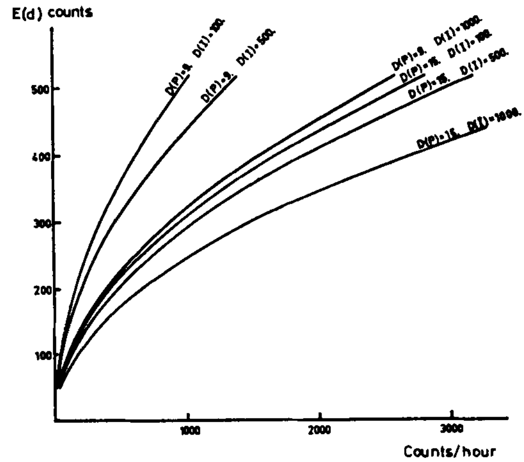

Fig. 3. The overestimation of $D^2(\varepsilon_x) + D^2(\varepsilon_y)$ by using (10) as functions of $E(d)$, $D(P)$ and $D(I)$.

Forming the means of a time series, where $\sum(\Delta P)$, $\sum(\Delta T)$ and $\sum(\Delta H) = 0$, give

$$E(x) = kE(I_y) + E(z). \tag{5}$$

$$E(y) = E(I_y) + E(z). \tag{6}$$

and the variances:

$$D^2(x) = k^2D^2(I_y) + D^2(z) + A[k^2E^2(I_y) + E^2(z) + k^2D^2(I_y) + D^2(z)] + 2kC(I_yz)[1 + A] + 2kD^2(P)E(I_y)E(z) + D^2(\varepsilon_x). \tag{7}$$

$$D^2(y) = D^2(I_y) + D^2(z) + A[E^2(I_y) + E^2(z) + D^2(I_y) + D^2(z)] + 2C(I_yz)[1 + A] + 2D^2(P)E(I_y)E(z) + D^2(\varepsilon_y). \tag{8}$$

$$C(xy) = kD^2(I_y) + D^2(z) + A[kE^2(I_y) + E^2(z) + kD^2(I_y) + D^2(z)] + (k + 1)[C(I_yz)\{1 + A\} + D^2(P)E(I_y)E(z)], \tag{9}$$

where $A = \alpha^2D^2(P) + \beta^2D^2(T) + \gamma^2D^2(H) + 2\alpha\gamma C(PH)$.

The difference $(x - y) = d$ between the two sections gives from (5) and (6)

$$E(d) = (k - 1)E(I_y)$$

and from (7), (8) and (9)

$$D^2(d) = (k - 1)^2[D^2(I_y) + \{E^2(I_y) + D^2(I_y)\}A] + D^2(\varepsilon_x) + D^2(\varepsilon_y), \tag{10}$$

where $(k - 1)^2 = \dfrac{[E(I_x) - E(I_y)]^2}{E^2(I_y)} = \dfrac{E^2(d)}{E^2(I_y)}$.

The ratio between two sections is

$$\frac{x_1}{y_1} = \frac{kI_{y1} + z_1 + (kI_{y1} + z_1)(\alpha\Delta P_1 + \beta\Delta T_1 + \gamma\Delta H_1) + \varepsilon_x}{I_{y_1} + z_1 + (I_{y_1} + z_1)(\alpha\Delta P_1 + \beta\Delta T_1 + \gamma\Delta H_1) + \varepsilon_y}.$$

and the variance (KENDALL, 1948):

$$D^2\left(\frac{x}{y}\right) = \frac{1}{E^2(y)}D^2(x) + \frac{E^2(x)}{E^4(y)}D^2(y) - 2\frac{E(x)}{E(y)E^2(y)}[E(xy) - E(x)E(y)],$$

Assuming

$$\left.\begin{aligned} & z \approx D(z) \approx 0, \\ & D^2\left(\frac{x}{y}\right) = \frac{1}{E^2(I_y)}[D^2(\varepsilon_x) + k^2D^2(\varepsilon_y)]. \end{aligned}\right\} \tag{11}$$

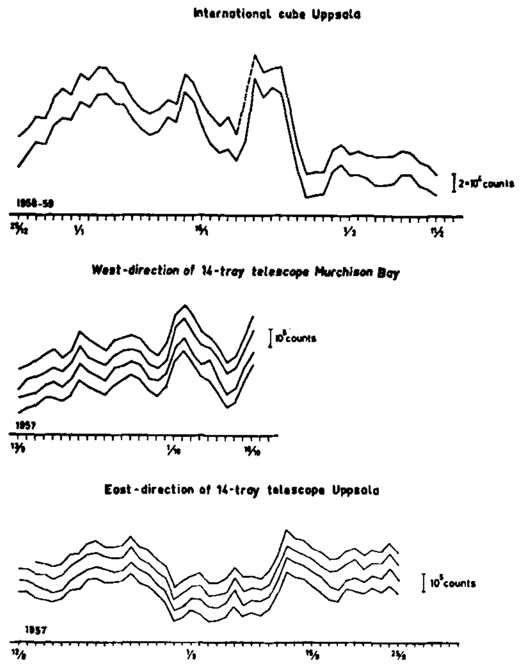

Fig. 4. Examples of the daily mean intensity changes from the sections of different counter telescopes during some periods used in the investigations.

The distribution of ε_x and ε_y describe the statistical fluctuations. (10) and (11) offer ways to estimate the sum of the error variances from two sections of an instrument by using time series data. Under certain conditions these estimates are to a great extent independent of the time variations.

Often the registered cosmic ray intensity has to be corrected for atmospheric pressure before being used in time series analysis. (3) is a simple formula for this correction. It can be written

$$N_{c1} = N_{r1}[1 - \alpha(P_1 - P_0)],$$

where N_c is the corrected and N_r the registered value; P_1 is the observed value and P_0 the mean during the period of observations.

Each observed value of the parameters can be written in the form

$$A_r = E(A) + \varepsilon_A,$$

and the error variance of the corrected cosmic ray intensity is

$$D^2(\varepsilon_{Nc}) = D^2(\varepsilon_{Nr}) + D^2(N_r, \Delta P, \alpha) - 2D^2(\varepsilon_{Nr})E(\alpha)E(\Delta P). \quad (12)$$

The variance of a triple product abc is

$$D^2(abc) = E^2(bc)D^2(a) + E^2(ac)D^2(b) + E^2(ab)D^2(c) + D^2(a)D^2(b)D^2(c).$$

All variables are assumed independent since they have only statistical fluctuations of the form ε_A.

Errors introduced by the method of registration

Cosmic ray data are often scaled and rounded off to make the following data process as simple as possible. The instruments discussed here all have a system of continuous registration with no zero resetting. Thus no systematic errors are introduced by the scaling. No rounding off is used. The corrections which must be applied to the variance of the scaled distribution to get the true distribution is rather complicated. The shape of the distribution, the scaling factor, the situations of the group intervals etc. are important. This problem will not be discussed here, all data used in this paper are being scaled with factors giving small corrections. The time of the registration process can also be assumed to be neglected. Photographic method and mechanical printing counters are employed in the registration. The mechanical counters have a printing time of a magnitude of 0.8 sec which will not introduce any important error.

Error variances of neutron monitor data

In order to get an estimation of the standard deviation of the statistical fluctuations for an IGY neutron monitor (Simpson *et al.*, 1953) data from three different stations, Uppsala, Murchison Bay and Mt. Wellington have been used (Table 1). The stations are all equipped with neutron monitors built after the IGY recommendations in two sections, each served by a complete set of electronics. Great care has been taken to use such periods for the calculation, when the two sections of the instruments have been working with the counting rate constant. Diagram 1 shows the mean of d for 10-day periods at the three stations. The differences d

TABLE 1.

Station	Lat.	Long.	Altitude	Instrument	Mean counting rate per hr	Number of sections or channels	Scaling factor
Uppsala	59°51′ N	18°15′ E	Sea level	IGY neutron monitor	25,000	2	1
				International cube	93,000	2	100
				Directional telescope	29,000	8	10
Murchison Bay	80°03′ N	17°55′ E	Sea level	IGY neutron monitor	26,000	2	1
				International cube	95,000	2	100
				Directional telescope	30,000	8	10
Mt. Wellington	42°55′ S	147°14′ E	725 m	IGY neutron monitor	36,000	2	64

TABLE 2.

Station	Period	$\bar{d}$	n	$s_a = s(d)$ acc. to (10)	$s(s_a)$	$s_p = s(d)$ acc. to (1)	$s(s_p)$	$\frac{s_a}{s_p}$	$s\left(\frac{s_a}{s_p}\right)$
Murchison Bay	11.9.57–10.12.58	+29.35	2175	190.27	2.88	161.45	2.45	1.179	0.025
Murchison Bay	11.12.57–10.6.58	−29.75	4305	182.20	1.96	155.94	1.68	1.168	0.018
Uppsala	1.8.57–30.2.58	+94.47	4907	188.20	1.90	157.76	1.59	1.193	0.017
Mt. Wellington	11.9.57–20.1.58	+479.71	3018	239.98	3.06	190.14	2.45	1.262	0.026

between the two sections are calculated for one-hour values uncorrected for atmospheric effects. The following formulas are being used:

$$\bar{d} = \frac{1}{n}\sum d,$$

$$E(\bar{d}) = E(d),$$

$$s^2(d) = \frac{1}{n}\sum (d - \bar{d})^2,$$

$$E(s^2) = \frac{n-1}{n} D^2(d).$$

For large values of n is s^2 a good estimate of D^2. This is fulfilled in this paper. The standard errors of the characteristics can be obtained from

$$D^2(\bar{d}) = \frac{D^2(d)}{n},$$

$$D^2(s^2) = \frac{2(n-1)}{n} D^4(d).$$

These equations are valid when the variable is drawn from a normal distribution. Diagram 2 shows the distribution of d for one period from Murchison Bay with the normal distribution fitted. As seen the distribution can be assumed to be approximately normal.
(10) shows

$$D^2(d) > D^2(\varepsilon_x) + D^2(\varepsilon_y) \quad \text{for } E(d) \neq 0.$$

In diagram 3 $E(d)$ is drawn as a function of the over-estimation when the following assumptions are made

$\beta \approx \gamma \approx 0$
$E(I_y)$ = 12500 counts/hr
α = −0.73 per cent/mb
$D(I_y)$ = 100, 500, 1000 counts/hr
$D(P)$ = 9, 12, 15 mb

For small values of $E(d)$ the overestimate is negligible. Accepting an overestimate of 1 per cent (for Murchison Bay and Uppsala ~400 counts/hr) employs

$E(d) \leqslant 200$ counts/hr.

TABLE 3.

Station	Period	$\bar{k}$	n	$s_a = s\left(\frac{x}{y}\right)$ acc. to (11)	$s(s_a)$	$s_p = s\left(\frac{x}{y}\right)$ acc. to (1)	$s(s_p)$	$\frac{s_a}{s_p}$	$s\left(\frac{s_a}{s_p}\right)$
Mt. Wellington	11.9.57–20.1.58	0.9739	3013	0.012836	0.000165	0.010242	0.000132	1.253	0.023

TABLE 4.

Estimate of standard deviation	Experimental value from a two-counter monitor (McCracken)	Estimated value IGY monitor (12 counters) (McCracken)	Mean of Murchison Bay and Uppsala values obtained from differences	Mt. Wellington obtained from the ratios
$D(\varepsilon_N)$	1.13 $N^{\frac{1}{2}}$	$\sim$1.2 $N^{\frac{1}{2}}$	(1.18 $\pm$ 0.01) $N^{\frac{1}{2}}$	(1.25 $\pm$ 0.02) $N^{\frac{1}{2}}$

Table 2 shows the results. They are in poor agreement with the theoretical value given by the Poisson distribution. According to McCRACKEN (1958) this is due to multiplicity of particle production in the atmosphere and to star production in the monitor as well. He has shown that the multiplicity gives a bigger standard deviation than expected from the Poisson distribution. This result is compared with the ones obtained in this paper (Table 4). The Mt. Wellington value is not directly comparable since d is too large. z being only a small fraction of the counting rate (McCRACKEN, 1958) the ratio between the two sections of the Mt. Wellington neutron monitor is calculated. The error variance is obtained from (11) and shown in Table 3.

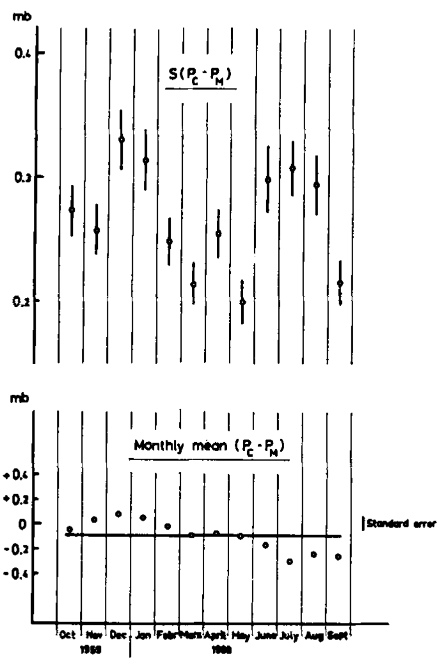

Fig. 5. The variations of the estimated monthly mean and standard deviation of $(P_q - P_m)$ Oct. 1959 to Sept. 1960.

The results from the neutron monitors of Murchison Bay and Uppsala are in good agreement. The instruments have also similar appearance and electronics. The value from Mt. Wellington is larger. One must bear in mind that the data from Mt. Wellington are scaled with a factor 64 while the data from Uppsala and Murchison Bay are unscaled. However, the correction which must be applied to the scaled variance to get the true variance will be comparatively small. Thus the results indicate that $D(\varepsilon_N)$ is an instrumental constant to some extent. The estimation of the error variance will include all statistical errors, the Poisson errors and those introduced by the method of registration and instrumentation. Different neutron monitors might differ in appearance and electronics which may cause a variability of $D(\varepsilon_N)$ from one station to another. Of course the actual condition of the instrument is an important factor. $D(\varepsilon_N)$ may be a function of time. The results obtained from Uppsala, Murchison Bay and Mt. Wellington show that this variability is comparatively small. They are also in good agreement with the result obtained by McCRACKEN.

Error variances of counter telescope data

The variance of the statistical fluctuations for the duplex cubical counter telescope and the directional telescopes are also calculated by means of (10). 2-hr values uncorrected for atmospheric effects from Uppsala and Murchison Bay (Table 1) are used. Further description of the instruments is found elsewhere (SANDSTRÖM *et al.*, 1960). As in the calculation of the neutron monitor data great care is taken to choose periods of observations where no changes in the counting rate occur. It is sometimes difficult to find such reliable periods for counter telescopes due to their construction.

Fig. 4 shows an example of the daily mean intensity of the sections from one international cube and one directional telescope during calculation periods.

To find the maximum value of $E(d)$ for an acceptable over-estimation by using (10) we make the following rough assumptions for the international cube:

$\alpha = -0.15$ per cent/mb		
$\beta = +0.1$ per cent/°C		(200 mb level)
$\gamma = -6$ per cent/km		(200 mb level)
$D(P) = 10$ mb		
$D(T) = 10$°C		(200 mb level)
$D(H) = 0.5$ km		(200 mb level)
Correlation coeff. $(PH) = 0.5$		
$E(I_y) = 10^5$ counts/2 hr		
$D(I_y) = 5000$ counts/2 hr		

For a maximal over-estimation of about 1 per cent

$E(d) \leqslant 900$ counts/2 hr.

The results from the international cube are shown in Table 5. The mean estimates of $D(\varepsilon_N)$ covering all periods for each station give

Uppsala $D(\varepsilon_N) = (1.12 \pm 0.01)N^{\frac{1}{2}}$,
Murchison Bay $D(\varepsilon_N) = (1.28 \pm 0.02)N^{\frac{1}{2}}$.

The calculations of the standard errors are based upon the uncertainty in the values from which the means are obtained.

The directional telescopes have eight channels, four odd-numbered pointing east and four even-numbered pointing west. Making the same assumptions as for the international cube apart from

$D(I_y) = 3 \times 10^4$ counts/2hr
$D(I_y) = 1500$ counts/2 hr

employ an acceptable over-estimation of 1 per cent for

$E(d) \leqslant 500$ counts/2 hr.

The four channels of each direction give six sets of differences. Only two are independent. Thus there are six equations and four variables ε_x, ε_y, ε_z and ε_v. By the least square method the variables are estimated. The results are shown in Tables 6 and 7. The mean estimates of $D(\varepsilon_N)$ covering all periods for each station give

Uppsala $D(\varepsilon_N) = (1.10 \pm 0.03)N^{\frac{1}{2}}$,
Murchison Bay $D(\varepsilon_N) = (1.17 \pm 0.03)N^{\frac{1}{2}}$.

The influence of the multiplicity (MCCRACKEN, 1958) is smaller for a counter telescope than for a neutron monitor. The resolving time is not sufficient for separating particles derived from the same primary. The local production of mesons is also small. Thus we can expect a closer agreement to the Poisson distribution for a counter telescope than for a neutron monitor. Still the results do not indicate this and we further find a difference in the variance of ε_N from Uppsala and Murchison Bay and between different channels of the same instrument and

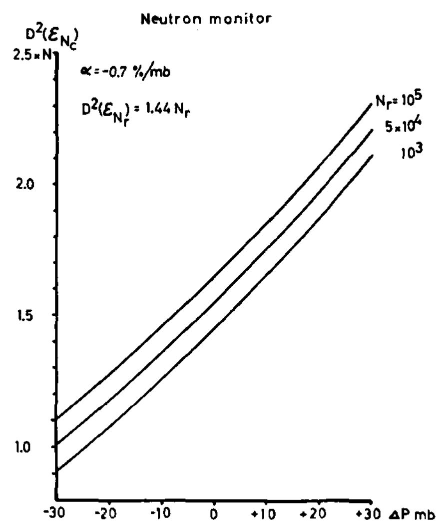

Fig. 6. Error variances of neutron monitor pressure corrected data as functions of counting rate and atmospheric pressure. Error variance of the atmospheric pressure is estimated to 0.04 mb².

TABLE 5.

Station	Period	d	n	$s_a = s(d)$ acc. to (10)	$s(s_a)$	$s_p = s(d)$ acc. to (1)	$s(s_p)$	$\frac{s_a}{s_p}$	$s\left(\frac{s_a}{s_p}\right)$
Uppsala	25.12.58–11.2.59	−133	562	527.2	15.7	433.8	12.9	1.218	0.051
	26.6–26.8.59	−726	747	458.2	11.8	428.2	11.1	1.069	0.036
	9.9–4.10.59	−623	308	464.8	18.7	432.7	17.4	1.074	0.061
Murchison Bay	8.11.57–5.1.58	−726	670	576.8	15.5	445.6	12.2	1.294	0.049
	17.1–2.4.58	−728	835	554.7	13.6	431.4	10.8	1.286	0.044
	6.9–26.9.57	−544	330	538.9	21.0	442.0	17.2	1.219	0.067

TABLE 6.

a	12.8–1.11.1957	*f*	21.8–15.10.1957	*k*	12.5–14.6.1958
b	5.11–9.12.1957	*g*	25.12–25.1.1957–58	*l*	13.9–10.10.1957
c	25.12–9.2.1957–58	*h*	31.7–1.9.1958	*m*	4.1–28.2.1958
d	2.3–2.4.1958	*i*	14.9–10.10.1957	*n*	18.4–17.5.1958
e	10.6–27.9.1958	*j*	29.12–21.2.1957–58		

Station	Channels	Period	d	n	$s_a = s(d)$ acc. to (10)	$s(s_a)$	$s_p = s(d)$ acc. to (1)	$s(s_p)$	$\frac{s_a}{s_p}$	$s\left(\frac{s_a}{s_p}\right)$
Uppsala	3–1	*a*	+460	951	259.2	5.9	240.7	5.5	1.077	0.035
		b	+607	418	265.8	9.2	238.9	8.3	1.113	0.054
		c	+600	532	289.7	8.9	242.9	7.5	1.193	0.052
		d	+581	356	298.5	11.2	241.1	9.0	1.238	0.066
		e	+551	1252	252.2	5.0	239.6	4.8	1.053	0.030
	7–5	*a*	+596	974	291.1	6.6	239.6	5.5	1.215	0.039
		b	+563	418	237.1	8.2	238.5	8.3	0.994	0.050
		c	+555	563	239.0	7.1	241.1	7.2	0.991	0.042
	4–2	*f*	−483	668	246.5	6.7	238.4	6.5	1.034	0.039
		b	−350	419	256.4	8.8	236.8	8.2	1.083	0.052
		g	−514	381	313.8	11.4	241.2	8.6	1.301	0.067
		h	−637	363	249.7	9.2	239.6	8.9	1.042	0.055
	8–6	*f*	+778	669	259.1	7.1	240.0	6.6	1.080	0.042
		b	+481	419	262.1	9.0	238.2	8.3	1.100	0.053
		g	+490	378	255.7	9.3	243.4	8.8	1.051	0.055
		h	−359	391	269.7	9.6	237.6	8.5	1.135	0.057
Murchison Bay	3–1	*i*	−572	311	294.6	11.8	241.0	9.4	1.222	0.066
		j	−299	636	257.2	7.2	245.0	6.8	1.050	0.041
		k	−141	396	271.7	9.7	240.7	8.6	1.129	0.056
	7–5	*i*	+297	324	256.7	10.0	241.8	9.5	1.062	0.059
		j	+253	675	312.3	8.8	245.3	6.8	1.273	0.051
	4–2	*l*	−665	316	290.2	11.5	242.0	9.3	1.199	0.067
		m	−621	637	294.4	8.2	245.5	6.7	1.190	0.047
	8–6	*l*	+160	322	291.3	11.5	244.1	9.4	1.193	0.066
		m	+170	643	280.5	7.8	243.6	6.5	1.151	0.045
		n	+280	354	286.2	10.7	239.2	8.9	1.196	0.063

from period to period. This indicates that the standard deviation of ε_N is an instrumental constant. The counter telescope, which is equipped with GM-counters, univibrators, sharpeners, coincidence circuits and scalers, introduces errors due to resolving time, spurious counts etc, and are comparatively sensitive to changes in the power line and trimming conditions. These types of errors are more prominent on a counter telescope than on the neutron monitor.

The station of Murchison Bay was situated in an area with hard weather conditions and its electric source was diesel-engined generators. Compared to the more quiet conditions of Uppsala it is not surprising to find a larger variance of ε_N for Murchison Bay.

It must be stressed that the periods of calculations are very carefully chosen. As a rule one is forced to accept data with less accuracy to get continuous registrations. Thus it is obvious that the Poisson distribution at counter telescopes gives an underestimation in calculating the standard errors of the data.

Error variances of atmospheric parameters

Correction of cosmic ray data for atmospheric effects can be made by (3). The most important parameter is the atmospheric pressure. The registration of the pressure is made by different methods at cosmic ray stations. At the Uppsala cosmic ray station for example the pressure is measured by a precision aneroid barometer which is photographed simultaneoulsy with the cosmic ray recording. To obtain an estimate of the variance of ε_P the differences between the recordings from the cosmic ray station and the Meteorological Institute of Uppsala University are calculated. The two stations lie about 3 km apart and at the same altitude. The stations are assumed to register the same atmospheric pressure. The data from the Meteorological Institute are determined from a 24-hr barograph which is corrected by readings of a good mercurial barometer three times a day at 08.10, 14.10 and 20.10 L.T. In the calculation only the values at 08, 14 and 20 L.T. have been used to get the best determined values. The registrations are independent and thus

$$D^2(P_q - P_m) = D^2(\varepsilon_{Pq}) + D^2(\varepsilon_{Pm}),$$

where q indicates the cosmic ray station and m the meteorological station. Fig. 5 shows the monthly mean and variance of $(P_q - P_m)$ for the year Oct. 1959 to Sept. 1960. The mean shows a tendency to have a yearly variation. However, no possible reasons for this will be discussed here. For the whole year:

$$s(\varepsilon_{Pq}) + s(\varepsilon_{Pm}) = 0.300 \pm 0.007 \text{ mb}.$$

The method of recording the atmospheric pressure is supposed to be more accurate at the meteorological station. The estimate is obtained:

$$D(\varepsilon_{Pq}) = 0.2 \text{ mb}.$$

This value calculated for the Uppsala cosmic ray station can probably be used for other stations. As a rule a mean pressure is determined over a period. The standard error of such pressure data will be dependent on how many observations the mean is calculated from, and the time variations of the atmosperic pressure during the period. However, the figure above will mostly be a useful estimate.

Error variances of pressure corrected cosmic ray data

Estimation of the error variances of pressure corrected cosmic ray data can be obtained from (12) by using the results in this paper. Following estimates are assumed:

Neutron monitor $= -0.7$ per cent/mb,

$$D^2(\varepsilon_\alpha) \approx 0,$$
$$D^2(\varepsilon_{Nr}) = 1.44\, N_r.$$

Counter telescope $\alpha = -0.15$ per cent/mb,

$$D^2(\varepsilon_\alpha) \approx 0,$$
$$D^2(\varepsilon_{Nr}) = 1.30\, N_r.$$

Atmospheric pressure $D^2(\varepsilon_p) = 0.04$ mb^2.

Forming corrected data over a period where $(\Delta P) = 0$ give in (12) the estimates:

Neutron monitor: $$D^2(\varepsilon_{Nc}) = 1.44\, N_r + 2 \times 10^{-6} N_r^2. \quad (13)$$

Counter telescopes: $$D^2(\varepsilon_{Nc}) = 1.30\, N_r + 0.1 \times 10^{-6} N_r^2. \quad (14)$$

(13) and (14) are not valid for single observations as $D(\varepsilon_{Nc})$ is dependent on the actual value of ΔP. When calculating standard errors for single observations or for shorter periods out of a bigger material the estimate must be obtained from (12). Fig. 6 shows the error variance of neutron monitor data corrected for pressure as functions of the counting rate and the atmospheric pressure.

Conclusion

(10) and (11) show methods to estimate the variance of the statistical fluctuations of cosmic

TABLE 7.

a	12.8–1.11.1957	*f*	21.8–15.10.1957	*k*	29.12–21.2.1957–58
b	5.11–9.12.1957	*g*	5.11–9.12.1957	*l*	12.5–14.6.1958
c	25.12–9.2.1957–58	*h*	25.12–25.1.1957–58	*m*	13.9–10.10.1957
d	12.3–2.4.1958	*i*	31.7–1.9.1958	*n*	4.1–28.2.1958
e	10.6–27.9.1958	*j*	14.9–10.10.1957	*o*	18.4–17.5.1958

	Uppsala			Murchison Bay		
Channel	Period	$\frac{s_a(\varepsilon)}{s_p(\varepsilon)}$	$s\left[\frac{s_a(\varepsilon)}{s_p(\varepsilon)}\right]$	Period	$\frac{s_a(\varepsilon)}{s_p(\varepsilon)}$	$s\left[\frac{s_a(\varepsilon)}{s_p(\varepsilon)}\right]$
1	*a*	1.16	0.03	*j*	1.10	0.06
	b	1.14	0.05	*k*	1.02	0.04
	c	1.07	0.05	*l*	1.05	0.05
	d	1.06	0.05			
	e	1.12	0.03			
2	*f*	1.05	0.04	*m*	1.10	0.06
	g	1.21	0.06	*n*	1.35	0.05
	h	1.38	0.07	*o*	1.18	0.06
	i	1.10	0.05			
3	*a*	1.04	0.03	*j*	1.33	0.07
	b	1.12	0.05	*k*	1.08	0.04
	c	1.34	0.06	*l*	1.21	0.06
	d	1.39	0.07			
	e	1.00	0.03			
4	*f*	1.07	0.04	*m*	1.38	0.08
	g	0.93	0.05	*n*	1.04	0.04
	h	1.20	0.06			
	i	1.00	0.05			
5	*a*	1.19	0.04	*j*	1.12	0.06
	b	1.03	0.04	*k*	1.16	0.05
	c	1.03	0.04	*l*	1.28	0.06
6	*f*	1.08	0.04	*m*	1.24	0.07
	g	1.08	0.05	*n*	1.11	0.04
	h	1.08	0.05	*o*	1.27	0.07
	i	1.08	0.05			
7	*a*	1.20	0.04	*j*	0.99	0.05
	b	0.98	0.05	*k*	1.37	0.05
	c	0.99	0.04			
	d	1.02	0.05			
	e	1.11	0.03			
8	*f*	1.12	0.04	*m*	1.28	0.07
	g	1.11	0.05	*n*	1.29	0.05
	h	1.10	0.05	*o*	1.12	0.06
	i	1.21	0.06			

ray intensity data by using time series data. The obtained estimation gives the error variance of the mean intensity during the period of observation.

The results from the three different neutron monitors show that the error variances of neutron monitor data uncorrected for atmospheric effects have a small variability in time and between different instruments. However, the error variance is expected to be an instrumental constant. A figure of $D^2(\varepsilon_N) = 1.44\ N$ seems to be a good overall estimate.

The error variances of the counter telescopes at Uppsala and Murchison Bay show a greater variability from time to time and between the different sections and instruments. A study of an increased number of instruments will probably confirm this. The condition of the instrument is important. The investigation shows no significant difference between the duplex cubical telescope and the directional telescopes.

The results clearly state that the use of the Poisson distribution in estimating standard errors of cosmic ray intensity data gives an underestimate. For the neutron monitor this is largely due to multiplicity (McCracken, 1958) but for counter telescopes it is probably caused by instrumental errors.

The standard errors of atmosphericly corrected data have some interesting features. The statistical error introduced by the inaccuracy of the atmospheric pressure recording is relatively small as long as no systematic errors are present, but it can nevertheless not be neglected. $D(\varepsilon_{Nc})$ is found to be a function of both the counting rate and the atmospheric pressure.

Counter telescopes data must often be corrected for additional atmospheric parameters. Different methods can be used for this correction. It is important to know the standard errors of such corrected data. It will not be discussed here but in a following investigation.

It must be pointed out that there is a need for a correct error calculation. The accuracy of the standard error estimate depends on the type of analysis made. Fine structure interpreting needs careful calculations but on the other hand rough estimates can often be satisfactory. The labour of the calculations must be weighted against the gain of the accurate estimate. However, the assumption that data in cosmic ray investigations are drawn from Poisson distributions is often unsatisfactory.

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