The Effect of Meanders on the Kinetic Energy Balance of the Gulf Stream

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Abstract

Calculations have been made, using surface velocity observations, of the horizontal transfer of kinetic energy from meanders to mean flow at two separate localities in the Gulf Stream System. In both cases, it was found that there was a surface flow of kinetic energy from the meanders to the mean flow, exactly opposite to what would be expected if the meanders were frictionally driven. The observations suggest that the meanders derive energy from sources other than the kinetic energy of the mean flow. The role of the meanders in relation to the mean Gulf Stream appears analogous to that of large-scale eddies in relation to the general circulation of the atmosphere.

1. Introduction

Calculations have been made, for two separate regions, of the horizontal surface transfer of momentum by meanders in the Florida Current section of the Gulf Stream System. In both regions, it was found that the meanders transferred momentum against the velocity gradient, exactly opposite to what would be expected if they were frictionally driven. Or, in other words, there was at the surface, a net transfer of kinetic energy from the meanders to the mean flow.

Since no other such observations have been made of the transfer of kinetic energy in ocean currents, it has not been possible to determine the source of kinetic energy of meanders. It has been supposed by some (e.g.: Rossby, 1936, p. 6; Stommel, 1958, p. 107; von Arx, 1954) that the meanders draw their energy from the kinetic energy of the mean flow and represent a mechanism for frictional dissipation of the mean flow. The observations here suggest that the opposite is true: that the mean flow is enhanced by the kinetic energy of meanders, and that the meanders should, therefore, derive their kinetic energy from sources other than the kinetic energy of the mean flow.

2. Observations

The data used to calculate eddy momentum fluxes were obtained from two separate sections across the axis of the Gulf Stream. One section is located off Onslow Bay, North Carolina, near Cape Hatteras, and the other is located in the Straits of Florida, running across the channel between Miami and Gun Cay, Bahamas. Figure 1 shows the location of these sections and their relation to the mean surface

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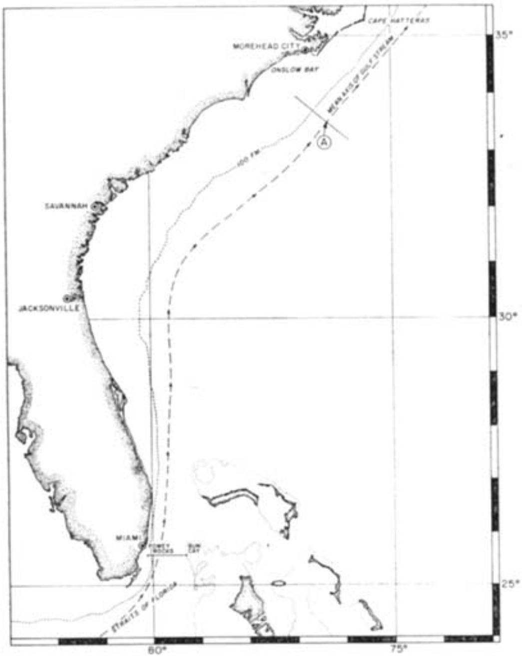

Fig. 1. Florida Current region.

axis of the Gulf Stream, as estimated by the United States Coast and Geodetic Survey.

GEK data. The necessary velocity measurements were made with the geomagnetic electrokinetograph (GEK), first described by VON ARX (1950). These measurements are usually less than the true surface velocity, depending mainly on the depth of moving water in relation to the total depth of water (LONGUET-HIGGINS, STERN, and STOMMEL, 1954). It is general practice to determine the reduction of GEK velocities for any region by calculating the average ratio between surface velocities measured by dead-reckoning methods and surface velocities measured with the GEK. This ratio (called "k") has been determined for the Onslow Bay area from several hundred measurements by VON ARX, BUMPUS, and RICHARDSON (1955), who found for k, 1.46 ± 0.09. In the Straits of Florida it has been calculated by HELA and WAGNER (1954) to be 1.68 ± 0.30. That the Straits of Florida have a somewhat higher k-value than does Onslow Bay is a result of the shallower depth of water in the Straits. All GEK observations presented here have been corrected by the appropriate value of k.

Onslow Bay. The data off Onslow Bay were collected by W. S. von ARX, D. F. BUMPUS, and C. G. DAY on Cruise 18 of the research vessel CRAWFORD during May and June, 1958. The structure of the meanders observed during this period has been described by WEBSTER (1961). The CRAWFORD sailed back and forth along a single line at right angles to the mean axis of the Stream. An observation of the surface water velocity was made each hour by means of the GEK. During the twenty-eight days of the cruise, 620 separate measurements, or fixes, of the surface water velocity were made. Between fixes, the component of surface velocity at right angles to the course of the ship was recorded continuously. Since the ship's track was across the mean axis, this component was equivalent to the downstream surface velocity. For this portion of the Stream, "downstream" means in the direction 040°T.

The time-average downstream surface velocity ($\bar{v}$) profile determined from CRAWFORD Cruise 18 data has a cyclonic shear of approximately 4.4×10^{-5} sec^{-1}, and an anticylonic shear of approximately 3.0×10^{-5} sec^{-1}. The Coriolis parameter at this latitude has a value of 8.0×10^{-5} sec^{-1}. Because the averages of the cross-stream surface velocity ($\bar{u}$) are generally less than their standard deviations, the averages are not significantly non-zero. If there are mean cross-stream motions, they are less than 4 cm/sec.

Florida Straits. Between 1952 and 1958, the Marine Laboratory of the University of Miami was engaged in part of a long-term program to determine the characteristics of the Florida. Current as it flows through the channel between Fowey Rocks, Florida (near Miami), and Gun Cay, Bahamas. The channel at this point is about 43 nautical miles (80 kilometers) wide. A total of 632 GEK observations of velocity in the surface layer from 42 of the cruises made by the Marine Laboratory were available for analysis (University of Miami, 1954, 1955, 1957, 1958).

The region surveyed by the University of Miami was divided into longitudinal zones. Only the observations made between latitudes 25°30′N and 25°59′N were used in calculating the averages for each zone. The zones and the region are shown in Figure 2. The limits of the zones are as follows:

Zone	From	To	Width
1	80°05′.5 W	80°01′.5 W	4′.0
2	01′.4	79°58′.5	3′.0
3	79°58′.4	55′.5	3′.0
4	55′.4	52′.5	3′.0
5	52′.4	49′.5	3′.0
6	49′.4	44′.5	5′.0
7	44′.4	39′.5	5′.0
8	39′.4	34′.5	5′.0
9	34′.4	29′.5	5′.0
10	29′.4	24′.5	5′.0
11	24′.4	19′.5	5′.0

The western boundary of the channel, Fowey Rocks, is at 80°06′.5 W, and the eastern boundary, Gun Cay, is at 79°18′.5 W, so that the zones chosen cover nearly the entire channel, and include all the observations available. In the Straits of Florida region, one minute of longitude is equal to 0.911 nautical miles (1.69 km). Therefore the width of the widest zones is 4.55 nautical miles (8.45 km).

The averages of the northward component of surface velocity (v) and the eastward component (u) were calculated for each zone

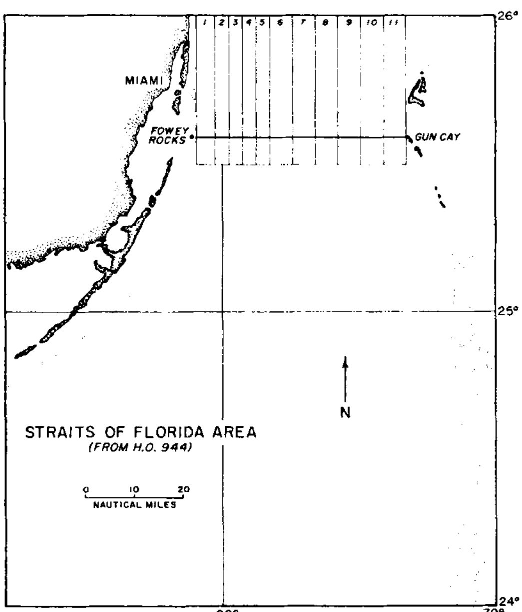

Fig. 2. Straits of Florida area, showing zones used to separate observations.

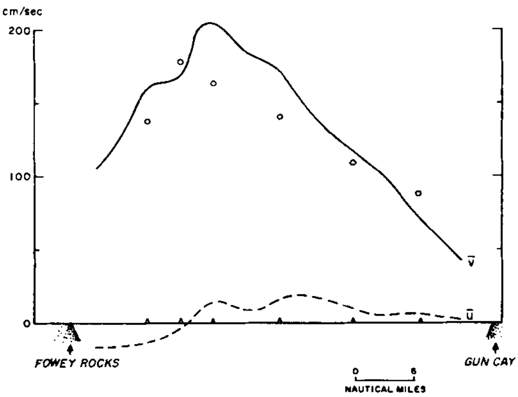

Fig. 3. Average velocity profile, Straits of Florida. Solid line is average downstream (north) velocity, dotted line is average cross-stream (east) velocity, and circles are velocity averages determined by Pillsbury.

using all the GEK observations lying in that zone. These averages are tabulated in Table II, and plotted in Figure 3.

The mean downstream velocity profile is asymmetrical, with a cyclonic shear region about 15 nautical miles (28 km) wide having a shear of approximately 3×10^{-5} sec^{-1}. The anticyclonic shear region is about 32 nautical miles (59 km) wide and has a shear of approximately 2×10^{-5} sec^{-1}. For comparison, the value of the Coriolis parameter at this latitude is 6.9×10^{-5} sec^{-1}. The cyclonic shear of the average stream is significantly less than that of the instantaneous stream, because variations in position of the latter produce a wide distribution of average velocity, and hence a profile more gentle than that which would be found on any particular crossing.

3. Energy Calculations

The transfer of kinetic energy from the eddies to the mean flow can be expressed as

$$\varrho \overline{u'v'} \frac{\partial \bar{v}}{\partial x} \qquad (1)$$

where a bar represents a time average of a quantity, a prime represents a deviation from the time average, u and v are the velocity components in the cross-stream (x) and downstream (y) direction, and ϱ is the density of the water. This method for treating perturbations of a mean flow was first developed by O. Reynolds (1895), and is outlined by Lamb (1932).

Table 1

Zone	$\bar{u}$ cm/sec		$\bar{v}$ cm/sec		$\overline{u'v'}$ cm²/sec²		$\frac{\partial \bar{v}}{\partial x}$ × 10⁻⁵sec⁻¹	$\varrho \overline{u'v'} \frac{\partial \bar{v}}{\partial x}$ 10⁻² ergs/cm³/sec	
Onslow Bay					$k = 1.46$				
6	+ 5.4	± 6.3	39.6	± 7.9	+ 210.2	± 279	2.13	+ 0.45	± 0.6
7	+ 2.6	± 7.2	53.6	± 13.0	— 72.1	± 503	2.68	— 0.19	± 1.3
8	+ 10.4	± 5.5	70.2	± 12.7	+ 236.3	± 341	4.35	+ 1.03	± 1.5
9	+ 6.9	± 6.6	95.9	± 9.1	+ 482.0	± 481	4.35	+ 2.10	± 2.1
10	— 4.8	± 8.0	118.1	± 8.8	+ 640.6	± 558	4.35	+ 2.79	± 2.4
11	+ 8.0	± 9.8	143.5	± 7.4	+ 648.8	± 364	3.84	+ 2.49	± 1.4
12	— 11.2	± 10.1	156.8	± 6.1	+ 335.6	± 409	0.71	+ 0.24	± 0.3
13	+ 2.2	± 6.0	157.2	± 6.4	+ 268.5	± 271	— 0.36	— 0.10	± 0.1
14	— 12.8	± 7.2	153.4	± 7.9	— 74.3	± 349	— 1.10	+ 0.08	± 0.4
15	+ 6.9	± 5.8	139.3	± 8.9	— 169.4	± 279	— 3.00	+ 0.51	± 0.8
16	— 3.8	± 5.4	126.0	± 8.3	— 61.9	± 253	— 2.21	+ 0.14	± 0.6
17	— 11.7	± 6.9	114.9	± 11.1	+ 72.4	± 349	— 2.85	— 0.21	± 1.0

Average Energy Flux: 79 × 10⁻⁴ ergs/cm³/sec.

Onslow Bay. For the Onslow Bay region, the data were divided into twelve zones across the current, each zone being 3 nautical miles wide. The eddy momentum transfer, $\overline{u'v'}$ was calculated by applying Simpson's rule to the value of $u'v'$, over the total time of observation, for each of the twelve zones. The value of $\frac{\partial \bar{v}}{\partial x}$ was calculated for each zone from the profile of average velocity ($\bar{v}$). Table 1 shows for each zone: $\overline{u'v'}$ the average transport of eddy momentum; $\frac{\partial \bar{v}}{\partial x}$ the shear of the average velocity; and the term (1) representing the production of mean kinetic energy by meanders. (ϱ was assumed constant, and equal to one gram per cubic centimeter.)

Figure 4 shows the distribution of (1) across the width of the current off Onslow Bay. The cross-stream integral of (1) is positive, indicating a net transfer of kinetic energy from the meanders to the mean current.

Florida Straits. The averages for the Florida Straits section were calculated by taking ensemble averages in each of the zones. That is, a barred quantity was evaluated as:

$$\overline{(\quad)} = \frac{1}{N} \sum_{i=1}^{N} (\quad)_i$$

where N is the total number of observations in the zone. The results of the calculations are tabulated in Table 2, and the profile of the term (1) across the current is shown in Figure 5. In this region, although there are points in the current where (1) is positive, the net production of mean kinetic energy across the whole stream is not significantly non-zero.

Statistical significance of results. The standard error of the means, $\bar{u}$, $\bar{v}$, and $\overline{u'v'}$ are given for each value in Tables 1 and 2. The standard error of a mean is defined, for large N, as $\sigma/\sqrt{N}$, where σ is the standard deviation of the sample from which the mean is calculated, and N is the number of individual observations.

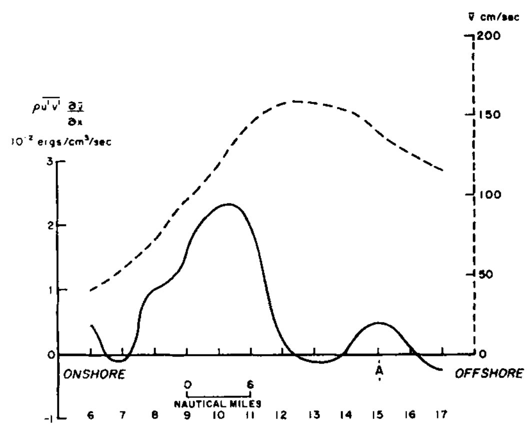

Fig. 4. Onslow Bay. Solid line: production of mean kinetic energy. Dotted line: mean downstream velocity.

Table 2

Florida Straits							$k = 1.68$			
Zone	$\bar{u}$ cm/sec		$\bar{v}$ cm/sec		$\overline{u'v'}$ cm²/sec²		$\frac{\partial \bar{v}}{\partial x}$ $\times 10^{-5}$ sec⁻¹	$\varrho \overline{u'v'} \frac{\partial \bar{v}}{\partial x}$ 10^{-2}ergs/cm³/sec		
1	−18.1	±3.5	103.1	±8.9	−411.7	±184	+4.27	−1.76	±0.78
2	−17.6	±6.4	129.9	±10.4	+259.0	±371	+5.68	+1.47	±2.11
3	−11.2	±5.7	160.7	±12.3	+270.0	±392	+3.60	+0.97	±1.41
4	−6.4	±4.5	164.8	±5.8	+975.4	±277	+4.27	+4.16	±1.18
5	+11.0	±6.7	202.8	±9.2	+357.0	±498	+1.97	+0.70	±0.98
6	+9.9	±4.2	188.0	±4.5	+333.2	±322	−2.35	−0.78	±0.76
7	+16.5	±5.7	167.0	±7.4	+15.2	±361	−3.34	−0.05	±1.21
8	+14.4	±4.2	131.4	±5.5	−13.2	±217	−3.55	+0.05	±0.77
9	+5.1	±4.4	107.2	±6.6	+140.6	±171	−3.43	−0.48	±0.59
10	+5.8	±4.5	73.5	±6.9	+122.7	±123	−3.75	−0.46	±0.46
11	+2.1	±4.4	43.9	±7.4	+281.2	±102	−3.49	−0.98	±0.36

Average Energy Flux: 3.0×10^{-4} ergs/cm³/sec.

4. Discussion of results

Scale of perturbations. The conclusions which can be derived directly from the evaluation of (1) across the Florida Current apply only to the surface layer of moving fluid, and only to the time scale of perturbations which the u' and v' characterize. That is, the results obtained say nothing directly about the energy balance below the surface nor about the eddy motions of other time scales.

PILLSBURY (1891) made a detailed study of the Florida Current at six anchor stations between Fowey Rocks and Gun Cay. His average surface velocities are plotted as circles in Figure 3 where they can be compared with the averages from the University of Miami

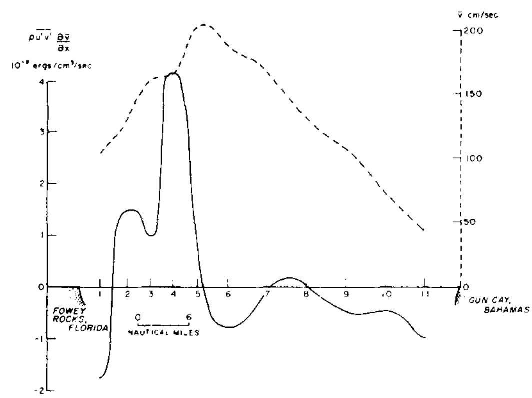

Fig. 5. Florida Straits. Solid line: production of mean kinetic energy. Dotted line: mean downstream velocity.

GEK measurements. His observations show that the subsurface velocities are not greatly less than those at the surface, and suggest a qualitative similarity between surface and subsurface velocity fields. If so, then the energy transfer between eddies and mean flow calculated for the surface layers from GEK data may be representative of most of the current depth in the Florida Straits.

PILLSBURY also made a study of the time variations of current position, velocity, and width in the FloridaStraits, and found monthly variations related to the declination of the moon, as well as daily, tidally-influenced oscillations. He concluded that the tidal components of the variations were significantly larger than those which could be attributed to non-tidal causes.

PARR (1937) analyzed a set of five anchor stations which were successively occupied for 24-hour periods in the Straits of Florida between Fowey Rocks and Gun Cay. He found that lunar periodicities were strongly indicated in the data, and he even combined data from station to station, by referring them to corresponding lunar hours.

More recently, MURRAY (1952) analyzed velocity fluctuations in the same region. These fluctuations, as determined with the GEK, had periods between a few hours and a day. Murray was unable to confirm Pillsbury's conclusion that there was a relationship between the transport and the declination of the moon, and found only inconclusive evidence of lunar effects. A further analysis of additional GEK data by HELA and WAGNER (1954) indicated that some tidal variations in velocity did exist, but that they were strongly masked by non-tidal effects. WERTHEIM (1954) from electromagnetic measurements of total transport, found conclusive evidence of diurnal tidal influence in the transport through the Florida Straits.

In conclusion, although it is probably rash to ascribe the velocity fluctuations of the current through the Straits of Florida predominantly to tidal causes, the periods of the fluctuations are of the order of a day, and the deviations from the mean which are used here to calculate the eddy transport of momentum most likely are representative of meandering motion.

In Onslow Bay, the data were taken continously for a month, in contrast to the Florida Straits, where the data were obtained at intervals over the course of several years. The eddy momentum transfer calculated from the perturbations has periods ranging between a few hours and a week; that is the range of meander periods which were observed (WEBSTER, 1961). Unfortunately, no systematic velocity measurements at depth, similar to those of Pillsbury in the Florida Straits, have ever been made in the Onslow Bay region.

We may conclude that both in the Florida Straits and in Onslow Bay, the calculated lateral surface transfer of momentum is produced by perturbations having periods of a day or longer. No observations have been made of perturbations of other time scales, in particular of small-scale perturbations, having periods which are small compared with the length of a day. Consequently, there is no evidence to suggest that small-scale perturbations would transport momentum in a manner similar to the meanders.

Regeneration Time. The magnitude and the lateral scale of the eddy kinetic energy release off Onslow Bay are similar to those in the Florida Straits. This apparent similarity between the two profiles suggests that there might be a characteristic scale of energy release throughout the whole region between the Straits of Florida and Cape Hatteras. However, a calculation of the time scales of the energy transfer reveals that the kinetic energy plays a different role in the mean kinetic energy balance in each region.

If the curves are integrated over their lengths, the average rate of surface transfer of energy from the meanders to mean flow is 79×10^{-4} ergs/cm^3/sec for Onslow Bay and 3.0×10^{-4} ergs/cm^3/sec for the Florida Straits. If the kinetic energy of the mean surface flow is averaged by integrating across the stream, the result is 7.3×10^3 ergs/cm^3 for Onslow Bay and 8.55×10^3 for the Florida Starits. If no other actions were present, the calculated rate of energy transfer would double the mean surface kinetic energy in 11 days in Onslow Bay and in 329 days in the Florida Straits.

The difference between the regeneration time of the surface kinetic energy in Onslow Bay and that in the Florida Straits may perhaps be explained partly by the confining channel of the latter. Near each shore there is a region where the meanders draw energy from the mean flow. This boundary layer effect is not so noticeable in the Onslow Bay region where the current is not closely confined by physical barriers.

In addition, the current through the Florida Straits may be more directly driven by a downstream pressure gradient. If so, then the meanders would contribute less to the mean flow than they do farther downstream where, possibly, the current is maintained by the cross-stream density field through the mechanism of meanders. At present, the measurements necessary to evaluate the role of downstream pressure gradients are not available.

Atmospheric Similarities. There is a similarity between the role of meanders in the Gulf Stream and the role of large-scale eddies in the atmosphere. Figure 6, adapted from STARR (1953) shows the production of zonal kinetic energy in arbitrary units, and the relative angular velocity as a function of latitude. When Figure 6 is compared with Figures 4 and 5, it can be seen that, for both systems, the maximum countergradient flow of momentum occurs in the region of maximum shear.

Several years' study of atmospheric transfer processes have established that the mean zonal flow is sustained by large-scale disturbances (STARR, 1953), and that the necessary meridional transports of momentum are effected by

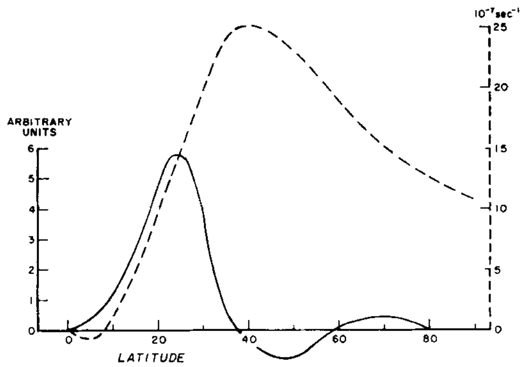

Fig. 6. Solid line: production of zonal kinetic energy, atmosphere. Dotted line: relative angular velocity. (from Starr, 1953, Figure 1.)

horizontal eddy exchange processes (STARR, 1954). The question of whether mean current systems in the ocean are maintained in an analogous manner cannot be answered until further observations are made.

Frictional models of the Gulf Stream (STOMMEL, 1948; MUNK, 1950) require some sort of eddy dissipation which could conceivably be supplied by perturbations of meander scale. A coefficient of lateral eddy viscosity, or Austausch coefficient, may be defined on the assumption that the perturbations of the mean flow are analogous to an eddy frictional mechanism. However, the perturbations of the Gulf Stream flow, observed here, act exactly opposite to frictional effects, so that a viscosity coefficient calculated using them would be negative, and have questionable physical significance. It is possible that the necessary frictional dissipation is carried out by perturbations of a scale smaller than the meanders. If so, then the energy balance is analogous to that in the atmosphere, where the mean zonal flow is sustained by large-scale eddies, but dissipated by small-scale eddies and molecular viscosity.

5. Kinetic Energy Equation for the Mean Flow

The equation expressing the flux of energy in a turbulent flow was originally developed by O. Reynolds for a parallel non-rotating dissipative flow (LAMB, 1932). The corresponding general equation for the atmosphere has been given by KUO (1951). The equations for ocean current systems are similar to those for the atmosphere because the coordinate frame for both systems is rotating, but dissimilar because the atmosphere is cyclic and centered about the axis of rotation, whereas ocean currents are a more local phenomenon. The action of winds upon the surface of the ocean is a mechanism whose atmospheric analogue is negligible.

The following model is a simple version of the Reynolds model, intended to provide an orientation. Better models may follow later.

Let us consider an ocean current, with a mean flow in the y-direction, having velocity components u, v, and w in the x, y, and z directions, x being directed to the right of y, and z being directed downward.

The momentum equations are

$$\varrho \frac{\partial u}{\partial t} + \varrho \mathbf{v} \cdot \nabla u = \varrho f v - \frac{\partial p}{\partial x} - X \qquad (1)$$

$$\varrho \frac{\partial v}{\partial t} + \varrho \mathbf{v} \cdot \nabla v = -\varrho f u - \frac{\partial p}{\partial y} - Y \qquad (2)$$

where ϱ is the density of the water, f is the Coriolis parameter, p is the pressure, and X and Y are components of a dissipative force. $\mathbf{v}$ is the three-dimensional velocity vector, $u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$, and ∇ is the three-dimensional vector operator, $\mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$. We shall omit explicit reference to the wind stress on the surface. With the aid of the continuity equation, $\frac{\partial \varrho}{\partial t} + \nabla \cdot \varrho \mathbf{v} = 0$, we may write (1) and (2) as:

$$\frac{\partial \varrho u}{\partial t} + \nabla \cdot \varrho u \mathbf{v} = \varrho f v - \frac{\partial p}{\partial x} - X \qquad (3)$$

$$\frac{\partial \varrho v}{\partial t} + \nabla \cdot \varrho v \mathbf{v} = -\varrho f u - \frac{\partial p}{\partial y} - Y \qquad (4)$$

The quantities u, v, and w can be separated into mean motion and deviations as:

$$u = \bar{u} + u'$$
$$v = \bar{v} + v'$$
$$w = \bar{w} = w'$$

where

$$\overline{(\ )} = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} (\ )\, dt.$$

The primes denote deviations from the time average. Terms of the form $\overline{u v}$ become $\bar{u}\,\bar{v} + \overline{u'\,v'}$.

Let us now multiply (3) by $\bar{u}$ and (4) by $\bar{v}$, and take time averages:

$$\bar{u}\frac{\partial}{\partial t}\varrho\bar{u} + \bar{u}\nabla\cdot\varrho\bar{u}\,\bar{\mathbf{v}} + \bar{u}\nabla\cdot\varrho\overline{u'\mathbf{v}'} =$$

$$= \varrho f \bar{u}\,\bar{v} - \bar{u}\frac{\partial\bar{p}}{\partial x} - \bar{u}\,\bar{X} \qquad (5)$$

$$\bar{v}\frac{\partial}{\partial t}\varrho\bar{v} + \bar{v}\nabla\cdot\varrho\bar{v}\,\bar{\mathbf{v}} + \bar{v}\nabla\cdot\varrho\overline{v'\mathbf{v}'} =$$

$$= -\varrho f\bar{u}\,\bar{v} - \bar{v}\frac{\partial\bar{p}}{\partial y} - \bar{v}\,\bar{Y} \qquad (6)$$

Equations (5) and (6) may be added, and with the aid of the continuity equation, written:

$$\varrho\frac{\partial}{\partial t}\bar{k} = -\mathbf{v}\cdot\nabla\bar{k} - \bar{u}\nabla\cdot\varrho\overline{u'\mathbf{v}'} - \bar{v}\nabla\cdot\varrho\overline{v'\mathbf{v}'}$$

$$-\bar{u}\frac{\partial p}{\partial x} - \bar{v}\frac{\partial p}{\partial y} - d \qquad (7)$$

where d is the dissipative force, $\bar{u}\,\bar{X} + \bar{v}\,\bar{Y}$, and $\bar{k} = \frac{\bar{u}^2 + \bar{v}^2}{2}$ is the total kinetic energy of the horizontal mean current. By definition, $\frac{\partial}{\partial t}\bar{k} = 0$.

If (7) is integrated over a cross-section of the current in the x-z plane, the result gives the balance of horizontal kinetic energy of the mean flow in a unit cross-section of the current:

$$0 = -\overset{①}{\iint \mathbf{v}\cdot\nabla\bar{k}\,dx\,dz}$$

$$-\overset{②}{\iint \bar{u}\nabla\cdot\varrho\overline{u'\mathbf{v}'}\,dx\,dz}$$

$$-\overset{③}{\iint \bar{v}\nabla\cdot\varrho\overline{v'\mathbf{v}'}\,dx\,dz}$$

$$-\overset{④}{\iint\left(\bar{u}\frac{\partial\bar{p}}{\partial x} + \bar{v}\frac{\partial\bar{p}}{\partial y}\right)dx\,dz}$$

$$-\overset{⑤}{\iint d\,dx\,dz} \qquad (8)$$

With the limited amount of information available now, it is not possible to evaluate each term in equation (8) for the Gulf Stream. However, an estimate of the role of some of the terms can be made.

First, consider terms ② and ③:

$$\bar{u}\nabla\cdot\varrho\overline{u'\mathbf{v}'} + \bar{v}\nabla\cdot\varrho\overline{v'\mathbf{v}'} = \bar{u}\frac{\partial}{\partial x}\varrho\overline{u'u'} + \bar{u}\frac{\partial}{\partial y}\varrho\overline{u'v'}$$

$$+\bar{v}\frac{\partial}{\partial x}\varrho\overline{u'v'} + \bar{v}\frac{\partial}{\partial y}\varrho\overline{v'v'}$$

$$+\bar{u}\frac{\partial}{\partial z}\varrho\overline{u'w'} + \bar{v}\frac{\partial}{\partial z}\varrho\overline{v'w'}$$

The axes were chosen so that the mean current is in the y-direction, so that $\bar{v} \gg \bar{u}$. In addition, the perturbations travel in the y-direction, so that for the time averaged velocity perturbation terms, $\frac{\partial}{\partial y} \approx 0$. Hence, terms ② and ③ become:

$$u\nabla\cdot\varrho\overline{u'\mathbf{v}'} + \bar{v}\nabla\cdot\varrho\overline{v'\mathbf{v}'} \cong \bar{u}\frac{\partial}{\partial x}\varrho\overline{u'u'} + \bar{v}\frac{\partial}{\partial z}\varrho\overline{u'v'}$$

$$+\bar{u}\frac{\partial}{\partial z}\varrho\overline{u'w'} + \bar{v}\frac{\partial}{\partial z}\varrho\overline{v'w'}$$

The terms in $\overline{u'w'}$ and $\overline{v'w'}$ represent contributions of kinetic energy to the mean flow by vertical perturbations. Since observations of vertical motions are not available, this term has not been calculated. It might represent an important contribution to the mean down stream kinetic energy.

The term $\bar{v}\frac{\partial}{\partial x}\varrho\overline{u'v'}$ includes the expression used in the surface calculation of this study. It may be re-written as

$$\overset{(a)}{\frac{\partial}{\partial x}(\varrho\bar{v}\cdot\overline{u'v'})} - \overset{(b)}{\varrho\overline{u'v'}\frac{\partial\bar{v}}{\partial x}}$$

The term $\frac{\partial}{\partial x}(\varrho\bar{v}\cdot\overline{v'u'})$ represents an eddy advection across the boundaries of mean kinetic energy and can be integrated easily across a stream to become:

$$[\varrho\bar{v}\,\overline{v'u'}]_A - [\varrho\bar{v}\,\overline{v'u'}]_B$$

If the stream is bounded by walls, u' will be zero at the walls, or if the mean stream velocity drops to zero at each side of the current, $\bar{v}$ will be zero, and the term (b) is zero. Off Onslow Bay, where the current is not bounded by walls, and the observations did not cover the whole width of the current, calculation of the term (a) has revealed that its integral across the width of observations is zero. Hence the cross-stream integral of the term:

$$\varrho\overline{u'v'}\frac{\partial v}{\partial x} \qquad (1)$$

is a measure of the increase of mean kinetic energy at the expense of kinetic energy of horizontal eddy motion.

Maximum values of $\frac{\partial}{\partial x}\varrho\overline{u'u'}$ at the surface were observed to be almost as large as average values of $\frac{\partial}{\partial x}\varrho\overline{u'v'}$. However, since $v \gg u$, the term $u\frac{\partial}{\partial x}\varrho\overline{u'u'}$ is negligible in comparison with $\bar{v}\frac{\partial}{\partial x}\varrho\overline{u'v'}$.

Term ① represents the advection across the boundaries of the region under consideration of mean horizontal kinetic energy. Equation (8) states that this boundary advection ① must be balanced by the generation of mean kinetic energy by meanders, ② and ③, pressure forces, ④, and by the frictional dissipation, ⑤.

Proper measurements do not exist which will permit a calculation of the contribution of the pressure forces, ④, to the mean kinetic energy balance. Indications are that the downstream pressure gradient is important in the Florida Straits, but little is known about it in Onslow Bay. More reliable measurements are needed in both areas.

The frictional dissipation term, ⑤, was included in the equation to represent the general action of frictional stresses. Included in the term is the effect of wind stress at the surface. An order of magnitude calculation shows that the energy provided by the wind stress is one or two orders of magnitude less than the transfer of kinetic energy from the meanders to the mean flow. That is, if we assume a maximum wind stress of half a dyne/cm^2, a maximum velocity of 50 cm/sec, and a current depth of 500 meters, the energy transfer will be 5×10^{-4} ergs/cm^3/sec, for total correlation between winds and currents. Assuming only partial correlation, or more characteristic values of wind stress and meander velocity, the energy contributed to the meanders by the wind is about 1×10^{-4} ergs/cm^3/sec. In comparison with the transfer of energy between meanders and mean flow of 80×10^{-4} ergs/cm^3 sec, the wind would appear to be eliminated as a significant source of kinetic energy for the meanders.

6. Concluding Remarks

The similarity between these results and those in the atmosphere provides hope that a general study of the energy, momentum, and heat transfers in ocean currents would be as fruitful as such studies in the atmosphere have been. Perhaps, to some extent, meanders are to an ocean current what cyclones are to atmospheric circulation: a mechanism which sustains the mean flow and which provides transfers necessary for climatological balances.

A further program of systematic measurements in the Gulf Stream System is necessary to illuminate the large scale balances. Downstream from Cape Hatteras, especially, the tortuous course of the Gulf Stream has only recently been discovered, and the processes determining its nature are unknown. An extensive observation program might be undertaken initially with GEK surface velocity measurements. Hydrographic station data probably would not be suitable for velocity measurements because non-geostrophic and/or non-baroclinic velocity components might be an important constituent of the transfer process. A system which at present seems promising, and which ultimately might provide the vast amount of data necessary for an oceanic study comparable to that in the atmosphere, is that of anchored buoys, which record data at predetermined time intervals. Such a system could be scattered throughout the ocean, and could record data at any depth.

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