Numerical Tests of a Method for Dynamic Analysis in Regions of Poor Data Coverage[1]

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Abstract

This paper describes a series of numerical experiments which were done to test a theoretical method for dynamic analysis in regions of poor data coverage. Theoretically, if vorticity were conserved, the exact vorticity field in such a region could be determined by Lagrangian advection of vorticity from areas of known data. The technique was applied to a field of rather dense data where a "hole" was created by withholding all observations within a given area.

The first experiments, assuming the atmosphere to be purely barotropic, resulted in a field within the hole that closely approached, after a reasonable time, the field obtained by a barotropic forecast using *all* the data. The remaining experiments, under simulated operational conditions, tested the effects of: 1) baroclinic processes, 2) size of hole, 3) frequency of observation, and 4) allowing a single strip of data across the hole. The results were much as predicted by the theory and reasonably accurate analyses were obtained.

1. Introduction

For many years meteorologists frequently blamed inaccuracies in upper air analysis on "lack of data". Since World War II, however, it has become increasingly apparent that actually there exists too much data to be readily assimilated and evaluated by individual or small groups of forecasters. The development of high speed electronic computers has, to a large extent, overcome this difficulty. We are again faced with "lack of data"-problems in many large areas of the world.

These areas with a low density of observations are distinct handicaps in any large scale forecasting technique because they prevent an accurate analysis of the existing meteorological situation. All synoptic, kinematic, dynamic and most statistical forecasting techniques are based on these analyses. The dynamic techniques are particularly sensitive to inaccurate analysis due to the mechanics of the method. A forecast at a given point depends not only on the data upstream but also on an ever widening band of data surrounding the upstream trajectory. This "region of dependence" is explicitly determined by the character of the equations used in the forecasting procedure, and may be, in some cases, extremely large. It is obvious, therefore, that forecasts for areas downstream from regions of low density of observations will be greatly affected by inaccuracies in the analysis.

P. D. Thompson (1961) reasoned that for all the inaccurate data "flowing *out*" of these low density observation areas, an almost equal amount of accurate data was "flowing *in*" on the other side. It remained thus to develop a

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technique for utilizing these accurate data in determining a more correct analysis in these regions. Thompson has developed a theory which showed that a more correct analysis could be obtained by conserving vorticity along forward extrapolated trajectories. In effect, the accurate data advects into the region and, to the extent that vorticity is conserved in actuality, replace the inaccurate data formerly there. He has also discussed the effect of baroclinicity, *i.e.* the creation of vorticity within the region. This effect is not well understood but its influence cannot be avoided.

Conventional barotropic forecasts are fairly accurate in regions of good data coverage. RMS vector wind errors average about 7 m.p.s. in 24 hour forecasts. In areas of poor data coverage, however, they have not been as accurate and in 24 hour forecasts, RMS vector wind errors average about 10 m.p.s.

This paper will describe a set of numerical experiments, using a barotropic model, which were done to test Thompson's theory.

2. Theory

Thompson has shown that the reduction of error in a sparse data region depends on: (1) the size of the region, (2) the speed of advection (this determines the amount of accurate data flowing into the region), and (3) the magnitude of the initial error. The sparse data region is termed a "hole" which, in theory, contains *no* observations. He further shows that when cyclogenesis is ignored, the dominant effect is due to advection of the accurate field by the *mean* wind $\mathbf{V}$ on the boundary of the hole. The contributions arising from deviations from $\mathbf{V}$ are of secondary importance in the advection and will be ignored in the following simplified theory.

General Definitions

A = area of "hole"
$\mathbf{r}$ = position vector inside A
C = boundary of A
ψ = stream function
$\zeta = \nabla^2\psi$

We assume that we know the distribution of the streamfunction ψ everywhere at all times outside the hole. Inside the hole, we approximate (by guess or "educated" guess depending on our knowledge of conditions surrounding the hole) a distribution of streamfunction ψ_a. We now define the mean error in the hole:

$$E = \frac{1}{A}\int_A (\nabla\varepsilon \cdot \nabla\varepsilon)\, dA \qquad (1)$$

where ε is the error in our approximate streamfunction $\psi_a - \psi$. Differentiating with respect to time, we obtain:

$$\frac{\partial E}{\partial t} = \frac{1}{A}\int_A \frac{\partial}{\partial t}(\nabla\varepsilon \cdot \nabla\varepsilon)\, dA \qquad (2)$$

Since ε is 0 everywhere outside the hole (and on the boundary), but is generally finite inside the hole, $\nabla\varepsilon$ will be discontinuous across the boundary C of A. Equation (2), therefore, will have contributions from the interior of A and from the boundary C. We shall solve for the boundary contribution first. Equation (2) may be transformed to:

$$\frac{\partial E}{\partial t} = \frac{2}{A}\int_A \nabla\varepsilon \cdot \nabla\frac{\partial\varepsilon}{\partial t}\, dA = \frac{2}{A}\int_A \nabla\cdot\varepsilon\nabla\frac{\partial\varepsilon}{\partial t} dA - $$
$$- \frac{2}{A}\int_A \varepsilon\nabla^2\frac{\partial\varepsilon}{\partial t}\, dA \qquad (3)$$

Noting the form of the first integral, we transform it to a line integral by Gauss' Theorem. The integrand then vanishes because ε is 0 on the boundary C. The second integral contains the term $\frac{\partial}{\partial t}\nabla^2\varepsilon$. Remembering that vorticity (defined as the Laplacian of a streamfunction) is conserved, we recognize this term as the local change of vorticity error which is merely the advection of $\nabla^2\varepsilon$ by the mean wind $\mathbf{V}$. Thus:

$$\frac{\partial E}{\partial t} = \frac{2}{A}\lim_{\Delta t\to 0}\frac{1}{\Delta t}\int_A \varepsilon(\mathbf{r})\,[\nabla^2\varepsilon(\mathbf{r}) - $$
$$- \nabla^2\varepsilon(\mathbf{r} - \mathbf{V}\Delta t)]\, d\mathbf{r} \qquad (4)$$

Considering the contribution of this integral at time t, we see that advection during a time Δt previous to time t crosses the boundary only on the inflow side. Therefore we may restrict our attention, in considering the boundary effect, to that part of the boundary

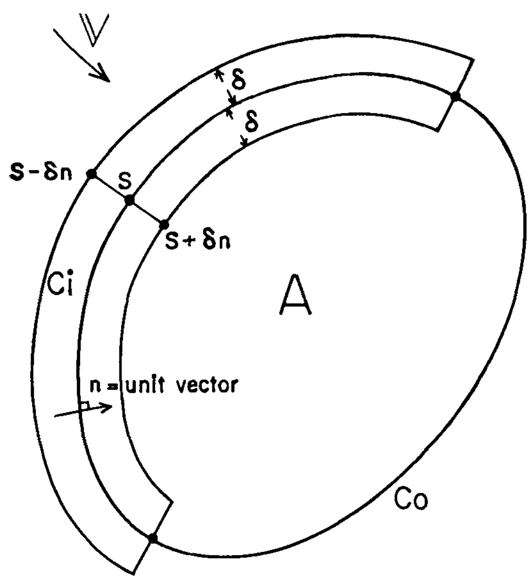

Fig. 1.

Ci across which there is flow into the hole (see Fig. 1).

This contribution comes from a strip along Ci whose width ($2\,\delta$) is vanishingly small. It is the behaviour of $\nabla^2\varepsilon$ on Ci that determines this contribution. Therefore, the first term in the integrand of (4) makes a contribution when $\mathbf{r}$ is integrated over the strip and the second term when $(\mathbf{r}-\mathbf{V}\Delta t)$ is integrated over the strip. Combining these two integrals and using ϱ as a position vector of integration, we obtain:

$$\left[\frac{\partial E}{\partial t}\right]_c = \frac{2}{A}\lim_{\Delta t\to 0}\frac{1}{\Delta t}\left\{\lim_{\delta\to 0}\int_{Ci}\int_{S-\delta n}^{S+\delta n}[\varepsilon(\varrho)\nabla^2\varepsilon(\varrho) - -\varepsilon(\varrho+\mathbf{V}\Delta t)\nabla^2\varepsilon(\varrho)]d\varrho\right\} \qquad (5)$$

where $\mathbf{n}$ is the inward-directed unit vector and $\mathbf{S}$ is the general position vector on Ci. Performing the inner integration and noting that $\frac{\partial\varepsilon}{\partial n}=0$ in the outward direction, we obtain:

$$\left[\frac{\partial E}{\partial t}\right]_c = \frac{2}{A}\lim_{\Delta t\to 0}\frac{1}{\Delta t}\int_{C_i}\left[\varepsilon(\mathbf{S})\frac{\partial\varepsilon}{\partial n}(S+) - -\varepsilon(S+\mathbf{V}\Delta t)\frac{\partial\varepsilon}{\partial n}(S+)\right]ds \qquad (6)$$

Taylor expanding the function $\varepsilon(S+\mathbf{V}\Delta t)$ about the point $\varepsilon(s)$ we obtain:

$$\varepsilon(S+\mathbf{V}\Delta t) = \varepsilon(S) + \mathbf{V}_i\Delta t\frac{\partial\varepsilon}{\partial n}(S+)$$

(for small t)

where $\mathbf{V}_i$ is the component of $\mathbf{V}$ parallel to $\mathbf{n}$. Therefore:

$$\left[\frac{\partial E}{\partial t}\right]_c = -\frac{2}{A}\int_{Ci}\mathbf{V}_i\frac{\partial\varepsilon}{\partial n}\cdot\frac{\partial\varepsilon}{\partial n}ds =$$

$$= -\frac{2}{A}\int_{C_i}\mathbf{V}_i\nabla\varepsilon\cdot\nabla\varepsilon ds \qquad (7)$$

The contribution from the interior of A must now be considered. Discontinuities in $\nabla\varepsilon$ on C can now be ignored. Equation (2) can be written:

$$\left[\frac{\partial E}{\partial t}\right]_A = -\frac{1}{A}\int_A\mathbf{V}\cdot\nabla(\nabla\varepsilon\cdot\nabla\varepsilon)dA$$

$$= -\frac{1}{A}\int_A\nabla\cdot(\nabla\varepsilon\cdot\nabla\varepsilon)\mathbf{V}dA \qquad (8)$$

Since $\mathbf{V}$ is non-divergent

$$\left[\frac{\delta E}{\delta t}\right]_A = +\frac{1}{A}\oint_c\mathbf{V}_n\nabla\varepsilon\cdot\nabla\varepsilon dS \qquad (9)$$

where $\mathbf{V}_n$ is the inward normal component of $\mathbf{V}$. Combining (7) and (9) we find that the total rate of change of E is:

$$\frac{\partial E}{\partial t} = -\frac{1}{A}\left\{\int_{C_i}\mathbf{V}_i\nabla\varepsilon\cdot\nabla\varepsilon dS + \int_{C_0}\mathbf{V}_0\nabla\varepsilon\cdot\nabla\varepsilon ds\right\} \qquad (10)$$

where $\mathbf{V}_0 = -\mathbf{V}_n$ on C_0 (see fig. 1).

Approximately $\frac{\partial E}{\partial t} = -\frac{1}{A}C\overline{|\mathbf{V}_n|}\,\overline{(\nabla\varepsilon\cdot\nabla\varepsilon)}$

where the bar denotes an average around C, therefore:

$$\frac{\partial E}{\partial t} = -\frac{\lambda C\overline{|\mathbf{V}_n|}E}{A} \qquad (11)$$

where λ is the ratio of the value of $\overline{\nabla\varepsilon\cdot\nabla\varepsilon}$ over the boundary to its value over A, and is assumed to be approximately constant.

3. Numerical Experiments

The numerical experiments were designed to serve two distinct purposes. *First*, the general validity of the theory must be checked. *Second*, a scheme that could be used in actual practice must be tested and evaluated. The first set of experiments, therefore, was performed without considering the application of the method on an operational basis. The second set, however, fulfilled the requirements that would have to be met in actual forecasting operations.

A. *Testing the Theory*

The simplest method for testing the theory appeared to be some technique whereby vorticity was actually conserved. It was decided, therefore, to use the one-parameter barotropic model and to assume that its results were perfect and represented the "true" state of the atmosphere. These "barotropic forecasts" provided "true" data *outside* the hole, which were used in making the "hole forecast", and "true" data *inside* the hole, which were used in evaluating the results of the "hole forecast".

Initially, we knew the observed distribution of ψ everywhere and could compute $\zeta = \nabla^2\psi$ everywhere. This information, with a suitable set of boundary conditions, was all we needed for the barotropic forecast. For the hole forecast however, we assumed that we knew nothing inside the hole and must depend only on initial observations *outside* the hole and subsequent "true" barotropic forecast data *outside* the hole. In our first computations, we guessed a value of relative vorticity $\zeta_a = 0$ at all points within the hole. This guessed field was then smoothed to eliminate discontinuities at the boundary of the hole. This smoothing was done by the following operator:

$$\zeta_a\,(\text{smoothed}) = \frac{1}{4+a}\,(\zeta_1 + \zeta_2 + \zeta_3 + \zeta_4 + a\,\zeta_0) \tag{12}$$

where the subscripts denote the positions of the gridpoints and a is a weighting factor applied to the center point. This equation may also be written:

$$\zeta_a\,(\text{smoothed}) = \frac{1}{4+a}\,(\zeta_1 + \zeta_2 + \zeta_3 + \zeta_4 + (4+a)\,\zeta_0 - 4\zeta_0) = \zeta_0 + \frac{1}{4+a}\,\nabla^2\zeta \tag{13}$$

This simplifies the computing procedures. A series of hand computations were made with this operator using values of 1, 2, and 3 for *n*. Three iterations were made. Based on the results of these computations, *n* was given the value 2 and 3 iterations were performed in the numerical experiments. In later computations, the approximate vorticity in the hole was computed from the known circulation on the boundary of the hole.

$$\overline{\zeta_a} = \frac{1}{A}\oint \mathbf{V}s\,ds \tag{14}$$

The smoothing operator was also applied to this field, but due to the limitations of our technique, no check was made to determine that the smoothed field agreed with the circulation.

We then had an "accurate" ψ-field and ζ-field *outside* the hole and an approximate ζ_a-field *inside* the hole. Using the "accurate" ψ-values on the boundary of the hole, we solved the Poisson equation $\nabla^2\psi = \zeta_a$ by relaxation and obtained the ψ_a-field inside the hole. This was all the information needed to compute the advection of vorticity inside the hole.

The advection was computed in a quasi-Lagrangian way. The components of velocity were computed at each grid point in the hole from the ψ_a-field. Assuming that these components represent a mean velocity of advection during the 45 minute time step, they determined the origin of a trajectory which terminated at the grid point at the end of the time step. The forecast vorticity at time t at the grid point, therefore, will be the same as that at the origin at time $(t-45)$ min. The absolute vorticity at this origin was computed by linear extrapolation from the vorticity values at the adjacent grid points.

We then computed a 45 minute forecast over the entire region, using all the observed data, with the barotropic model. This gave us a forecast ψ-field and ζ-field everywhere. We again relaxed the ζ_a-field inside the hole (using the "accurate" forecast ψ on the boundary) to get the ψ_a forecast field. We then had all the required information to repeat the advection computation inside the hole. This procedure was repeated until the desired length of forecast was obtained. Print-outs of both the "accurate" barotropic fields and the hole data were made at periodic intervals. The error was evaluated from these print-outs.

The above procedure is summarized in the step-by-step outline below:

1. Determine the initial ζ-field from the observed ψ-field over the entire region.
2. Approximate an initial ζ_a-field inside the hole.
3. Smooth this approximate field to eliminate boundary discontinuities.
4. Determine an approximate ψ_a-field inside the hole from the approximate ζ_a-field and the known ψ-field on the boundary of the hole.
5. Compute the forecast ζ_a-field inside the hole by the quasi-Lagrangian advection method.
6. Compute the forecast ψ-field and ζ-field with the barotropic forecast.
7. Return to step 4 and use the new forecast values.

B. *Operational Computations*

The second set of experiments was designed to fulfill operational requirements; *i.e.* provision was made to utilize *new observed* data outside the hole as it became available. Also, the actual known data within the hole was never used at any time during the computations.

Initially, we used only the known ψ-field outside the hole. As before, we computed the vorticity ζ from this field. The approximate vorticity field ζ_a inside the hole was again guessed or computed from the circulation, and these fields were "faired" at the boundary by using the smoothing operator. The ψ_a-field inside the hole was determined, as before, by relaxing the ζ_a-field and using known ψ-values on the boundary. Note that the vorticity values on the boundary of the hole were determined only from the initial approximate values in the hole with some influence (during the smoothing) coming from the known vorticity along the first grid row outside the hole. This did not guarantee that the boundary vorticity values would be in agreement with the ψ-field. Therefore, we computed a new ζ'-field from the combined ψ- and ψ_a-fields. This did not change the vorticity values outside the boundary of the hole or within the hole, only those along the boundary. These new values on the boundary then agreed with the ψ-distribution and we proceeded with the quasi-Lagrangian advection as before.

When the computations reached a new observation time, all the data were discarded except the forecast ζ_a-field inside the hole. This, as before, was used with the new observed ψ-values on the boundary of the hole to determine the ψ_a-field within the hole. The new ζ'-field was computed from the combined ψ- and ψ_a-fields and we proceeded to a new observation time.

The above procedure is summarized in the step-by-step outline below:

1. Determine the initial ζ-field outside the hole from the observed ψ-field outside the hole.
2. Approximate an initial ζ_a-field inside the hole.
3. Smooth this approximate field to eliminate boundary discontinuities.
4. Determine an approximate ψ_a-field inside the hole from the approximate ζ_a-field and the known ψ-values on the boundary of the hole.
5. Compute an "adjusted" ζ'-field from the combined ψ- and ψ_a-fields (to insure compatibility).
6. Compute a forecast ζ-field by quasi-Lagrangian advection over the entire field.
7. Relax the forecast ζ-field to obtain a forecast ψ-field over the entire region.
8. Return to step 6 and repeat until a new observation time is reached.
9. At the new observation time, retain *only* the ζ_a-field inside the hole, return to step 4 and repeat.

4. Results

A. The first set of experiments, to test the theory, was based on the observed data at 1200Z on 25 April 1960. The hole was chosen over the North Sea, Northern Germany, and Southern Scandinavia. A larger hole was subsequently chosen in the same area. This particular set of data was chosen because the one-parameter model barotropic forecasts were particularly accurate during this period. The hole locations were chosen to have a relatively dense network of observations around the boundaries. A section of the 16×16 grid,

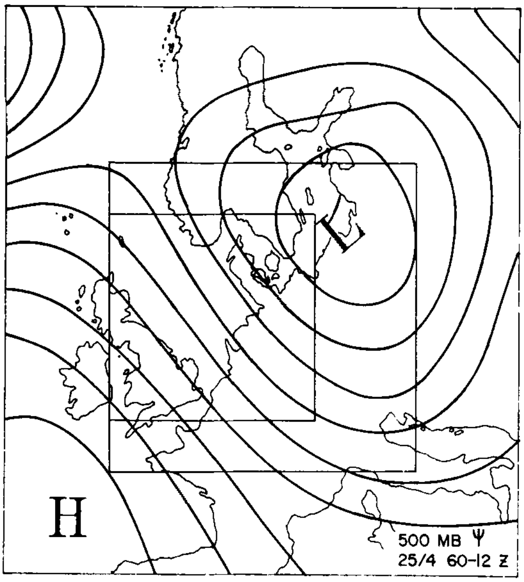

Fig. 2.

the location of the holes, and the initial synoptic pattern are shown in figure 2.

Early computations in the 3 × 3 hole (1200 × 1200 km) were encouraging in that

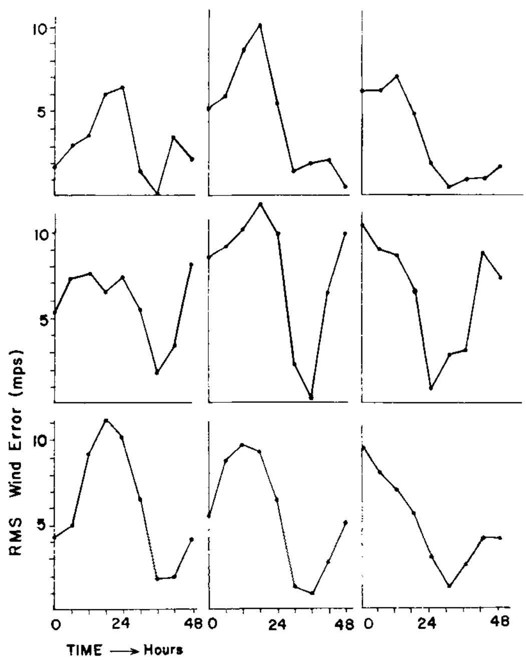

Fig. 3. The change of RMS wind error with time at each grid point of a 3 × 3 hole. The wind flow is from upper right to lower left.

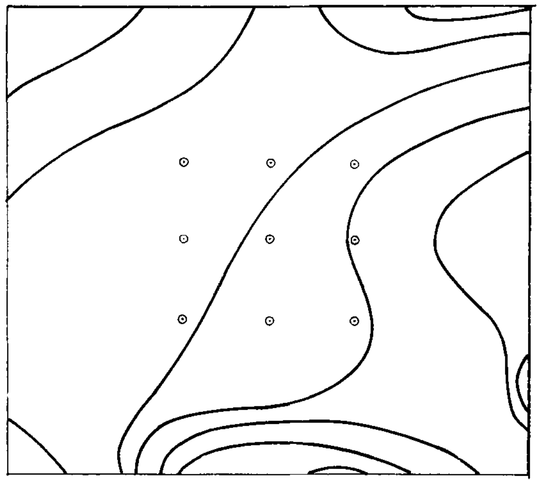

Fig. 4. The absolute vorticity field (as computed by quasi-Lagrangian advection) after 5 hours. Grid points in the hole are circled.

the decrease of error advected across the hole just as predicted in the theory (see fig. 3). The decrease occurred first at the points along the inflow boundary and the points along the outflow boundary showed the decrease last. At most points, however, this decrease was only temporary and the error grew rapidly almost immediately after having decreased to a very small value. These results were very similar to those experienced by WELANDER (1956) in a similar experiment.

A careful analysis was made of all the barotropic fields and the hole forecast fields to find an explanation of this growth of error. It was immediately apparent that the fields generated by the quasi-Lagrangian method were very smooth and all the small scale perturbations had been smoothed out. On the other hand, the fields generated by the barotropic forecast (using an Eulerian scheme), were definitely not smooth and many small scale perturbations were present (see figs. 4 & 5). Remembering that these fields must match at the boundary of the hole, it was apparent that truncation errors alone could cause the technique to fail. In some cases, the uneven field of barotropic data resulted in an erroneous change of sign during the advection.

The theory required that the vorticity be conserved along *forward extrapolated trajectories*. Therefore, the only alternative was to modify the barotropic forecast routine to use the

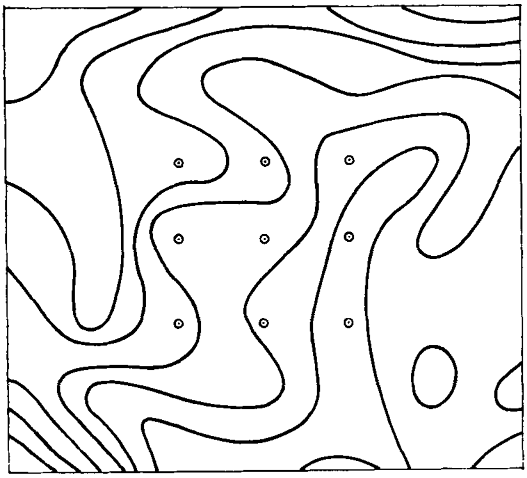

Fig. 5. The barotropic Eulerian absolute vorticity field after 6 hours.

type of advection rather than the usual Eulerian. This proved to be the correct solution (see fig. 6). The decrease in error still progressed downstream, it approached zero after a reasonable length of time, and there was no tendency for later growth beyond that which could be due to round-off error.

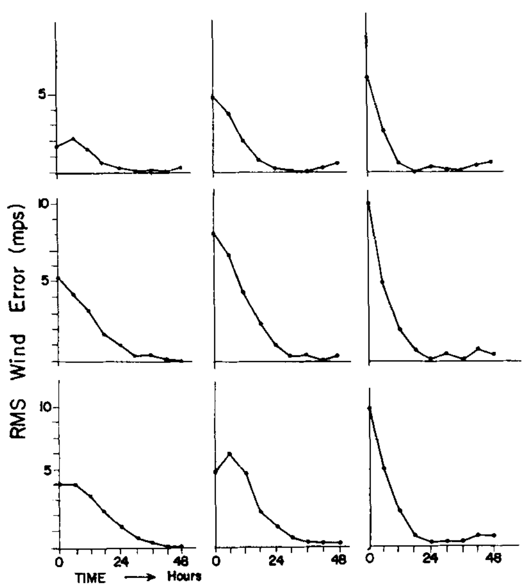

Fig. 6. The change of RMS wind error with time at each grid point of a 3×3 hole based on a quasi-Lagrangian forecast. The wind flow is from upper right to lower left.

Figures 7—9 show a comparison of the theoretical decrease of error (according to equation 11) and the observed decrease of error. The constant λ was evaluated from the data shown in figure 7, and then used to determine the theoretical curves in figures 8 and 9. The case shown by figure 9 differs from that shown by figure 8 *only* in the method of determining the initial approximate ζ_a-field in the hole. In the case shown by fig. 8, the initial relative vorticity was assumed to be zero. The other case was computed with an initial mean vorticity computed from the

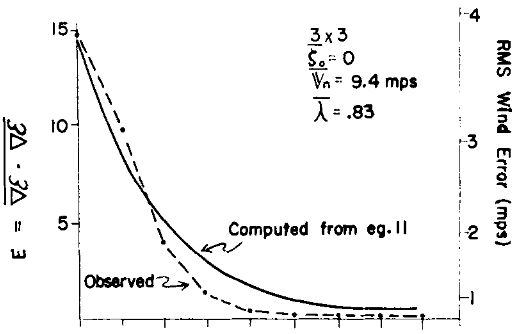

FIGURE 7

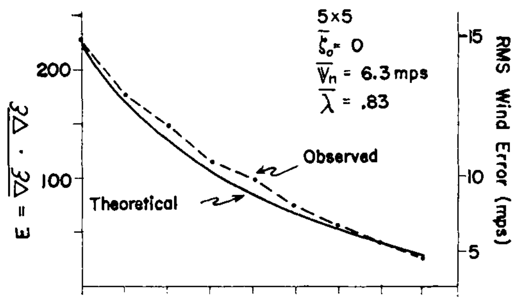

FIGURE 8

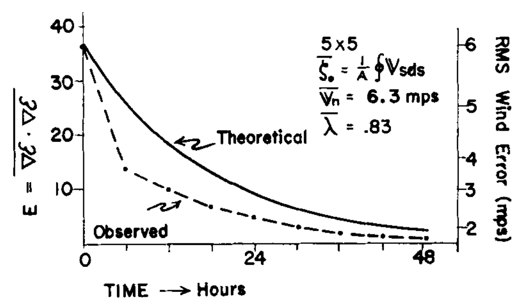

FIGURE 9

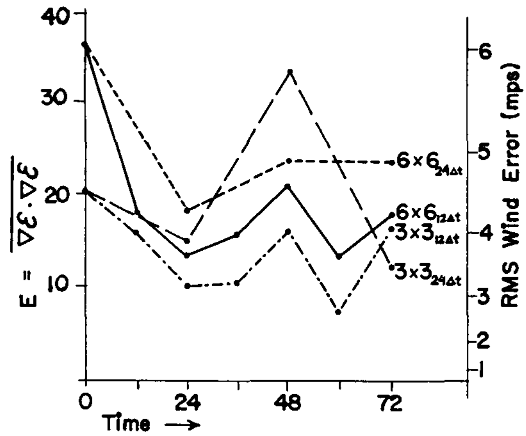

Fig. 10. Change of E with time for two hole sizes and two observation frequencies.

circulation around the hole. Note the *initial* RMS wind error was reduced from approximately 15 mps to 6 mps by this refinement only. Also, the latter case showed an even greater decrease of error than the theoretical one throughout the forecast period.

B. The second set of experiments, to simulate actual operational use, was based on the observed data for the period 0000Z 14 November 1960 to 1200Z 16 November 1960. As before, this period was chosen because the barotropic forecast had been quite successful during this time. Two holes, 3×3 grid points (1200×1200 km) and 6×6 grid points (2100×2100 km) were chosen in the same region as before. The total field consisted of 30×30 grid points.

There were two important differences between these and the experiments described earlier. First, as stated in section III b, new observed data were introduced into the scheme at the international observation times. Second, the results of the hole forecast were compared with actual observed data to compute the error field. Therefore, we did not expect the error to decrease to zero as vorticity is not actually conserved in the atmosphere. Rather, we expected the error to decrease to and oscillate about some minimum value.

In addition to varying the hole size, we also varied the time interval between the introduction of new observed data. This would give us an idea of the effect of hole size and frequency of observation on the method. In all, four complete computations were made, each for a period of 72 hours.

Figure 10 shows the results of these computations. As expected, the 12 hour observation frequency produced better results, both in amount and rate of error decrease. The effect of the hole size did not behave exactly as expected. In fact, the percentage decrease of error was slightly larger in the case of the larger hole and also, the rate of decrease was slightly greater. This does not really contradict the theory however, for the initial guess of vorticity (based on the circulation) would be much better in the small hole. Had we started the computations with zero vorticity in both cases, the smaller hole case would probably have been better.

The unexpectedly large error (larger than the initial value) at 48 hours in the $3 \times 3 - 24$ hour time interval case was not completely understood. There was a general increase of error between 24 and 48 hours in all cases, but it was excessively large in only the one instance. Possibly the natural oscillation of the error and some other effects (due to the long period of time between observations) both acted "in phase" to create a resonance. Also, this particular case may have possessed some unusual and unknown characteristics which made it misbehave. Although identical programs, tapes, and data were used in all cases, this one was very difficult to compute. Several times during the computations, extremely large relaxation residuals occurred. Also, on two separate occasions, the computer stopped suddenly after having completed many time steps successfully. The reason for this stop could not be determined, and it was assumed to be due to machine error. If so, it is possible that the final results were influenced by some unknown machine error and are not realistic. No unusual trouble was experienced in any of the other computations.

C. An additional pair of computations were made in which a strip of observed data was inserted across the 6×6 hole at each observation time. The first computation was made with the strip normal to the wind, the second with the strip along the wind. This single strip, consisting of 6 points, does not provide any vorticity values in the hole. Its main influence is in providing additional boundary values when the approximate vorticity field is relaxed to obtain the streamfunction field in the hole. The results are shown in figure 11.

These "strip" computations were made with the same data that were used for the 6 × 6 case (with a 12 hour frequency of observation) which is also included in figure 11. Comparing these two results, we see that the introduction of this strip resulted in a reduction of error initially and at later times. The case where the strip was normal to the wind increased the error slightly between 12 and 24 hours but showed a consistent decrease of error elsewhere and the error was still decreasing at 48 hours. The case where the strip was along the wind, however, showed a consistent increase in error after 24 hours.

Both cases were definitely better than in the case of the hole without the strip and, unless the error growth continued in the second case, they were even better than the 3 × 3 cases. This is a little surprising because the hole with the strip still had 30 points where the data were unknown while the small hole had only 9. Also, the initial RMS wind error for the strip case was almost one m.p.s. larger than for the small hole. Apparently, for very small holes (of the order of 1000 km or less) the circulation and smoothing technique for obtaining the initial vorticity field results in a fairly close approximation to the true values and further improvement is difficult. We are not advocating the establishment of large holes but are merely pointing out that, where large natural holes already exist (oceans, uninhabited areas, etc.), the judicious application of this technique could be helpful. If additional observing sites were to be established, then these results (or other results based on tests in the area in question) could indicate the most advantageous sites to be chosen.

The results of these strip experiments suggest that aerial reconnaissance would be an excellent method for obtaining observations. In areas with sufficient air traffic, these observations could be made on a routine basis by commercial and other regular aircraft. In areas with little or no traffic, this function could be performed by special weather reconnaissance aircraft, just as is done at the present time. The latter have a distinct advantage in that the tracks flown can be varied on a seasonal (or synoptic situation) basis so as to produce the best results.

D. A statistical analysis of the operational

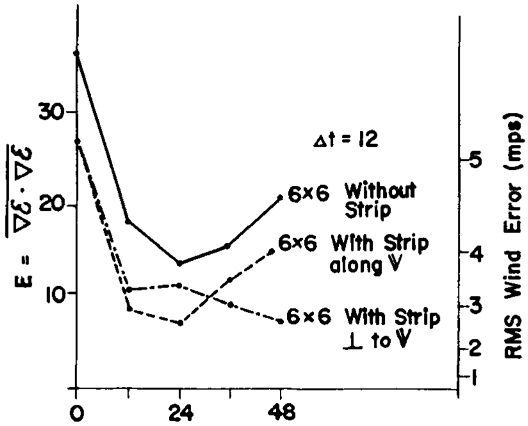

Fig. 11. Change of E with time for a 2100 × 2100 km hole with one row of observed data across the hole. The upper curve is for the same hole without the strip.

computation results is shown in Table 1. The actual mean wind velocity $\mathbf{V}'$ in a square hole will be larger than the mean normal velocity by an average factor of $\frac{\pi}{2}$. Based on this value and the values of errors in the predicted field in the hole, the root mean square error of wind velocity σ_v, and the root mean square error of wind direction σ_Θ (assuming no error in speed) were computed.

Table 1

Case*	$\mathbf{V}'$ (mps)	σ_v (mps)	σ_Θ (degrees)
$6 \times 6_{24}$	11.5	4.7	23.6
$6 \times 6_{12}$	11.5	4.0	20.0
$6 \times 6_{12}$ with strip ⊥ to V	11.5	3.1	15.5
$3 \times 3_{24}$	10.8	4.3	23.0
$3 \times 3_{12}$	10.8	3.5	18.7

* The case notation is the same as in figures 10 and 11.

4. Conclusions

It is evident that the method described in the paper does result in an overall decrease of error in region of low density observations. It is felt, therefore, that it should be applied by those organizations who are engaged in routine large scale weather analysis and forecasting by numerical methods. It is also evident that the 12 hour frequency of observation was superior to the 24 hour frequency during these tests. It would have been interesting to make similar tests using observations at more frequent intervals to determine this effect more accurately. Unfortunately, however, such data over an adequately large area could not be located.

The largest hole used during these tests was approximately half as large as the natural hole in the North Atlantic Ocean. If we assume that similar results would be obtained in a hole as large as the North Atlantic, it appears that with sufficient boundary data surrounding this hole, we could expect an accuracy of the order of 5 m.p.s. RMS wind error. We were not able to carry out tests involving holes that large, partly because of the limited capacity of the BESK computer and partly because we could not locate a hole that large (in the data fields we used) with sufficiently dense and extensive known data on the boundaries.

It was noted that, generally, there was no great trend for improvement in the results after 24 hours. Assuming the actual advective wind velocity to be 10 m.p.s., advection from the inflow boundary would only penetrate about 800 km into the hole in 24 hours. This suggests that boundary influence and "region of influence" effects also contribute in reducing the error.

It is interesting that the oscillation of the error had a period of approximately 24 hours. VEDERMAN AND HUBERT (1957) found a similar period in the non-conservation of absolute vorticity as computed by a finite difference technique. There appears to be no physical explanation for this. Perhaps it is an inherent property of the finite difference method which has not yet been isolated.

6. Suggestions for Further Investigation

During the course of these experiments, a number of ideas, suggestions, and questions were brought up. Some were discarded, some were attempted, and some were reluctantly by-passed because they were beyond the capability of the system we used. They are listed here for those who plan further work in this subject.

1. Extend the tests to larger size holes.
2. Test the effects of asymmetric holes.
3. Try shorter observation frequencies (either by generating a hypothetical set of observations or by using data such as the "Tornado Alley" observations in the U.S.).
4. Do experiments similar to the strip cases with only single points or small triangles (from which the vorticity can be computed) of known data in the hole.
5. Require that the mean vorticity in the hole agrees with circulation around the hole at all times, not just initially.
6. Trace the advection during each time step, make a new circulation calculation around that part of the hole which has not been affected by the advection, and revise the approximate vorticity field accordingly.

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