On the Vertical Velocity Field in a Steady Symmetric Hurricane

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Abstract

It is shown how a system of equations (motion, mass continuity and the first law of thermodynamics) can be integrated for a steady symmetric vortex with a prescribed tangential motion. The method of characteristics is used to obtain a solution for the vertical-velocity field. For the normally observed characteristic lines in a hurricane no vertical velocity is possible for a frictionless atmosphere, and therefore numerical experiments are carried out with different magnitudes of the momentum exchange coefficients. It is also shown that as long as the ratio of the lateral and the vertical exchange coefficients is kept fixed, one calculation of the vertical velocity field enables an examination of a wide range of variation of the individual coefficients. A consistent structure of an Atlantic hurricane is constructed from the National Hurricane Research Project data.

1. Introduction

Very few attempts have been made to obtain vertical velocity fields in tropical storms due to lack of data. The works of PALMÉN (1958), PALMÉN and RIEHL (1957), and later work by RIEHL and MALKUS (1960) with the aircraft reconnaissance data are attempts to vertically integrate the horizontal divergence field and involve considerable skill. The first two mentioned works deal with a large scale, the storm having a dimension of 1,000 km and the latter of a smaller scale storm having a size of over 100 km. Neither of these attempts gives details of the vertical velocity field. The vertical velocity field in a hurricane should show the following features:

(i) an eye,
(ii) an eye wall of tall clouds,
(iii) inward spiralling bands.

Features (i) to (iii) are generally observed in radar films. Reference may be made to the radar-composite charts of MALKUS (1960) that give these details.

The modern approach in Meteorology is to examine the growth of a system for prescribed initial conditions, and to seek for observed features in various models under different constraints. PHILLIPS' (1956) work on the general circulation and KASAHARA (1960) model of the hurricane are examples of such an approach. An initial distribution of the motion field and the heating is given and the system of equations are integrated.

Our interest here will be somewhat of an inverse one. We shall first show that for a prescribed tangential velocity field, a closed system of equations can be obtained that will completely define the structure of a steady, symmetric vortex.

An outline of the diagnostic system of equations that are used to obtain the structure of a symmetric vortex will be presented, and an application of the method will be made to determine the structure of Atlantic Hurricane Cleo of 1958.

2. The Fundamental Equations:

Cylindrical isobaric coordinates will be used with the radius vector r pointing outward Θ the angle increasing in the cyclonic sense. Table I lists the symbols used in this paper.

Table 1.

Symbol	Meaning of symbol
t	Time.
r	Radius increasing outward.
Θ	Angle increasing in the counter clockwise sense.
p	Pressure used as a vertical coordinate.
u	Tangential wind velocity positive along increasing Θ.
v	Radial wind velocity positive along r.
ω	Vertical velocity in isobaric coordinates.
Z	Height of constant pressure surface above sealevel.
T	Temperature.
H	Rate of nonadiabatic heating per unit mass of air.
C_p	Specific heat of air at constant pressure.
R	Gas constant of air.
ν	Coefficient of lateral momentum exchange.
k	Coefficient of vertical momentum exchange.
g	Acceleration of gravity.
f	Coriolis parameter.

The equations of motion may be written in the form:

$$v\left(\frac{\partial u}{\partial r}+\frac{u}{r}+f\right)+\omega\frac{\partial u}{\partial p}= =\frac{\partial}{\partial r}\left[\nu\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)\right]+\frac{\partial}{\partial p}\left(k\frac{\partial u}{\partial p}\right) \tag{1}$$

$$v\frac{\partial v}{\partial r}-u\left(\frac{u}{r}+f\right)+\omega\frac{\partial v}{\partial p}= =-g\frac{\partial Z}{\partial r}+\frac{\partial}{\partial r}\left[\nu\left(\frac{\partial v}{\partial r}+\frac{v}{r}\right)\right]+\frac{\partial}{\partial p}\left(k\frac{\partial v}{\partial p}\right) \tag{2}$$

$$0=-g\frac{\partial Z}{\partial p}-\frac{RT}{p} \tag{3}$$

The equation of continuity,

$$0=\frac{1}{r}\frac{\partial vr}{\partial r}+\frac{\partial\omega}{\partial p} \tag{4}$$

and the thermodynamic energy equation

$$H=C_p\left[v\frac{\partial T}{\partial r}+\omega\frac{\partial T}{\partial p}\right]-\frac{RT}{p}\omega \tag{5}$$

If u, the tangential velocity, is prescribed at each point then, in principle, we can solve for v, ω, Z, T and H, provided we have knowledge about the distribution of ν and k.

Instead of assigning certain reasonable values for ν and k and solving the equations, an alternate method is presented.

The system of equations (1) to (5) are non-dimensionalized through the following choice of fundamental units:

Length L_0
Pressure p_0
Time $t_0=\frac{2}{f_0}$ where f_0 is the mean Coriolis parameter.

The following equations may be considered as the non-dimensional equivalents of equations (1) to (5):

$$v\left(\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{2f}{f_0}\right)+\omega\frac{\partial u}{\partial p}= =\frac{\partial}{\partial r}\left[\nu\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)\right]+\frac{\partial}{\partial p}\left(k\frac{\partial u}{\partial p}\right) \tag{6}$$

$$v\frac{\partial v}{\partial r}-u\left(\frac{u}{r}+\frac{2f}{f_0}\right)+\omega\frac{\partial v}{\partial p}= =-g\frac{\partial Z}{\partial r}+\frac{\partial}{\partial r}\left[\nu\left(\frac{\partial v}{\partial r}+\frac{v}{r}\right)\right]+\frac{\partial}{\partial p}\left(k\frac{\partial v}{\partial p}\right) \tag{7}$$

$$0=-g\frac{\partial Z}{\partial p}-\frac{RT}{p} \tag{8}$$

$$0=\frac{1}{r}\frac{\partial vr}{\partial r}+\frac{\partial\omega}{\partial p} \tag{9}$$

$$H=C_p\left[v\frac{\partial T}{\partial r}+\omega\frac{\partial T}{\partial p}\right]-\frac{RT}{p}\omega \tag{10}$$

Table 2 shows the one to one correspondence between each term of equations (6) to (10) with each term of equations (1) to (5).

3. Solutions of the System:

Since u is prescribed, equations (6) and (9) can be treated as two equations for the unknowns v and ω. Equation (6) can be rewritten as

$$v=b+a\omega \tag{11}$$

Table 2.

Relation between dimensional and non-dimensional terms		
Dimensional term	= Constant	× Nondimensional term
r	L_0	r
p	p_0	p
t	$2/f_0$	t
Z	L_0	Z
u	$L_0 f_0/2$	u
v	$L_0 f_0/2$	v
ω	$p_0 f_0/2$	ω
ν	$L_0^2 f_0/2$	ν
k	$p_0^2 f_0/2$	k
g	$L_0 f_0^2/4$	g
RT	$L_0^2 f_0^2/4$	RT
$C_p T$	$L_0^2 f_0^2/4$	$C_p T$
H	$L_0^2 f_0^3/4$	H

where

$$b = \frac{\partial/\partial r\left[\nu\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)\right]+\frac{\partial}{\partial p}\left(k\frac{\partial u}{\partial p}\right)}{\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{2f}{f_0}} \tag{12}$$

and

$$a = -\frac{\partial u/\partial p}{\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{2f}{f_0}} \tag{13}$$

Eliminating v from (9) and (11) the following equation in ω may be obtained:

$$\frac{\partial \omega}{\partial p}+a\frac{\partial \omega}{\partial r}+\frac{\omega}{r}\frac{\partial ar}{\partial r}+\frac{1}{r}\frac{\partial br}{\partial r}=0 \tag{14}$$

The two newly introduced quantities a and b possess certain interesting properties.

a is the slope of the characteristic lines of equation (14) and determines the slope of the vortex lines in the vertical plane. b is a term representing the role of the lateral and vertical friction in the determination of the vertical velocity field.

Equation (14) may be written in the form:

$$\frac{\delta \omega}{\delta p}+\frac{\omega}{r}\frac{\partial ar}{\partial r}+\frac{1}{r}\frac{\partial br}{\partial r}=0 \tag{15}$$

where $\delta\omega$ will denote a change in ω along the characteristic line of slope a.

We shall now show that equation (15) reveals certain interesting features of the vertical velocity field.

Let

$$\frac{1}{r}\frac{\partial ar}{\partial r}=x$$

$$-\frac{1}{r}\frac{\partial br}{\partial r}=y$$

An explicit solution of equation (15) can be written in the form:

$$\omega(p)=\omega(p_0)+e^{-\int x dp}\int_{p_0}^{p} e^{\int x dp}\, y dp \tag{16}$$

If $\omega(p_0)=0$ then for $y=0$, $\omega(p)=0$ and we get the important result that, *in a steady symmetric frictionless system no vertical motion is possible if the vertical velocity is zero at any point of a characteristic line.*

In examining the characteristic lines in various atmospheric systems, the author found that these lines are usually vertical, i.e., $a\approx 0$, and they intersect the 1,000 mb surface usually vertically. A boundary condition of the type $\omega=0$ at $p=1{,}000$ mb therefore would lead to no vertical motion for a steady, symmetric, frictionless system. In such a system therefore one must have friction to have any vertical motion.

We shall next, let $\nu/k=N$
and $\nu=M$

A machine programme can be easily set up to investigate the field of vertical velocity for different values of N and M. We shall in this paper discuss the structure of steady symmetric vortices for $N=1$ and M taking different values.

Once $\omega(p)$ is determined for any value of M, equation (11) may be used to obtain a corresponding field of v.

If $Z(L_0, p)$ is prescribed at $r=L_0$ then Z $Z(r, p)$ can be obtained from equation since

$$\frac{\partial Z}{\partial r}=-\frac{1}{g}\left\{v\frac{\partial v}{\partial r}-u\left(\frac{u}{r}+\frac{2f}{f_0}\right)+\omega\frac{\partial v}{\partial p}-\frac{\partial}{\partial r}\left[\nu\left(\frac{\partial v}{\partial r}+\frac{v}{r}\right)\right]-\frac{\partial}{\partial p}\left(k\frac{\partial v}{\partial p}\right)\right\} \tag{17}$$

Further, $T(r, p)$ can be obtained from the hydrostatic equation (8) for the calculated

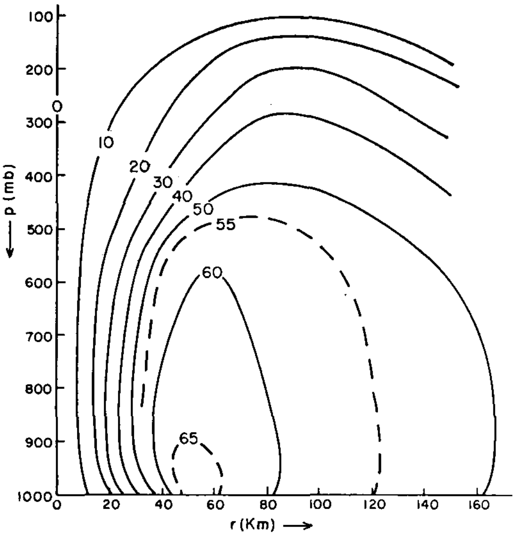

Fig. 1. Vertical cross-section of mean tangential velocity for Hurricane Cleo 1958. Units knots.

$Z(r, p)$ field. And the right hand side of equation (10) can be computed to give a distribution of $H(r, p)$, the non-adiabatic heating.

Now we shall make an application of the suggested model for the data of Hurricane Cleo of 1958.

4. Application to Hurricane Cleo Data:

Hurricane Cleo was over the Atlantic between August 12 and 21, 1958. The National Hurricane Research Project made several flights into this storm on August 18. The three flights at levels 820, 580 and 237 mb surfaces were very useful. Fig. 1 shows a cross-section of u obtained by averaging the tangential velocity around the storm, using the data of these three levels. It may be noted that since u is a larger component of the total wind, the per cent errors in the evaluation of u are small.

In a review of the hurricane season of 1958, WEATHER BUREAU (1958), mention that the central pressure of Hurricane Cleo was around 960 mb for about three days, suggesting a quasi-steady state. Data do not justify the symmetry assumption; in fact, there is always a velocity maximum in the right side of the storm. But a hurricane has been treated as a symmetric disturbance by many modern meteorologists, KASAHARA (1960), KUO (1959) etc. Here the symmetry assumption has been made to carry out the proposed analysis. The author will soon present an extension of this approach by including the asymmetries.

The following fundamental units were used for calculations:

$$L_0 = 150 \text{ km},$$
$$p_0 = 1{,}000 \text{ mb},$$
$$f_0 = 0.8 \times 10^{-4}/\text{sec},$$
$$t_0 = \frac{2}{f_0} = 2.5 \times 10^4/\text{sec}.$$

Since L_0 is only about a degree and a half latitude units in length, the Coriolis parameter f is assumed to be constant $= f_0$.

A tabulation of $U(r, p)$ was made with

$$\Delta r = 7.5 \text{ km},$$
$$\Delta p = 50 \text{ mb}.$$

a. *The characteristic lines of Hurricane Cleo:*

The advantages of the proposed method will become clear if charts of intermediate calculations are presented. With this in view, Figs. 2 and 3 are presented. Fig. 2 shows a cross-section of $\frac{\partial u}{\partial p}$, the vertical shear in non-dimensional units. Below 1,000 mbs, $\frac{\partial u}{\partial p}$ is

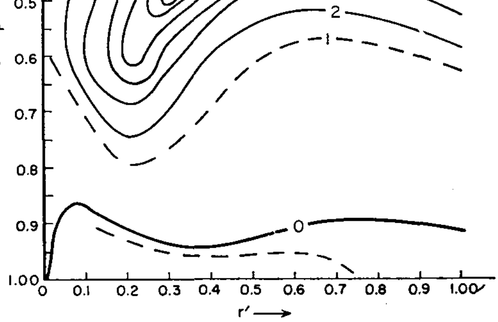

Fig. 2. Vertical cross-section of vertical shear, $\partial u/\partial p$. Non-dimensional units.

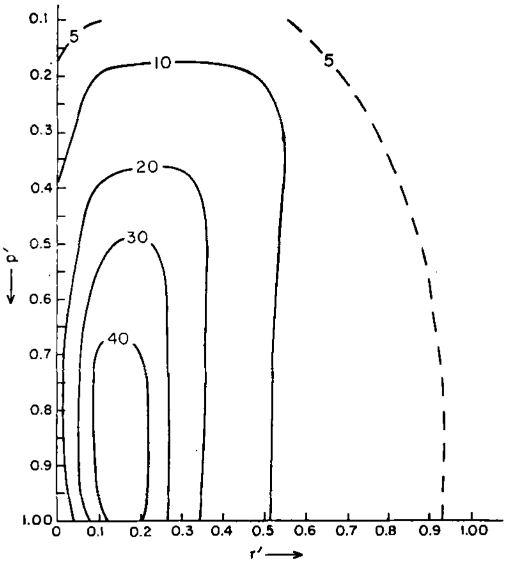

Fig. 3. Vertical cross-section of absolute vorticity, ζa, non-dimensional units. To get in units of 10^{-4} sec^{-1}, multiply by 0.4.

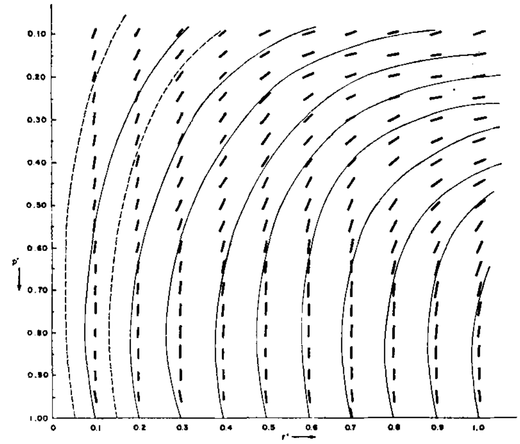

Fig. 4. Characteristic lines for Hurricane Cleo. The coordinates are expressed in non-dimensional units.

negative and in the rest of the chart it is positive. Fig. 3 is a cross-section of absolute vorticity, $\frac{\partial u}{\partial r}+\frac{u}{r}+2$ expressed again in non-dimensional units. One may obtain the dimensions by multiplying the numbers on Fig. 3 by $f_0/2$. The characteristic slope a can be obtained by dividing Figs. 2 and 3, since $a=-\frac{\partial u/\partial p}{\partial u/\partial r+u/r+2}$. The slope is negative everywhere except very near the 1,000 mb surface. In order to obtain the characteristic lines we must draw little line segments of slope a and draw free hand characteristic lines. Fig. 4 shows the characteristic lines for Hurricane Cleo.

b. *The calculation of the vertical velocity field:*

We shall first let $M=1$ and obtain a field of b the friction term. Fig. 5 shows an analysis of the field of b in non-dimensional units. All spatial derivatives were obtained over distances $2\,\Delta r$ and $2\,\Delta p$ except at the corners, where uncentered differences over Δr and Δp were evaluated.

The vertical velocity field can be obtained from equation (15) in a coordinate system along the characteristic lines.

Equation (15) can be written in a finite difference form as follows:

$$\frac{(\omega_{i+1}-\omega_{i-1})}{2\Delta p} = -\frac{1}{r_i}\,\frac{(\omega_{i+1}+\omega_{i-1})}{2}\,\frac{\Delta}{2\Delta r}\left\{\left(\frac{a_{i+1}+a_{i-1}}{2}\right)r_i\right\} - \frac{1}{r_i}\,\frac{\Delta}{2\Delta r}\left\{\frac{(b_{i+1}+b_{i-1})}{2}\,r_i\right\} \qquad (18)$$

Equation (16) can be solved for ω_{i-1} in terms of ω_{i+1}.

$$\omega_{i-1}=\omega_{i+1}\left[\left\{\frac{1}{2\Delta p}-\frac{\Delta\left(\frac{a_{i+1}+a_{i-1}}{2}\right)r_i}{4r_i\Delta r}\right\}\Big/\left\{\frac{1}{2\Delta p}+\frac{\Delta(a_{i+1}+a_{i-1})r_i}{4r_i\Delta r}\right\}\right]$$
$$-\left[\left\{\frac{\frac{1}{r_i}\Delta\left(\frac{b_{i+1}+b_{i-1}}{2}\right)r_i}{2\Delta r}\right\}\Big/\left\{\frac{1}{2\Delta p}+\frac{\Delta(a_{i+1}+a_{i-1})\,r_i}{4r_i\Delta r}\right\}\right] \qquad (19)$$

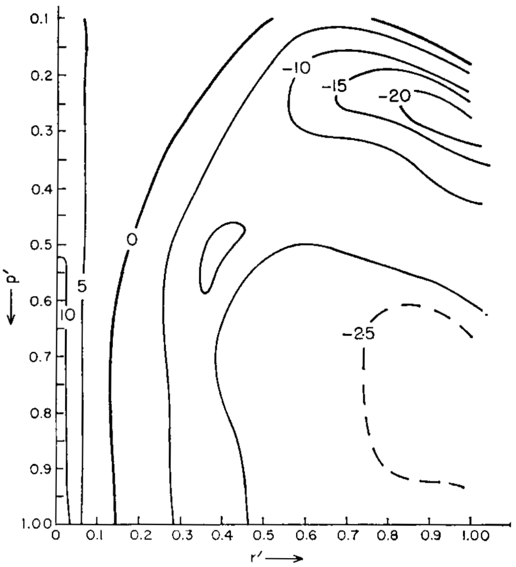

Fig. 5. Isolines of b the friction term in the ω-equation. Non-dimensional units.

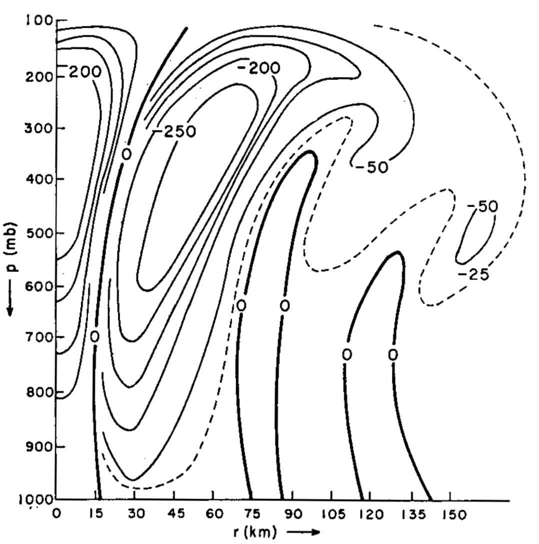

Fig. 6. Cross-section of vertical velocity in units of millibars per hour.

With $\omega = 0$ at p = 1,000 mbs, equation (17) may be used to obtain $\omega(r, p)$. It may be noted from equation (19) that for $b = 0$ we get $\omega = 0$ identically as was shown for equation (16).

The first calculation of the $\omega(r, p)$ field for $M = 1$ showed rather interesting features. *It may also be noted that as long as N is fixed* ($= 1$ *for instance*) *the field of* $\omega(r, p)$ *for different values of M will appear the same, though with a different strength.* The calculated field of $\omega(r, p)$ for $M = 1$ turned out to be of very great intensity with a maximum vertical velocity of 1,000 mbs per hour around the 400 mb surface. A choice of $M = 0.3$ was made as a next choice. Fig. 6 shows the field of $\omega(r, p)$ in units of mbs per hour.

The following features may be noted:

(i) Sinking motion near $r = 0$;

(ii) Very intense rising motion for r, around 25 km;

(iii) The magnitude of rising motion at $r = 150$ km is about an order less than that around $r = 25$ km;

(iv) The vertical velocity field shows a slope outward as the pressure decreases.

The boundary line between (i) and (ii), namely the $\omega = 0$ line, can be designated as the boundary of the eye wall, inside of which the air sinks. Very intense motion of (ii) may be designated as the eye wall. The air moves up generally except in two other regions. This calculation of the ω-field is quite interesting. The magnitudes of ω may not still be quite reasonable but, the pattern does seem to be.

In order to see more explicitly the role of the various terms of the ω-equation, a detailed calculation of the vertical velocity field at the 700 mb surface was made. The results are presented in Table 3. It may be observed that features like the eye and the eye wall are obtained by the inclusion of the lateral viscosity. The inclusion of the vertical viscosity coefficient gives rise to slow upward motion everywhere. The total vertical velocity at the 700 mb surface is thus primarily due to the lateral friction. It may be mentioned that Kuo (1959) had made this observation in his discussion of the dynamics of the hurricane eye.

The following dimensional values correspond to $M = 0.3$.

$\nu = 2.7 \times 10^9$ C.G.S. Units
$k = 1.2 \times 10^7$ C.G.S. Units

These magnitudes should not be taken literally as the values for a storm. The author believes that further experiments on a similar line

Table 3.

Vertical Velocity (Nondimensional units), Stead symmetric Hurricane Cleo at 700 Millibar

r (Km)	Radial Friction Only	Vertical Friction Only	Total
0	0.352	−0.0145	0.338
10	0.351	−0.0144	0.337
20	−0.401	−0.0211	−0.422
30	−0.416	−0.0192	−0.435
40	−0.399	−0.00758	−0.407
50	−0.118	−0.00313	−0.212
60	0.0104	−0.00284	0.00733
70	−0.00407	−0.00319	−0.00726
80	−0.00211	−0.00103	0.00314
90	−0.0176	−0.00360	−0.0212
100	−0.0475	−0.00373	−0.0512
110	−0.0459	−0.00889	−0.0548
120	0.00703	−0.00585	0.00117
130	0.0406	−0.00411	0.0365
140	0.00116	−0.0106	−0.00944
150	0.00116	−0.0106	−0.00944

should be carried out to understand the space variation of v and k.

We shall now use the $\omega(r, p)$ field of Fig. 6 and obtain corresponding fields of v, Z, T and H by the method outlined earlier.

c. *The radial velocity field and the Stokes stream function:*

It is convenient to depict the $\omega(r, p)$ field by a diagram of a Stokes stream function ψ, that satisfies the mass continuity equation. It may be defined by the following pair of equations:

$$\frac{\partial \psi}{\partial r} = \frac{2\pi}{g}\,\omega r$$

$$\frac{\partial \psi}{\partial p} = -\frac{2\pi}{g}\,v r$$

Then

$$\psi = \int \frac{\partial \psi}{\partial r}\,dr + \int \frac{\partial \psi}{\partial p}\,dp$$

with a choice of $\psi = 0$ at $r = 0$, $\psi(r, p)$ can be readily computed from the vertical velocity distribution. Fig. 7 shows the results of this calculation. We notice a deep inflow layer, an intense outflow in the upper troposphere. The sinking branch inside the eye being connected by a rising streamline. The magnitude of ω, in the sinking branch in the eye was quite large, but as we see from Fig. 7, actually it is a very small amount of mass that circulates inside the eye, this is because of the smaller area. At first sight it is hard to accept a deep inflow layer for a symmetric vortex with friction. Experiments on symmetric dishpan experiments have in fact demonstrated a shallow inflow layer, FALLER (1959). It must be remembered however that the above calculation was performed for k = constant. If k had been made to vary with pressure, a shallower inflow layer could have been obtained.

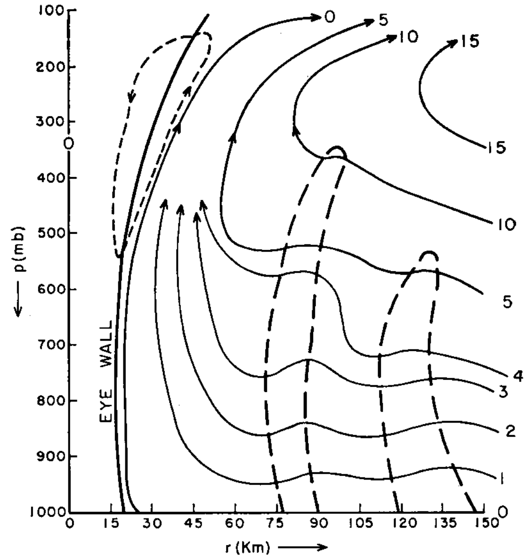

Fig. 7. Cross-section of the streamlines of Stokes stream function. Units 10^{+12} grams/sec.

In this connection, mention may be made to the presentation of the mean hurricane data by PFEFFER (1958) which shows a deep inflow layer and a shallow outflow layer. Observations of mean storm by PALMÉN and RIEHL (1957) and individual storms by various workers have shown that over a deep layer below the 500 mb surface the tangential wind u does not vary much with height, if one should impose $\frac{\partial u}{\partial p} = \frac{\partial^2 u}{\partial p^2} = 0$ rigorously in the equations of motion, then we would obtain $\frac{\partial v}{\partial p} = \frac{\partial^2 v}{\partial p^2} = 0$ and a deep inflow layer can be calculated for a steady disturbance.

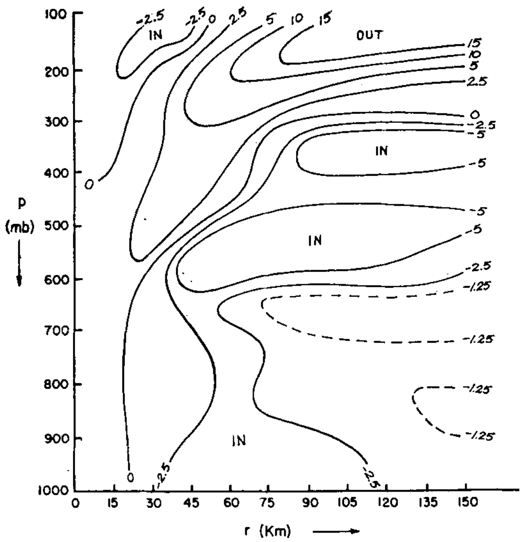

Fig. 8. Cross-section of radial velocity v in units of meters/sec.

Fig. 8 shows the corresponding distribution of v (r, p). The zero lines divide the inflow and outflow regions and possess the property mentioned above.

d. *The height field:*

Using $Z(L_0, p)$ from JORDAN's (1954) mean hurricane sounding for the outer boundary, the field of $Z(r, p)$ was calculated using the equation

$$Z(r,p) = Z(L_0,p) + \int \frac{1}{g}\left\{ v\frac{\partial v}{\partial r} + \omega\frac{\partial v}{\partial p} - \frac{u^2}{r} - \frac{2uf}{f_0} - \frac{\partial}{\partial r}\left[v\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)\right] - \frac{\partial}{\partial p}\left[k\frac{\partial v}{\partial p}\right]\right\} dr \quad (20)$$

The field of $Z(r, p)$ shown in Fig. 9 shows lower heights nearer the axis, this anomaly extends through the troposphere.

The height field along the 1,000 mb surface shows negative heights for $r \leq 45$ km, showing that, in this region the surface pressure would be less than 1,000 mbs. Since the true boundary condition at $r = L_0$ is not known, it is hard to estimate the central pressure of the storm. One can however see that the calculation has the correct sign, and appears to be very reasonable.

e. *The temperature field:*

Given Z (r, p), the temperature field T (r, p) can be evaluated from the hydrostatic relation, equation (8). Fig. 10 shows the results of the calculations. It is possible to obtain a warm core, and the increase of temperature from the outside $r = L_0$ to inside $r = 0$ is comparable to what was reported for Hurricane Daisy of 1958 by JORDAN (1960), from actual observations.

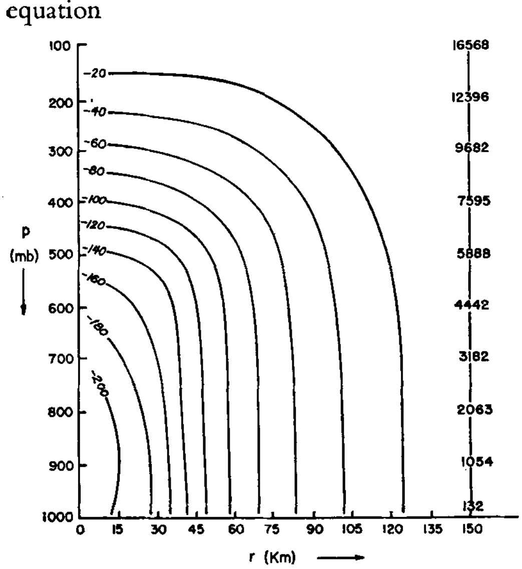

Fig. 9. Cross-section of height Z of constant pressure surfaces. Units meters.

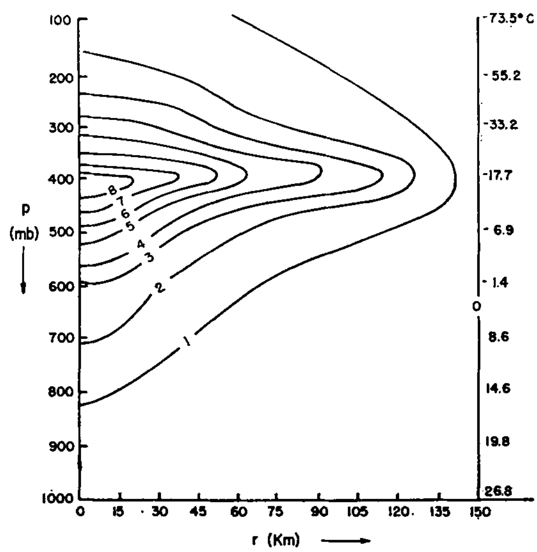

Fig. 10. Cross-section of temperature T. Units degrees Centigrade.

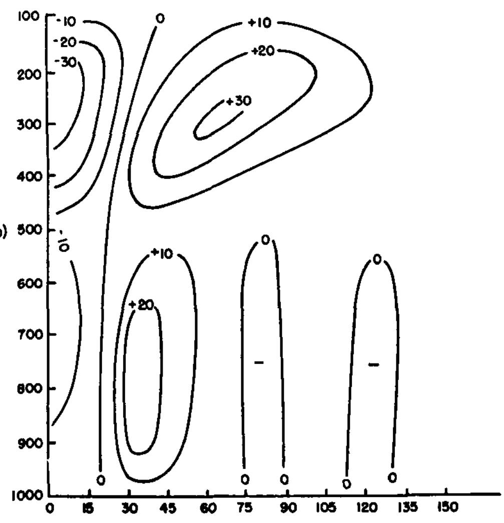

Fig. 11. Cross-section of non-adiabatic heating in units of 10^{-4} cal/(gm. sec.).

f. *The field of non-adiabatic heating:*

Equation (10) can be used to evaluate the heating, since we have the fields of v, T and ω. Fig. 11 shows the results of the calculation. We have succeeded in obtaining here the distribution of heating inversely by constructing consistent fields of various atmospheric variables. Such a field can be of great use in setting up the prescribed heating for initial-value problems.

In hurricanes the principle heat source is the release of latent heat. A rough comparison of the H-field with the release of latent heat can be made as follows: Along the moist-adiabats of a Tephi-gram, a computation of the change in specific humidity Δq for particles rising with the speed ω can be made. Then $L\frac{\Delta q}{\Delta t}$, the release of latent heat, should compare with H for a heat balance. In Table 4 a typical calculation is demonstrated. The balance is fairly reasonable, suggesting that this model which is based on a simple type of lateral and vertical friction coefficients gives a consistent picture of heating distribution.

The effect of the surface boundary layer has not appeared explicitly in the foregoing analysis. One must note that, in almost all the work on hurricanes much attention has been given to this layer. The following explanation may perhaps be in place here: The input winds have been influenced by surface friction and therefore this effect is in part included. As mentioned earlier, the magnitude of the vertical momentum exchange coefficient could have been made larger near the ground and decreasing with height. This would have been more realistic insofar as it would have provided a shallower inflow layer. It may however be noted that, in the framework of the Navier-Stokes equation presented here, the role of the vertical exchange coefficient appears to be important only as a means for providing an exchange of angular momentum with the ground. Above the friction layer its role may therefore be of insignificant value.

Table 4.

Nonadiabatic heating and the release of latent heat in the eye wall
Mean vertical velocity between 400 and 500 mbs $\omega = 250$ mb/hour
Change in specific humidity along moist adiabat $\Delta q = 3.2$ gm/kgm
Release of latent heat $L\frac{\Delta q}{\Delta t} = 12.9 \times 10^{-4}$ Cal/(gm. sec)
Computed heating $H = 15 \times 10^{-4}$ Cal/(gm. sec)

5. Conclusions

In this paper we have obtained the structure of a steady symmetric vortex, for a prescribed tangential wind velocity field. It was possible to show that the vertical velocity field in hurricanes is primarily determined by the inclusion of radial friction coefficient. The vertical momentum exchange coefficient gives rise to a very slow upward motion without any details. The choice of $N=1$, and $M=0.3$ is one of the many possibilities. Further work on similar lines should be done to understand the role of friction. This paper may be considered as a first attempt in this direction. A natural extension of such an approach would be the inclusion of asymmetries. Considerable difficulties arise in the formulation of the asymmetric system of closed equations. In a later paper the author will discuss this system, and an application of the model to the Atlantic Hurricanes.

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