Forecasting Thunderstorms and Showers by the Slice Method

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Abstract

The author gives a further development of the slice method proposed by J. Bjerknes in 1938. The formulae obtained use data from aerological soundings to compute the vertical thickness of convective clouds, their optimum amount, and the velocity of vertical development. The paper deals with the way of forecasting thunderstorms and showers based on the slice method.

Introduction

The slice method developed by J. BJERKNES (1938), S. PETTERSSEN (1939) and A. F. DUBUK (1945), permits computation of the energy resources of instability in the atmosphere spent on convection. Unlike the particle method, the slice method applied to the development of convective clouds takes into account both the amount of energy associated with the ascent of cloud air and the amount of energy spent on the compensating descent of air outside the clouds.

N. BEERS (1945) proposed a method of forecasting thunderstorms and showers based on the slice method. The method was tested by S. PETTERSSEN and others (1946) in England and by S. BANERJI (1950) in India.

Computations were based on the determination of mean kinetic energy per unit air mass participating in convection. Computations were made separately for dry and saturated slices, the saturated slices being defined as those with relative humidity of 70 % or more. However, the methods of computation were too inaccurate and the method of forecasting proposed by N. Beers gave unsatisfactory results.

The author proposes a new method of forecasting thunderstorms and showers on the basis of a certain development of the slice method SHISHKIN (1957). The method has been tested by many forecasting units of the Soviet Union and showed a high percentage of accuracy for the air mass thunderstorms and showers.

Theoretical Foundations of the Slice Method

J. BJERKNES (1938) established thermodynamic conditions for the development of convective clouds. He discussed an atmospheric layer containing clouds and cloudless air. The change in the heat content of the layer per unit time during the development of clouds is expressed by the formula

$$\frac{\partial Q}{\partial t} = C_p\left(M_d\frac{\partial T_d}{\partial t} + M_s\frac{\partial T_s}{\partial t}\right) \qquad (1)$$

where C_p is the specific heat of air at constant pressure, M_d and M_s – masses of dry and saturated air participating in circulation, T_d and T_s – temperatures of dry and saturated air, respectively. The right hand side of expression (1) may be transformed if we add and subtract the term $M_s\frac{\partial T_d}{\partial t}$. Then we obtain

$$\frac{\partial Q}{\partial t} = C_p\left[(M_d+M_s)\frac{\partial T_d}{\partial t} + M_s\left(\frac{\partial T_s}{\partial t} - \frac{\partial T_d}{\partial t}\right)\right] \qquad (2)$$

The first term in square brackets gives the change of heat energy in the slice due to the general warming (or cooling), while the second term shows excessive heating (or cooling) of cloud air, i.e. the heat energy of circulation.

When we take into account the law of conservation of mass

$$M_d v_d + M_s v_s = 0 \qquad (3)$$

and make a number of simplifying assumptions, we obtain the following expression for the change of heat energy of convection when the cloud air ascends through a slice ΔH with lapse-rate γ (SHISHKIN, 1959):

$$\Delta Q = C_p \bar{\varrho}\, S_s \left[(\gamma - \bar{\gamma}_s) - \frac{S_s}{S_d} (\gamma_d - \gamma) \right] (\Delta H)^2 \qquad (4)$$

or, for a unit mass of cloud air,

$$\Delta q = C_p \left[(\gamma - \bar{\gamma}_s) - \frac{S_s}{S_d} (\gamma_d - \gamma) \right] \Delta H \qquad (5)$$

In the above formulae $\bar{\gamma}_s$ is the mean value of saturated lapse-rate in the slice ΔH, γ_d – dry-adiabatic lapse-rate, S_s and S_d – amounts of clouds and cloudless air, respectively (10 tenths are considered a unit), $\bar{\varrho}$ – mean density of air in the slice. It is assumed that the lapse-rate γ is within the range $\bar{\gamma}_s < \gamma < \gamma_d$.

The right hand sides in formulae (4) and (5) are positive if the following condition is fulfilled:

$$\frac{S_s}{S_d} \leqslant \frac{\gamma - \bar{\gamma}_s}{\gamma_d - \gamma} \qquad (6)$$

The inequality (6) is called the Bjerknes criterion. It gives the condition for the development of convective clouds.

Let us now compute the work done when the development of convective clouds occurs in a slice with a lapse-rate γ.

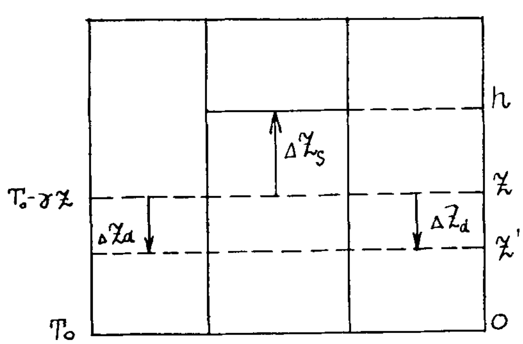

Fig. 1. Scheme of development of the convective cloud.

Fig. 1 shows schematically a small portion of the slice containing one convective cell. Here h is the vertical thickness of a cloud at a time t. When ascending, the cloud mass becomes warmer than the surrounding stable air. The ascending force related to the unit mass at the height z is:

$$f_s = g \frac{T_s - T}{T} \qquad (7)$$

where g is acceleration due to gravity, $T_s = T_0 - \gamma_s z$ is the temperature of the rising cloud air, $T = T_0 - \gamma z$ is the temperature of stable air at the height z, T_0 is the air temperature at the lower boundary of the slice. It is assumed that the air ascends adiabatically.

Substituting values T_s and T into (7) we obtain

$$f_s = \frac{g\,(\gamma - \gamma_s)\, z}{T_0 - \gamma z} \qquad (8)$$

By the time the upper boundary of the cloud reaches the height h, the following amount of cloud air will pass through the horizontal plane at the height z:

$$\Delta M_s = \varrho_s\, S_s\, (h - z)$$

where ϱ_s is the density of cloud air.

In accordance with the law of continuity, the same amount of dry air must descend. The thickness of the slice of dry air that has descended, Δz_d, may be determined from the condition

$$\varrho_s\, S_s\, (h - z) = \varrho_d\, S_d\, \Delta z_d$$

where ϱ_d is the density of dry air. Neglecting the difference between the densities of cloud air and dry air we obtain

$$\Delta z_d = \frac{S_s}{S_d} (h - z)$$

Let us now consider the way by which the compensating descent of dry air takes place. When the cloud air ascends due to buoyancy, the dry air situated above the cloud is forced to move upward. If the dry air ascends adiabatically it becomes colder than the stable air at the corresponding level. The motion is retarded and the ascending air is spreading out. Being colder, the dry air displaced aside begins to descend in the cloudless space. In its turn, the ascent of cloud air in the lower part of the slice promotes the development of a descending flux in the surrounding dry air. Thus a closed circulation appears.

If we do not take into account turbulent mixing of the air and other non-adiabatic processes, then the dry air which descends near the upper boundary of the cloud will return to its initial level, where it began ascending, with its same initial temperature. Thus, we may consider that the dry air which descends from a level z has an initial temperature $T = T_0 - \gamma z$.

At the level $z^1 = z - \Delta z_d$ when the descent is dry-adiabatic, the air has the temperature $T_d = T_0 - \gamma z + \gamma_d \Delta z_d$. The buoyancy force applied to a unit mass of dry air at the level z^1 (in relation to the stable air) will be therefore

$$f_d = g\,\frac{T_d - T}{T} = g\,\frac{(\gamma_d - \gamma)\,\Delta z_d}{T_o - \gamma\,(z - \Delta z_d)} \qquad (9)$$

This force, as well as the buoyancy force for the saturated air, is directed upward.

Comparison of formulae (8) and (9) shows that the correlation of buoyancy forces is different at different levels. Having made an equation of these expressions, we shall find the level where the buoyancy forces for the cloud and dry air are equal to one another:

$$Z_K = \frac{h}{1 + \dfrac{S_d}{S_s}\,\dfrac{\gamma - \gamma_s}{\gamma_d - \gamma_s}} \qquad (10)$$

We have $f_s > f_d$ above the critical level and $f_s < f_d$ below the critical level, if we assume that during the development of circulation the temperature at the lower boundary of the cloud remains constant. Thus, in the lower part of the slice retardation of circulation takes place during the development of the cloud.

However, it is not the relation between f_s and f_d that determines the development of circulation as a whole. The motion is determined by the integral action of buoyancy forces of all the masses participating in the circulation. The integral sums of the buoyancy forces affecting the cloud and dry air are:

$$F_s = \int f_s\, dM_s \quad , \quad F_d = \int f_d\, dM_d$$

Substitution of expressions (8) and (9) into the above expressions gives on integrating:

$$F_s = \frac{g}{2T_0}\,(\gamma - \bar{\gamma}_s)\,\bar{\varrho}\, S_s h^2,$$

$$F_d = \frac{g}{2T_0}\,(\gamma_d - \gamma)\,\bar{\varrho}\, S_s h^2 \qquad (11)$$

The work done by buoyancy forces in forming clouds ΔH thick is:

$$\Delta A = \int F_s\, dh_s + \int F_d\, dh_d$$

Since $dh_d = -\dfrac{S_s}{S_d}\,dh_s$ we obtain on integrating:

$$\Delta A = \frac{g\bar{\varrho} S_s}{6T_0}\left[(\gamma - \bar{\gamma}_s) - \frac{S_s}{S_d}(\gamma_d - \gamma)\right](\Delta H^3) \qquad (12)$$

Comparing this with (4) and taking account of $g \approx C_p \gamma_d$ we obtain the formula:

$$\Delta A = \frac{T_0 - T_d}{6T_0}\,\Delta Q \qquad (13)$$

as $\gamma_d \Delta H = T_0 - T_d$, where T_d is the temperature at the upper boundary of the layer of thickness ΔH, when the ascent is dry-adiabatic.

Consequently, the value

$$\eta = \frac{T_0 - T_d}{6T_0} \qquad (14)$$

is the coefficient of transformation of heat energy into kinetic energy. It has been obtained under the assumption that at the given level the vertical velocity of the cloud is uniform. The same assumption is used for the velocity of the compensating motion of dry air.

If we assume that the distribution of velocities of vertical motions in convective clouds and in the environment is approximately the same as with regulated convection, we may then use the following relation: mean density of heat flux in a convective cell is equal to 1/6 of density of heat flux in the centre of the cell (Shishkin, 1954).

Thus, if the ascending flux is the same along all its cross-section as in the central part of the cells and if the velocity of the descending flux be correspondingly greater, then the work done would be six times greater.

The maximum coefficient of transformation of heat energy of convection into kinetic energy is:

$$\eta = \frac{T_0 - T_d}{T_0} \qquad (15)$$

Let us now find the velocity of development of convective cloudiness.

The amount of work done in developing the convective cloud ΔH thick may be considered equal to the kinetic energy of circulation:

$$E_{KIN.} = M_s \frac{v_s^2}{2} + M_d \frac{v_d^2}{2}$$

Using the continuity equation (3) we obtain:

$$v_d = -\frac{M_s}{M_d} v_s$$

If we assume the clouds to be of cylindrical shape and make simple transformations we obtain the expression:

$$E_{KIN.} = \bar{\varrho} \frac{S_s}{S_d} \frac{v_s^2}{2} \Delta H \tag{16}$$

Equating the right hand sides of (12) and (16) we obtain the values for the kinetic energy of unit cloud mass at the moment when the thickness of the cloud reaches the value ΔH:

$$\frac{v_s^2}{2} = \frac{g S_d}{6 T_0} \left[(\gamma - \bar{\gamma}_s) - \frac{S_s}{S_d} (\gamma_d - \gamma) \right] (\Delta H)^2 \tag{17}$$

Equation (17) leads to the condition of development of convective clouds (6) obtained by J. Bjerknes.

Allowing for the energy expended in lifting dry air above developing clouds, the condition of appearance of convective clouds becomes

$$\gamma \geqslant \frac{\gamma_s + \gamma_d}{2} \tag{18}$$

For practical purposes it is convenient to introduce a substitution in (17):

$$(\gamma - \bar{\gamma}_s) \Delta H = T_s - T, \quad (\gamma_d - \gamma) \Delta H = T - T_d,$$

where T is the temperature of stable air at the level ΔH. T_s and T_d are temperatures of the dry- and saturated-adiabatically ascending air, respectively, when the ascent occurs from the lower boundary up to the level ΔH.

Considering $S_d = 1 - S_s$ and replacing $g = C_p \gamma_d$, we ultimately obtain:

$$\frac{v_s^2}{2} = C_p \left[(T_s - T) - S_s (T_s - T_d) \right] \frac{T_0 - T_d}{6 T_0} \tag{19}$$

When the cloud is developing through several atmospheric layers with different values of the lapse-rate γ_k, it is necessary to sum up the changes of heat energy for the unit cloud mass:

$$\Delta q_K = C_p \left[(T_s - T)_K - S_s (T_s - T_d)_K \right]$$

and to multiply the result by the coefficient of transformation of heat energy into kinetic energy:

$$\eta_n = \frac{T_0 - Td_n}{6 T_0}$$

where Td_n is the air temperature at the upper boundary of the layer n when dry-adiabatic ascent from the condensation level occurs.

We obtain:

$$\Delta \left(\frac{v_s^2}{2} \right) = \frac{T_0 - T_{dn}}{6 T_0} \sum_{K=I}^{n} C_p \left[(T_s - T)_K - S_s \cdot (T_s - T_d)_K \right] \tag{20}$$

This is an approximate formula since it does not take an accurate account of the correlation between temperatures of the ascending and descending air when convection develops through several atmospheric layers.

In order to compute the velocities of vertical motions in clouds it is necessary to make the sums by slices beginning from the condensation level. The conditions for development of clouds through several atmospheric layers are most favourable in the case when maximum amount of kinetic energy is guaranteed.

In order to find out the corresponding amount of cloud (which may be termed the optimum amount), we have to determine the variation of the work done, given by equation (12), as S_s varies.

Assuming the derivatives to be zero we obtain the optimum amount of clouds:

$$S_0 = 1 - \sqrt{\frac{T - T_d}{T_s - T_d}} \tag{21}$$

When convective clouds develop through several atmospheric layers,

$$S_0 = 1 - \sqrt{\frac{\sum_K (T - T_d)_K}{\sum_K (T_s - T_d)_K}} \tag{22}$$

Substitution of $S_s = S_0$ into (20) gives the velocities of the ascending air motions in convective clouds under the optimum condi-

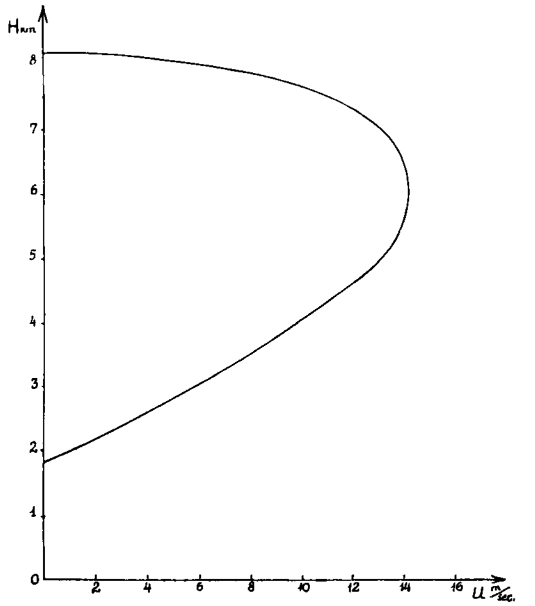

Fig. 2. Change of velocity of ascending flux in clouds with altitude on Juni 6, 1959, after computation data.

tions of development. Fig. 2 shows the computed change of velocities of the ascending motions in convective clouds with height. The data used refer to the morning radio-souding in Ruispiri (East Georgia, U.S.S.R.) on June 6th, 1959. We can see that the velocity could reach the value of 14 m/sec at the height of 6 km.

The optimum amount of clouds varies for clouds of different vertical thickness, i.e. for clouds in different stages of development.

In the case when it changes little with height, conditions are favourable for the continuous development of the same convective clouds. When S_0 changes with height by a factor of two or more, the development of some clouds promotes the dissipation of others. Therefore, under such conditions, cloud growth proceeds in a rather unstable manner: some clouds are intensively developing while others are rapidly dissipating.

With an unstable atmospheric stratification, the velocity of descending motions in those convective clouds which are dissipating, may also be computed using formula (20). In this the initial value of T_0 should be taken as the temperature at the upper boundary of the dissipating clouds. The computation of velocity is carried out by slices from the cloud top to the cloud base.

Computation of Vertical Thickness of Convective Clouds

Two necessary conditions for the development of convective clouds are the accumulation of energy of atmospheric instability and sufficient humidity. The energy released by convection may be computed by summing the expression on the right hand side of formula (20) for each slice. Different empirical rules are used to evaluate the moisture supply; some of them will be treated later on.

The amount of convective clouds is a value that changes greatly with time. With sufficient air humidity it depends mainly on the lapse-rate in a layer, several hundred meters thick, above condensation level. In order to solve the problem of forecasting convective cloudiness in the most general way, we may compute the cloud thickness by setting the amount of cloud in (20) equal to 1, 2, 3 ... tenths, i.e. by giving S_s the values 0.1, 0.2, 0.3 ... Thus we can find the possible thickness of clouds for different cloud amounts, given the temperature stratification.

Let us now consider the problem of forecasting the diurnal variation of convective cloud from the morning aerological sounding.

The basic data for the computation are as follows: (1) data of aerological sounding[1], (2) daily forecast of maximum temperature at the ground. The forecast of dew point for the period of maximum development of clouds is also desirable. When it is not available, forecasters may use the actual value of the dew point near the ground at the moment of forecasting or the maximum value of the dew point (taken, as a rule, either at the ground or at the upper point of ground inversion) from the data of the morning aerological sounding.

The condensation level is given by the point of intersection of the specific humidity curve corresponding to the chosen dew point and the dry-adiabatic curve passing through the forecast ground temperature.

[1] It should be considered that the aerological sounding made during rain or soon after it does not give representative data on the lapse-rate. It should not be used in this work.

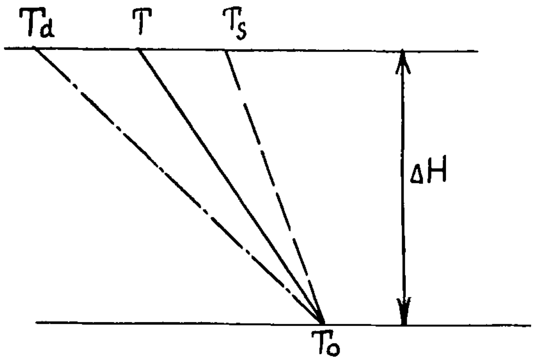

Fig. 3. Scheme of plotting on the aerological diagram.

If the point of intersection of these two curves lies to the left of the stratification curve by an amount exceeding some critical value (for the temperate climatic zone of the U.S.S.R. this critical value ranges from 3° C to 5° C), then the humidity is not sufficient to produce convective clouds. When humidity is sufficient, suitable slices for computation are chosen in accordance with points of inflection of the strafitication curve. Slices 50 – 100 mb thick prove to be the most expedient. To simplify the computation it is convenient to have an additional plotting on the aerological diagram: let us draw the dry and saturated adiabatic curves through the point on the temperature stratification curve at the slice base up to the slice top (see fig. 3). The values $(T_s - T)$ and $(T_s - T_d)$ can be measured off the diagram.

In order to determine the possible vertical thickness of clouds when their amount is S_s, we must sum up the expressions (20) from the condensation level to the height where the sum becomes zero[1]. This will be the level of the cloud upper boundary at maximum development.

Summing up the values $(T_s - T)$ by slices beginning with the condensation level, we obtain the greatest possible thickness of convective clouds when their amount is small $(S_s \to 0)$.

A comparision of computed maximum thicknesses of thunderstorm and shower clouds with a large number of the actually observed cases shows that the average computation error does not exceed 0.2—0.3 km.

For convective clouds of small thickness the difference between the computed and observed data is usually much smaller.

Forecasting of Thunderstorm and Showers

The computation data for vertical thickness of convective clouds may be used for the forecast of thunderstorms and showers.

According to the data of aircraft studies of convective clouds carried out over a few years at the Main Geophysical Observatory, U.S.S.R. the minimum vertical thickness of shower clouds for temperate zone summers is 2.2 km, while the minimum thickness of thunderstorm clouds is 4.0 km. We can determine empirically some critical values of the computed vertical thickness above which thunderstorms and showers may occur. The values may be different in various geographical conditions. Thus, for the zones of moderate and subtropical climate the critical values of thickness for shower clouds are likely to range between 2.2 km and 3.2 km[2] while for thunderstorm clouds they range between 4.0 km and 4.5 km. Even brief experience allows one to specify these criteria for particular districts.

Application of this method in the U.S.S.R. shows that the forecast of thunderstorms and showers is best justified for territories of 100 – 200 km radius. The forecast can be made for the day following the morning aerological sounding (or for a night after the evening sounding) performed at a station within the forecast territory. The range of forecasting may be extended to 1/2—1 day it we use the data of the evening or morning aerological soundings performed on the previous day at an upwind station within the same air mass.

Apart from the alternative forecast of thunderstorms and showers, the computation gives the portion of the territory where thunderstorms and showers may be expected. As a result of computation we know the amount of cloud when their thickness exceeds the critical values for thunderstroms and showers. It should be stressed that the results obtained do

[1] If the sum of terms in a slice is negative and in higher slices becomes positive again, then, as a rule, the upper boundary of clouds is considered the highest level where the sum becomes zero.

[2] If the computed upper boundary of convective cloudiness reaches —10°, then showers may be forecast for thickness of clouds less than 2.2 km.

not correspond to the amount of cloud seen from one station but give the relative proportion of the forecast area covered by clouds of super-critical depth. Thus, one tenth of cloudiness of thickness more than 4.5 km means that the thunderstorm cloudiness can develop over 10 % of the territory. In this case thunderstorms could be detected at 10 % of stations if there were no displacement of clouds. Forecasting experience shows that on account of cloud motion the calculated percentage of territory with thunderstorms and showers must be multiplied by an empirical coefficient 1.5—2.5 that varies in different geographical conditions. For the plains on the U.S.S.R. territory the coefficient is 2.0—2.5 while for the mountainous regions it is 1.5—2.0.

The mean error in forecasting the percentage of territory with thunderstorms and showers was $\pm$ 12 %.

The following empirical rules are used to take into account air humidity in the free atmosphere in the process of forecasting thunderstorms and showers:

1. Thunderstorms are not forecast if the total deficit of dew point[1] at the levels of 850, 700 and 500 mb exceeds some critical value (in temperate zone of the U.S.S.R. it various from 26° to 30°).

2. Showers are not forecast if the total deficit of dew point at the levels of 850 and 700 mb exceeds some critical value (in the above zone it varies from 18° to 20°).

If the test shows that the air humidity is small, computation may be omitted.

Test of the described method was made for a considerable part of the territory of the U.S.S.R. in 1956—57. During that period over 1,500 experimental forecasts were made. On the average, forecasts proved to be correct in 92 % of cases for air mass thunderstorms and showers, when the daily forecasts were based on data of the morning aerological soundings of the same day. For the extended forecasts the verification is somewhat poorer due to additional errors in forecasting air mass transfer and to neglecting air mass transformation. For the frontal thunderstorms and showers the verification of forecasts was 70—80 %.

Greatest success has been achieved for territories of the interior continental plains (up to 97 %). For the shores of seas and oceans the method gives good results when the air mass is transported from the land. When the air mass is transported from the sea, a considerable transformation of air during a day occurs and the method gives bad results. In mountainous regions the development of convective clouds has a number of peculiarities.

Not infrequently they develop over mountains even when the temperature stratification is stable in the lower layers of the air above an adjoining plain or a valley where clouds either do not develop at all or are of small thickness.

When forecasting, account of this phenomenon may be taken in the following way: if in a slice situated above the condensation level the lapse-rate is less than the saturated-adiabatic value ($\gamma < \gamma_s$), while higher the stratification becomes saturated-unstable, computation is made beginning with the lower boundary of the unstable layer. In this case the probable vertical thickness of convective clouds is found as the difference between the calculated upper boundary of clouds and the lower boundary of the unstable layer.

In mountainous regions the verification of forecasts is somewhat lower than it is for the plains (82—90 % for the air mass conditions).

Zones of thunderstorms and showers may be specified more accurately by interpolating values of maximum thickness after the data of several aerological stations.

Computation of one aerological sounding takes only 10—15 minuts. Thus, the proposed method may be applied in operational practice when forecasting thunderstorms and showers for the warmer part of the year.

[1] Deficit of dew point is the difference between actual air temperature and dew point.

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