Radiation to Actinometric Receivers in Its Dependence on Aperture Conditions

By ANDERS ÅNGSTRÖM, The Eppley Foundation for Research, Newport R.I., U.S.A.

(Manuscript received May 5, 1961)

Abstract

After an introduction on basic conceptions, concerning the influence of aperture conditions on the readings of actinometers, a short review is given on recent results as regards the circumsolar sky radiation. The combined effect of aperture and sky radiation on pyrheliometric readings, in the case of point receivers, is discussed. Conclusions are drawn as regards the influence of aperture conditions on (a) solar constant determinations and (b) on the difference between readings with aid of different pyrheliometers. The principles which ought to be applied in the construction of pyrheliometers for different kind of use, are briefly discussed.

1. When a pyrheliometer is pointed at the sun for measurement of the sun radiation, it takes in not only the direct radiation from the disk of the sun but also that from a portion of the sky in its immediate vicinity. The amount of this circumsolar sky radiation is dependent on the scattering properties of the atmosphere, and also on the geometrical construction of the receiver and of the pyrheliometer tube. The construction problem is often referred to as a question of the *opening angle* or the *aperture* of the instrument. With the aperture we understand, for an instrument with circular diaphragms, the top angle of the cone within which radiation from without is able to reach the central point of the receiver. The aperture can be defined through the radius R of the most narrow diaphragm, active in the instrument, and the distance (l) from its center to the center of the receiver (see fig. 1). If the aperture angle is denoted by φ, we have:

$$\operatorname{tg}\frac{\varphi}{2}=\frac{R}{l} \tag{1}$$

If the instrument is directed towards a surface of uniform radiation intensity I, covering the whole aperture angle, the radiation reaching the central point of the receiver and referred to a unit of surface is given by

$$\pi I \sin^2\frac{\varphi}{2} \tag{2}$$

If we, with Dubois, define the "surface angle ratio" (the Flächenwinkelverhältnis) ψ, as the ratio between what a receiver, whose aperture is limited by diaphragms, measures and what is measured at an unlimited exposure to the whole hemisphere of the same instrument, the named ratio Ψ, in the case that the receiver is concentrated in a point, situated on the axis through the center of the diaphragm is, given by:

$$\psi=\sin^2\frac{\varphi}{2} \tag{3}$$

This is an ideal case which only seldom is realized in the actual actinometers used for field measurements. In general the receiver has

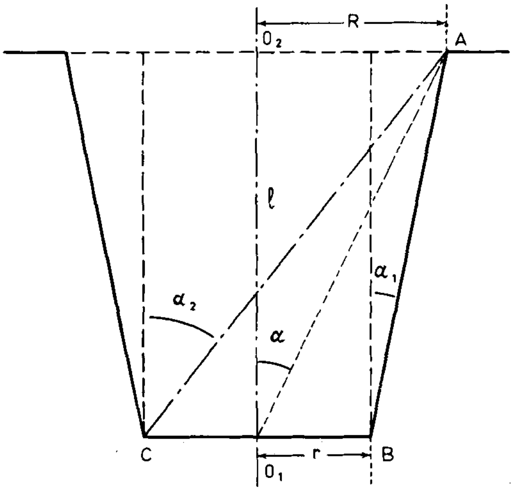

Fig. 1. Actinometer with circular diaphragm (schematic)
R = radius of diaphragm, r = radius of receiver,
l = distance between both α = opening angle $\left(\frac{\varphi}{2}\right)$
α_1 = slope angle, α_2 = limit angle.

dimensions so large that the laws derived above are not strictly applicable. An important case, first treated by J. H. LAMBERT (1760) is the following.

We consider an actinometer with circular receiver and circular diaphragms, schematically represented in Fig. 1; R is here the radius of the diaphragm, r the radius of the receiver and l the distance between diaphragm and receiver. If the instrument is pointed at a source with uniform luminosity, I, the problem of finding the radiation falling on the receiver is that of determining the radiation exchange between two circular surfaces, parallel with one another and with their centers on an axis perpendicular to them both. Lambert derived for the radiation Q falling through the diaphragm on the receiver the following expression:

$$Q = \frac{\pi^2}{4} I (AC - AB)^2 \tag{4}$$

where AC and AB have the meaning apparent from the figure. Using the geometrical relationships evident from fig. 1, we obtain from equation (4) for the average radiation Q_1, falling on the unit of surface of the receiver:

$$Q_1 = \pi I \frac{R^2}{\left(\frac{AC + AB}{2}\right)^2} = \pi I \psi \tag{5}$$

Here evidently:

$$\psi = \frac{R^2}{\left(\frac{AC + AB}{2}\right)^2}$$

where ψ is the surface angle ratio. For an instrument with the receiver concentrated at a point, we have $AC = AB$, and the equation (5) then coincides with equation (2), which thus represents the limit conditions of (5).

The equations presented above have a *basic interest* when we consider the results of measurements with actinometers used for observations of the radiation from limited parts of a sky of comparatively uniform radiation power. The practical limitation of the applicability of the formula (4) of Lambert and of equation (5) derived from it, is due chiefley to the fact that in most practical cases the receiver has not a uniform sensitivity for the radiation falling upon it. A simple case is that of the silver-disk pyrheliometer where a flat cylindrical disk is heated by radiation, the temperature being measured at the central part of the disk. It is evident that, if we apply the usual prescribed procedure of reading the pyrheliometer and let a given amount of radiation fall on different parts of the disk, we will in general, if we apply a given constant to the readings, get different values for the radiation. The values will vary with the distance of the illuminated spot from the center. If we introduce a function $F(r)$ for the sensitivity in its dependence on the distance r from the center of the receiver, we are able to modify the law of Lambert, and derive formulas valid for actinometers of various construction. L. BOSSY and R. PASTIELS (1948, 1959) have, in papers of rather fundamental importance, treated this subject in a strict mathematical way and shown how the theory may be applied to practical cases. In many cases actually occurring, however, an approximation on the basis of the original formula of Lambert seems to give sufficient accuracy. LINKE (1931) has suggested the use of two parameters a and b, which he calls the *constants of normalization*, for defining the aperture conditions of actinometers with circular diaphragms. With the indications used above we have: $a = \frac{R}{r}$ and $b = \frac{l}{r}$, and consequently:

$$\frac{a}{b} = \frac{R}{l} = \operatorname{tg} \frac{\varphi}{2} \qquad (6)$$

Linke's constants of normalization may, if R and r, and R_1 and r_1 are used to denote the half length and half width of rectangular diaphragms, respectively receivers, be applied also to instruments of a rectangular construction. In this case we get two sets of constants of normalization, namely $a = \frac{R}{l}$, $b = \frac{l}{r}$ and $a_1 = \frac{R_1}{r,}$ and $b_1 = \frac{l_1}{r,}$.

For characterizing the aperture conditions we get two aperture angles defined by:

$$\operatorname{tg} \frac{\varphi}{2} = \frac{a}{b} \text{ and } \operatorname{tg} \frac{\varphi_1}{2} = \frac{a_1}{b_1} \qquad (7)$$

A computation of the radiation falling on the rectangular receiver leads to rather complicated formulas, if great accuracy is attempted. (See Bossy and Pastiels, loc. cit.)

2. When we consider the case of the *pyrheliometer* and ask how the readings are related to the energy falling on the receiver and to the radiation which we intend to measure, account must be taken not only of the aperture conditions, the dimensions and sensitivity function of the receiver, but also of the fact that the radiation to be measured is not uniformly distributed over the space covered by the aperture. The circumsolar sky covered by the aperture has naturally a much smaller luminosity than the sun's disk, but on the other hand it extends for most pyrheliometers over a solid angle about 50 times larger than the solar disk itself. The luminosity of the circumsolar sky has been subjected to a number of studies, among which here may be mentioned those of Dorno and Thilenius, of F. Linke and Dubois, of F. Linke and E. Ulmitz, of D. Stranz, of F. Volz, E. V. Piaskovskaia-Fenkova and of L. Bossy and R. Pastiels. Linke and Ulmitz (1940) expressed the radiation from the circumsolar sky through the equation:

$$I = kE_0\, e^{-c\alpha} \qquad (8)$$

where E_0 is the intensity at the center of the sun and α is the distance from the center in degrees. For the constant k Linke and Ulmitz gave a value which according to Bossy and Pastiels (1948) is about 50 times too high, due to an error in the calculation. Using the measurements of Linke and Ulmitz, Bossy and Pastiels derive for k a value of 0.000479 (instead of 0.0235) and thus arrive at the formula:

$$I = 0.00048 \cdot E_0\, e^{-0.58\,\alpha} \qquad (9)$$

On the basis of the formula (8) and with introduction of the constants of formula (9), Bossy and Pastiels have computed the energy of the circumsolar radiation entering the pyrheliometers of the Smithsonian Institution (Silver Disk) and of Linke-Feussner. In this computation they have taken into consideration also the sensitivity distribution of the receiver. The introduction of the last-named factor gives results which generally deviate with up to about 10 per cent from the value for the circumsolar radiation obtained under the assumption that the receiving element is concentrated in a point (a point-receiver). Now, as we will discuss further below, the formulas (8) and (9) give only a very approximate idea of the distribution of the circumsolar radiation. Not only are, as already was emphasized by Linke and Ulmitz, the constants k and c variable to the extent that they in special cases are more than the double or less than the half of those presented, but also the form of the equation (8) itself represents an ideal case, which only seldom is satisfied. In many cases it ought to be supplemented by a factor, showing maxima and minima with the distance from the sun.

It seems therefore justified, for practical purposes, to simplify the computations and to base a general consideration of the effects of aperture conditions on the simple case of a point receiver situated within pyrheliometers of different apertures. For the conclusions aimed at *here* such a limitation seems fully satisfactory.

The total circumsolar sky radiation D falling on a point receiver, the aperture being $\varphi = 2\alpha$, is given by:

$$D = 2\pi \int_{n=1}^{n} Ir dr \cos\alpha \qquad (10)$$

where r is the distance from the center of the sun, with the radius of the sun as unit. For the small angles α here in question ($\alpha < 5°$), we may put $r = 3.72\,\alpha$. A further simplification, here justified, is arrived at by replacing cos α

by 1.00. Both these approximations introduce an error less than one per cent in D, and are justified with regard to the fact that the constants of equations (9), as we will see, must be subjected to rather great variations from case to case.

An integration of (10), after these approximations, gives:

$$D = 0.0030 \left[\frac{e^{-\gamma}}{\gamma} \left(1 + \frac{1}{\gamma} \right) - \frac{e^{-\gamma r}}{\gamma} \left(r + \frac{1}{\gamma} \right) \right] E_0 \quad (11)$$

As the radiation Q from the whole solar disk, on the basis of values available, may be approximated to $0.8\, \pi E_0$, we get for the ratio: $\varepsilon = \dfrac{D}{Q}$

$$\varepsilon = 0.0012 \left[\frac{e^{-\gamma}}{\gamma} \left(1 + \frac{1}{\gamma} \right) - \frac{e^{-\gamma r}}{\gamma} \left(r + \frac{1}{\gamma} \right) \right] \quad (12)$$

Here γ has, in the case corresponding to the observations of Linke and Ulmitz, a value:

$$\gamma = \frac{c}{3.72} = \frac{0.58}{3.72} = 0.156$$

The following table gives the value of ε for different values of φ, $\left(\varphi = 2\alpha = 2 \cdot \dfrac{r}{3.72}\right)$, and for three different values of γ:

Table 1

γ \ φ	0	2°	4°	6°	8°
0.100	0	0.0050	0.020	0.035	0.051
0.156	0	0.0045	0.015	0.024	0.032
0.200	0	0.0040	0.013	0.019	0.023

Graphically the dependence of ε on the aperture angle (2α), in the case of a point receiver, is shown in fig. 2.

It is evident already from a superficial consideration of the process of scattering that the equation (9) can be valid only within a rather limited range. With the constants given, the equation (9) represents, as was emphasized by Linke and Ulmitz, mean values, from which the individual values deviate down to the half or up to the double. Linke and Ulmitz found for the ratio $\dfrac{D}{Q}$ values which were related to the product of air mass and turbidity (T) by an approximately linear relationship (for $\alpha < 5°$). This result has been confirmed by Volz, who also, as it is to be expected, finds an increase of ε with a decrease in wavelength. The turbidity factor T, which includes also the effect of water vapor absorption on the extinction, is not, however, a satisfactory measure of the scattering power of the atmosphere. W. SCHÜEPP (1949) has treated statistically a rather large mass of observations from Davos taken (1) with the aid of a pyrheliograph with about 2° aperture and (2) by means of a Michelson actinometer with a rectangular opening of 6° × 15°. On the basis of comparisons between both sets of values, Schüepp derives an expression for the percentage difference, given as a function of the product of the turbidity coefficient and the air mass. Translating from the turbidity coefficient B of Schüepp to the coefficient β, employed in the present paper, we obtain:

$$\varepsilon = 0.315\, m\, \beta \quad (13)$$

where m is the air mass. Assuming $m = 1.2$ and $\beta = 0.08$, which may reasonably apply to the average conditions under which the measurements of Linke and Ulmitz were made, we obtain $\varepsilon = 3.0$ per cent, a value which comes very near to the 2.8 per cent derived from the diagram (fig. 2) for the difference corresponding to actinometers with apertures of 8° and 2° respectively.

KALITIN (1933 and 1937) found that the sky luminosity was not subjected to a continuous decrease with an increasing distance from the sun, but showed minima at about 2°.5 and 4°.5 distance; LINKE and ULMITZ (1940) found

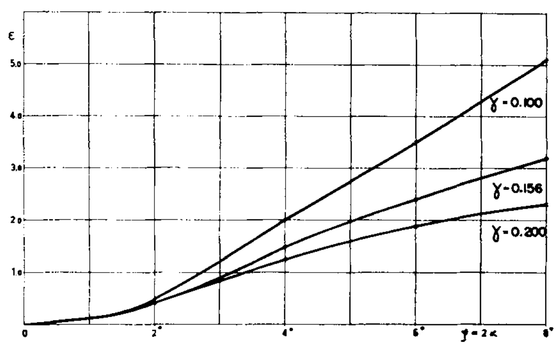

Fig. 2. Percentage increase, ε, of observed solar radiation, if measured by means of pyrheliometers (point receivers) with different apertures, φ. The coefficient γ is a measure of the decrease of circumsolar sky radiation with the distance from the sun.

Table 2. Apertures of pyrheliometers.

Pyrheliometer	aperture[1]
K. Ångström comp. pyrh.	$5° \times 13°$
Silverdisk	$6°$
Michelson	$5° \times 13°$
Moll-Gorzynski	$8°$
Linke-Feussner	$11°$
Eppley normal incid.	$5.7°$

[1] From IGY Instruction Manual, Vol. V, Part VI, Radiation Instruments and Measurements, Pergamon Press. 1958.

the decrease to turn into an increase at $6°.5$ distance from the sun. This is in conformity also with the results of D. Stranz (1940, 1942), who states that the scattered radiation from the sky, in the neighbourhood of the sun, is not, under many conditions, a simple decreasing function of the angular distance, but shows secondary maxima and minima, the location of which are dependent mainly on the nature and size of the scattering particles of the aerosole.

Recently G. A. Newkirk (1956) has made a photometric study of the solar aureole. No integral or absolute values are directly derived for the circumsolar radiation, but it may be concluded from his discussion, that, accepting the empirical formula (8), we must assume k as well as c to be dependent of the amount and character of the scattering particles of the atmosphere.

The conclusion must be that the values for the circumsolar sky radiation given in table 1 and fig. 2, can only represent average conditions, limited to a clear sky, a moderate or low turbidity, and a solar elevation corresponding to relative air masses less than about 2.5. At a solar elevation of, for instance 15°, the circumsolar sky radiation entering an actinometer of the type used in practical field work, may easily rise to about 10 per cent or even more.

3. Practical Considerations

It may be concluded from the previous discussion that, in the operation of the common pyrheliometers, K. Ångström, Silverdisk, Linke-Feussner, Eppley, Michelson, etc., we are never perfectly free from the effect of an additional sky radiation. Practically we may get rid of it by making the aperture less than about one degree, but such a construction has very evident practical drawbacks and we then must consider more closely what is to be gained by making the aperture very small.

a. *Solar constant determinations*

As the amount of circumsolar sky radiation is rather small and approaches zero in a linear way when the air mass decreases to zero, it seems evident that it can not be expected to influence to an appreciable degree the solar constant value, obtained through the so-called "long method". As this method is the basis also of the "short method" of Abbot it seems reasonable to assume that aperture effects do not introduce an appreciable error into solar constant determinations.

The error, which may sometimes occur from this cause, must in any rate be small compared with the error occasionally due to *variations in the atmospheric transmission* during the time covered by the set of radiation measurements. Such a change in the transmission must evidently affect the variation of Q as well as D and leads, when the values are extrapolated to air mass 0, to an error dependent on the variation of both the named quantities. Under conditions of a very clear atmosphere, as on high mountain tops in dry climates, and in cases where the aperture is smaller than 8°, the named error probably seldom amounts to more than a small fraction of 1 per cent.

b. *Standard scales*

As regards the difference between the standard scales, the Ångström and the Smithsonian, a slight effect of differences in aperture conditions may reasonably be suspected. On the other hand no influence of the air mass or solar elevation on the difference of the scales has, so far, been shown to exist, which ought to have been the case if the difference of the scales was due mainly to this cause. P. Courvoisier (1957) has presented a fundamental discussion on this question. It seems therefore probable, that for the average distribution of circumsolar sky radiation, the effective surface angle ratios of the pyrheliometers, which have represented the scales, are so closely equal as to eliminate differences occurring on this account[1]. It seems more probable that the variations of the difference between the two scales, which undoubtedly have been obtained on different occasions (about 3.5 ± 1.0 per cent), are due to other causes (ÅNGSTRÖM 1958).

If we with the true pyrheliometric scale understand the values Q_0 given by an exact pyrheliometer, which measures the direct sun radiation only, without any additional sky radiation, we must realize, that there is no possibility whatsoever to derive a strict evaluation of this scale simply from measurements with pyrheliometers characterized by aperture angles like those generally used in practical work (Ångström, Silverdisk, Michelson, etc. pyrheliometers). The measured value is here always given by:

$$Q = Q_0 + D(m, \beta, \lambda, \varphi) \tag{14}$$

where the circumsolar term D is a function of air mass m, turbidity β, wavelength λ and aperture φ, as the chief variables. We may guess at the value of D from measurements like those of Linke and Ulmitz, but as long as D is not directly measured at the instant of observing Q, and as the function D is not accurately known, all speculations as regards the differences between observed values and those of the true solar radiation itself are rather fruitless, at least if they aim at an accuracy greater than $\pm$ 1.0 per cent.

If we observe the radiation with, let us say, an Ångström pyrheliometer or a silver disk, the true value of the solar radiation *alone* may be between 1.0 and up to 5.0 per cent lower, dependent upon atmospheric and other conditions determining the variables in equation (14). Comparing two pyrheliometers characterized by different apertures, they may thus differ with one value today and another value tomorrow dependent upon the named variables. Strictly, therefore the true pyrheliometric scale value can never be given through applying a fixed constant percentage correction to the values obtained with pyrheliometers with aperture angles larger than a couple of degrees.

[1] In the case of the K. ÅNGSTRÖM compensation pyrheliometer the circumsolar sky radiation falling on the strip illuminated by the sun is partly compensated by the sky radiation reaching the screened strip. In view of the occurrence of secondary maxima, as mentioned above, it seems even possible that under extreme conditions an overcompensation may take place.

Practical average values for the correction may however be derived. For greater accuracy, when in rare cases a need for such occurs, we are dependent on a clearer knowledge of the function D, or preferably on measurements with more specialized instruments.

c. *Practical use*

In discussing the aperture effect and the question in regard to the desirability of its elimination, we must consider also the practical use, which the measurements are intended to serve. For many purposes, for instance when it comes to computing the heating effect on differently oriented surfaces, we wish to distinguish between the *directed radiation* from the sun, and the *diffuse radiation from the sky*, both groups differing fundamentally as regards their dependence on geometrical optical factors. For most practical evaluations it seems here reasonable to include a certain amount of circumsolar sky radiation in the solar radiation itself, as has been done in practice using pyrheliometers with apertures about 5° in size.

The circumsolar radiation naturally follows the sun and obeys the cosine law in its dependence on solar elevation and azimuth.

Acknowledgements

The present paper has been prepared under the Eppley Foundation for Research. For ample conveniences offered by the Eppley Laboratory the author is indebted to Mr Roy Anderson, president. He is also obliged to Mr A. J. Drummond, M. Sc., Chief Research Physicist of the Laboratory, and to Dr P. R. Gast for their contribution to many useful discussions on the matters here treated.

Appendix

In a report of the Sub-commission on Aperture conditions of the Radiation Commission of the International Association of Meteorology (IUGG) submitted to the Davos session of the Radiation Commission, in September 1956, the following parameters were recommended to specify the aperture conditions of a radiometer. (Compare fig. 1 and Linke's "Constants of normalization".)

α the "opening angle". ($\mathrm{tg}\,\alpha = \frac{a}{b}$)

α_1 the "slope angle". ($\mathrm{tg}\,\alpha_1 = \frac{a-1}{b}$)

α_2 the "limit angle" ($\mathrm{tg}\,\alpha_2 = \frac{a+1}{b}$)

Evidently all these parameters can be derived from the constants of normalization, a and b.

The subcommission realized that it would be unpractical to recommend an aperture angle so small that the circumsolar radiation falling on the receiver would be practically negligible, but regarded it desirable, in order that measurements with different standard instruments should be comparable with one another, that a certain normalization of the aperture conditions were established. For this reason it was recommended that "pyrheliometers constructed in future should have an angle of slope not less than 1°, and not greater than 2°, and a ratio $\frac{l}{r}$ equal to or greater than 15. These conditions imply that the opening angle should not be greater than 4° ($\varphi < 8°$).

As a practical instance of the extent to which some pyrheliometers with circular apertures conform to these rules, we may refer to the following table:

Abbot Silver disk pyrheliometer:

R = 18.5 mm	α = 2°.9	
r = 13.5 »	α_1 = 0°.8	b = 27
l = 370 »	α_2 = 4°.9	

Eppley standard pyrheliometer:

R = 10.3 mm	α = 2°.9	
r = 2.8 »	α_1 = 2°.1	b = 73
l = 206 »	α_2 = 3°.6	

Pyrheliometer Linke-Feussner:

R = 6.3 mm	α = 5°.1	
r = 5.0 »	α_1 = 1°.0	b = 14
l = 70.3 »	α_2 = 9°.1	

It will be noticed that the geometry of the pyrheliometers are in reasonable agreement with the named requirements. It is especially important that the aperture angle (2 × 2.9°) for both the first named pyrheliometers be well below the limit value 8°, and yet above about 4° ($\alpha > 2°$), which seems to represent a reasonable lower limit, when we are concerned with practical field observations within the meteorological station network.

REFERENCES

ÅNGSTRÖM, A., 1958: On pyrheliometric measurements, *Tellus*, **10**, pp. 342—354.

BOSSY, L., PASTIELS, R., 1948: Etude des propriétés fondamentales des actinomètres. *Institut Royal Météorologique de Belgique. Mémoires*, XXIX.

COURVOISIER, P., 1957: Contributions to Standard Pyrheliometry with an introduction by A. Ångström and W. Mörikofer, *Sveriges Meteor. & Hydrological Institute Communications, Series* **B**. No. 13.

KALITIN, N. N., 1933: De la radiation de la voute celeste autour du soleil pour les ondes longues et pour les ondes courtes. *Veröff. d. Sekretariats d. Intern. Meteor. Org.* **15**.

— 1937: Des erreurs pendant le mesurage de l'intensité de la radiation solaire importées par les nuages dans l'angle d'ouverture de l'actinomètre. *Secretariat de l'Organisation Meteor. Internationale*, **33**.

LAMBERT, J. H., 1760: *Photometria sive de mensura et gradibus luminis colorum et umbrae.*

LINKE, F., 1931: Die Bedeutung des Öffnungsverhältnisses eines Aktinometers für Messungen der Sonnen- und Himmelsstrahlung. *Strahlentherapie*, **39**, p. 351.

LINKE, F., ULMITZ, E., 1940: Messungen der zircumsolaren Himmelsstrahlung. *Meteor. Zscht*, **57**, p. 372.

NEWKIRK, G. A., Jr., 1956: Photometry of the Solar Aureole. *Journal of the Opt. Soc. of America*, **46**, 12.

PASTIELS, R., 1959: Contribution à l'etude du problème des méthodes actinométriques. *Inst. Roy. Meteor. de Belgique, Publications Serie* **A**, 11.

SCHUEPP, W., 1949: *Die Bestimmung der Komponenten der atmosphärischen Trübung aus Aktinometermessungen.* Inaug. Diss. Wien, pp. 320—321.

STRANZ, D., 1942: Die Himmelshelligkeit in unmittelbarer Nähe der Sonne. *Meteor. Zscht*, **59**, p. 120.

— 1940: Untersuchungen der Himmelshelligkeit in der Umgebung der Sonne *Veröff. Geophys. Inst. Univ. Leipzig*, **XII**, 4.

VOLZ, F., 1956: Report to the Radiation Symposium at Davos (mimeographed).