Techniques of Determinig the Turbidity of the Atmosphere[1]

By ANDERS ÅNGSTRÖM, The Eppley Foundation for Research, U.S.A.

(Manuscript received January 16, 1961)

Abstract

After a short survey I of the principles underlying the determination of the turbidity coefficient β, earlier introduced by the author as a measure of the atmospheric turbidity, a condensed summary II is given of earlier and also of more recent determinations of β. The variation of the turbidity with the time of the year, the airmass and with latitude is discussed. Finally a simple method of determining the wave length dependence of the extinction by atmospheric aerosol is outlined. The method is founded upon measurements of integral radiation values with aid of pyrheliometers and glass filters, III. Accuracy and probable error are considered, IV.

I. The transmission of solar radiation through the atmosphere is mainly dependent on three factors: the scattering by the molecules (Rayleigh scattering), the scattering and absorption by solid and liquid particles, and the selective absorption by gaseous constituents. If we with p_λ', p_λ'' and p_λ''' denote the transmission factors corresponding to these phenomena respectively, we may express the solar radiation I_λ at a given wavelength λ, reaching the earth's surface through the equation:

$$I_\lambda = I_{0\lambda}\,[p_\lambda']^{m_1}\cdot[p_\lambda'']^{m_2}\cdot[p_\lambda''']^{m_3} \qquad (1)$$

where $I_{0\lambda}$ is the solar constant for the wavelength λ, and m_1, m_2 and m_3 are the relative air masses corresponding to the attenuating factors.

For solar elevations above about 15°, we may put $m_1 = m_2 = m_3 = m$ and the equation simplifies into:

$$I_\lambda = I_{0\lambda}[p_\lambda'\cdot p_\lambda''\cdot p_\lambda''']^m \qquad (2)$$

The value of $I_{0\lambda}$ is known within fractions of one per cent from the investigations of the Smithsonian Institution and their treatment by various authors (BAUR, 1953, Table 95). Knowing the pressure at the place of observation, we can compute p_λ' on the basis of theoretical and experimental evidences. (BAUR 1953, table 100). For large parts of the solar spectrum p_λ''' is small and due to comparatively permanent constituents, and for these parts we may therefore put:

$$I_\lambda = E_\lambda\cdot[p_\lambda'']^m \qquad (3)$$

[1] Since the present paper had gone to print, the author has had opportunity to study an important paper by Dr Mircea Herovanu: *Determination des paramètres d'Ångström par des observations actinométriques courantes* (Geofisica pura e applicata, Vol. 44 1959/III). Evidently the method of Herovanu is fundamentally very similar to the one applied in the present paper, with the exception that Herovanu does not make the simplifying assumption that the two groups of radiation, corresponding to our λ_1, and λ_2, can be regarded as homogeneous. His computations then are more laborious and must be based on graphical integration. It can be shown however, that his results, with such an accuracy as the measurements are justifying, can be derived equally well, in applying equations perfectly similar to those given in the present paper. His investigation then furnishes a further support for the validity for practical work of the assumption as regards the the homogenety of the wavelength groups considered. I propose to considered this question more in detail in a following note.

where E_λ is a quantity which, when the air mass m is known, can be derived from known data. For p_λ'' we have on the basis of the law of Bouguer:

$$p_\lambda'' = e^{-f(\lambda)} \tag{4}$$

where $f(\lambda)$ is a function of the wavelength and of the qualities of the turbid medium producing the scattering and absorption. For the simple case that we know the form, size and material of the scattering particles in the atmosphere, $f(\lambda)$ can be derived on a theoretical basis, as has been done by Mie and his followers (MIE 1908, LA MER and collaborators 1943, FOITZIK 1938, HOLL 1946, 1948). In the practical case of the atmosphere, the particles originate from a number of different sources and consist mainly of (1) dust particles emanating from the ground, (2) water and ice particles arising through condensation, (3) nuclei of hygroscopic salts on which water is condensed to larger or smaller extent and finally, (4) to a comparatively small amount of meteoric dust, which under normal conditions scarcely produces more than some fractions of a per cent of the total scattering effect. To the variation in quality can be added a great variation in form and size, with a size distribution highly different for the different constituents of the aerosol (JUNGE 1952).

It therefore has seemed justified to approach the problem in a more empirical way and try to derive $f(\lambda)$ on an experimental basis. From, especially, the very extensive material of transmission coefficients collected by Abbot and his colleagues of the Astrophysical Observatory of the Smithsonian Institution, and also from material gained at K. Ångström's solar observatory at Uppsala the following empirical formula was derived (ÅNGSTRÖM 1929, 1930).[2]

$$p_\lambda = e^{-\frac{\beta}{\lambda^\alpha}} \tag{5}$$

The equation (5) defines a "turbidity coefficient" β and a wavelength exponent α, which both together, in a broad sense, are characterizing the scattering and absorbing aerosol.

It was found that α varies within a comparatively small range and that under average conditions, at a rather great variety of stations, it has a value close to 1.3 ± 0.2, being seldom below 0.5 or above 1.6. Under conditions when the atmosphere is polluted, by, for instance, volcanic outbreaks or forest fires, α may be reduced to a rather low value, 0.5 or less. Examples of this are furnished by the conditions after the outbreak of Mount Katmai at Alaska in 1912, when spectrobolometric records from Mt. Wilson in California, Bassour in Algeria and Uppsala in Sweden all show very low α values on disturbed days (ÅNGSTRÖM 1929). An extensive study of the variation of α and β has been made by WEMPE (1947) on the basis of a great mass of observations, spectrobolometric ones as well as also from photometric measurements on stars within the visible spectrum. A rather comprehensive summary of Wempe's results is given in the form of tables in a recently published book by FOITZIK and HINZPETER (1958). Statistical studies of the variations of α and β in their dependence on various variables are presented in an important thesis by W. SCHÜEPP (1949). Schüepp confirms in the main the idea about the constancy of α with wavelength within the visible spectrum, but points out that the constancy seems to be valid only up to an upper limit of wavelength – generally occurring in the near infrared – which is lower the smaller the turbidity coefficient. Some doubt must, however, be attached to the transmission coefficients determined from the spectrobolograms within or near the regions of the spectrum where the water vapor absorption takes place, and the same doubt must be attached to conclusions of Schüepp drawn from them as regards the scattering effect within these spectral regions.

Schüepp (loc. cit.) replaces the coefficient β defined by equation (5) with a coefficient B, fundamentally based on the same considerations, but referred to the base 10 instead of e and to the wavelength 0.5 μ instead of 1.0 μ, viz. the decimal scattering coefficient is B at the wavelength 0.5 μ. The relation between β and B is then given by the equation:

$$e^{-\frac{\beta}{\lambda^\alpha}} = 10^{-\frac{B}{(2\lambda)^\alpha}} \tag{6}$$

$$B = \beta \cdot 2^\alpha \cdot \log e \tag{7}$$

For $\alpha = 1.3$ this gives:

$$B = \beta \cdot 1.07 \tag{8}$$

This transformation is evidently a purely mathematical one. It speaks in favor of the transformation of Schüepp that his B refers to

[2] In the following we put $P_\lambda'' = P_\lambda$.

a wavelength which represents the central part of the visible spectrum instead of the wavelength 1.0, where the scattering is of a more abstract interest. The advantage, however, is highly reduced by the exchange of the natural basis against the decimal one. As long as β is not too large we can namely put:

$$e^{-\beta} = 1 - \beta \tag{9}$$

In this case β evidently gives us directly an idea about the fraction of the incident radiation, which is scattered and absorbed by diffusing particles. Multiplying β by 2 we get an approximate idea of the scattering within the visible.

Schüepp has given in his thesis an excellent survey of the different sources of error which should be avoided or corrected for in actinometric research, especially when we wish to determine from such measurements the parameters of atmospheric transmission, like the turbidity coefficient and the wavelength exponent α. The method most widely used at present for such determinations is founded upon measurements with so-called cutoff filters (ÅNGSTRÖM 1929, 1930). Measuring the radiation behind such filters and also the total radiation without filter, we are able to distinguish between two integrals of energy, namely one affected mainly by the scattering, the other by scattering as well as selective absorption. Through evaluation of these integrals, one can by means of tables or graphical representations derive values for β or B and, by a method suggested by Schüepp, also values for α.

The tables or diagrams are generally made up on the basis of known values for:

(1) The extraterrestrial radiation at mean solar distance corresponding to different wavelengths;
(2) The extinction of the atmosphere through the Rayleigh scattering by the molecules;
(3) The absorption of ozone within the visible and ultraviolet spectrum;
(4) The relative and absolute air masses m and m_h.

Further the extinction by dust and solid particles is assumed to take place according to the formula (5), in specialized cases with the introduction of a value for α equal to 1.3. We enter with the measured values of the total radiation and of the radiation measured behind filters, properly reduced to represent standard conditions, into the named tables and derive from them values of the parameters of turbidity (IGY INSTRUCTION MANUAL 1958).

Some corrections or reductions must be applied to these values, and others are desirable if high accuracy is attempted. The former ones refer mainly to the following conditions.

(a) The measured solar radiation ought to be reduced to the mean distance R_0 between sun and earth by multiplication with the factor $\left[\frac{R}{R_0}\right]^2$, where R is distance at the time of observation.
(b) The filter measurements must be reduced to the standard conditions of the respective filters, because it is to these standard conditions that the tables refer.
(c) The relative air mass, which is determined from the time equation, from the directly observed solar elevation, or from the tables, must be corrected to represent absolute air mass. This is generally done by multiplication with a factor $\frac{p}{p_0}$ where p_0 is the standard pressure at the earth's surface and p is the pressure at the place and time of observation of the sun's radiation. This means that the air mass 1.0 is not, at determinations of β or B from tables, the vertical air mass, but an air mass which is $\frac{p_0}{p}$ times the vertical one, a fact which is especially to be observed. The values of β or B, which are obtained from the tables, consequently do not refer to the vertical air mass. In order to correct them to vertical air mass they must be multiplied by the factor $\frac{p}{p_0}$.

The correction for pressure must generally be applied at stations for which the altitude is higher than about 200 m, where the mean pressure is more than 30 millibars less than at sea level. At stations not far from sea level, a correction for the daily and annual pressure variations can generally be neglected, so long as the pressure deviations are below about $\pm$ 20 millibars (compare IGY INSTRUCTION MANUAL 1958).

Further corrections, which have been suggested, refer to the following conditions. The relative air mass is slightly different for the different absorbing constituents ($m_1 \div m_2 \div m_3$

for values of m above about 4.0). A correction for this can in most cases be neglected. It must be kept in mind that the theory of atmospheric extinction is generally founded upon the idea that the atmosphere is homogeneous within concentric air layers. We have generally very small guarantee that this condition is fulfilled when we go to relative air masses of 4.0 or more.

Other corrections refer to temperature of filters. In recent papers by the present author and A. J. Drummond, we have investigated in detail the transmission of the standard filters used within solar research in its dependence on wavelength, thickness of filter, and temperature (ÅNGSTRÖM and DRUMMOND 1959, 1960). It was shown that in order to obtain the radiation integrals tabulated for determination of β from measurements behind the filters, we must apply (a) *a filter factor*, which seems to be rather constant for different standard filters, even if they are from different melts, and (b) in general also, *an additive correction*, ΔI, which is mainly dependent on the position of the cutoff of the filter, in comparison with the standard specimens (ÅNGSTRÖM and DRUMMOND 1961). It is evident from these investigations that a correction to standard specimen filter conditions cannot be made simply through applying, as has been generally done up to now, a constant "filter factor" to the measurements behind the filter (IGY INSTRUCTION MANUAL 1958), and that the additive term causes a constant filter factor, if applied under various atmospheric conditions, to introduce errors up to $\pm$ 5 per cent, or under extreme conditions even more. Changes in temperature produce a shift in the cutoff of up to 5 mμ, but this shift is comparatively small in comparison with the shifts up to 15 or 20 mμ, which up to now have occurred between specimens of different melt of the standard filters in use. The conclusion is, that if a very accurate knowledge of the location of the cutoff of the filter is not acquired, there is not much use in applying corrections for filter temperature. An instance of that is given inadvertently by SCHÜEPP (1949). He derives through extrapolation a value for the extraterrestrial radiation corresponding to his OGl-filter, of 0.548 gram calories, whereas the correct value is about 0.510 (derived from Nicolet and others). An error of 8 per cent is introduced through the application of a faulty standard filter factor. Under such circumstances an introduction of small corrections of the order of size of less than a couple of per cent seems rather superfluous.

One must keep in mind, when values of β or B and α are derived, that the definition of these values are founded on a number of approximations. They are accepted as means of obtaining a broad view of the transmission conditions of the atmosphere, and they are originally meant to furnish possibilities to get general climatological and synoptical concepts about the transmission conditions of the atmosphere. The exponent α represents merely a theoretical simplification of the fact, that the scattering has more or less a certain complicated relationship to wavelength. As soon as we come to attempts to fix its value within a few per cent, from integral measurements like those here considered, we go into minor considerations not justified by the general definitions behind them. It seems in this field of research to be rather important that a certain simplicity of the technical procedure be maintained.

II. With these considerations in mind, it appears justified to give a short survey of the result of recent derivations of the turbidity, expressed through the coefficient β. Minor corrections have in many cases been neglected in these derivations. In general, however, the results satisfy their aim, namely to give broad ideas about the atmospheric turbidity in its dependence on air-mass, time of the year, latitude and local conditions.

As an introduction two diagrams from early measurements may be referred to. The first one gives β as a function of time of the year at a number of stations (Fig. 1). The result is derived from measurements or computations by ABBOT and FOWLE, KIMBALL, HOELPER (1939), TRYSELIUS (1936) and A. ÅNGSTRÖM (1929, 1930). They show as a rule a rather pronounced maximum of β in the early summer. There seems to be rather few exceptions from this general rule, which evidently implies, that the number of scattering solid and liquid particles in the atmosphere has a maximum at that time of the year. The obvious cause of this must be that on account of the rapid heating of the ground in spring and early summer, the atmosphere shows its greatest vertical instability at that time, and that consequently dust and combustion products are then carried most effectively

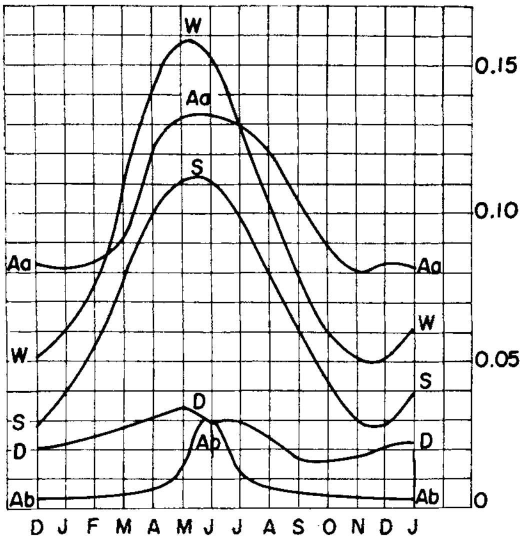

Fig. 1. Annual variation of turbidity coefficient β, for Aachen (Aa), Abisko (Ab), Davos (D), Stockholm (S), and Washington (W) according to HOELPER (1935) and TRYSELIUS and ÅNGSTRÖM (1934). From TRYSELIUS (1936).

from ground to atmosphere. It may be concluded, perhaps, from this variation that in the summer, at continental stations, the scattering nuclei, which originate from the solid surface of the earth, produce an effect which is at least the double of that produced by nuclei of other origin.

An interesting diagram (Fig 2), which demonstrates how the β-values may be used for synoptic purposes, is reproduced from a paper by E. SCHULMAN (1943). The diagram brings out the difference between polar continental and modified pacifique or polar air masses *Np* as regards turbidity. The turbidity coefficient was occasionally found as high as 0.25 for Tropical air (from the Gulf of Mexico, Tg), whereas in some cases it was below 0.03 for Polar air. For Stockholm mean values of 0.06 and 0.09 were found for Polar and Tropical air respectively (ÅNGSTRÖM 1930). All these results are founded on a rather limited mass of observations.

We may add a diagram giving a broad survey on the variation of β with latitude (ÅNGSTRÖM 1930). The diagram is of interest more as a starting point for supplementary comments than as a final result of detailed measurements. For the Tropics the diagram suggests annual mean values of β of about 0.12, with a rapid fall off down to about 0.06 with increasing latitude within the temperate zones. The value for the Arctic region was founded merely on a series of measurements at Spitzbergen, but more detailed information is now available through the measurements of OLSSON and TRYSELIUS (1936) and especially through the very comprehensive measurements and observations of Liljequist

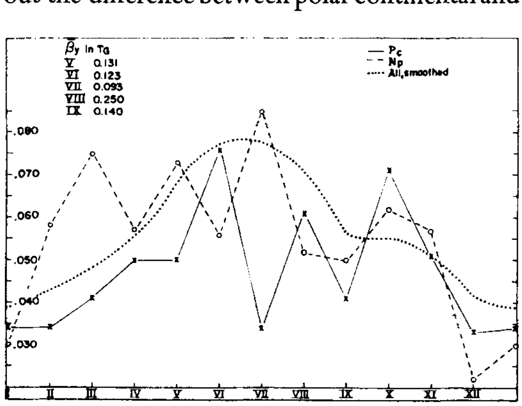

Fig. 2. Annual variation in turbidity at Blue Hill Observatory (Lat. 42° 14′44, long. 71° 6′53, elev. 195 m), July 1934—December 1938, according to air mass type. From SCHULMAN (1943)

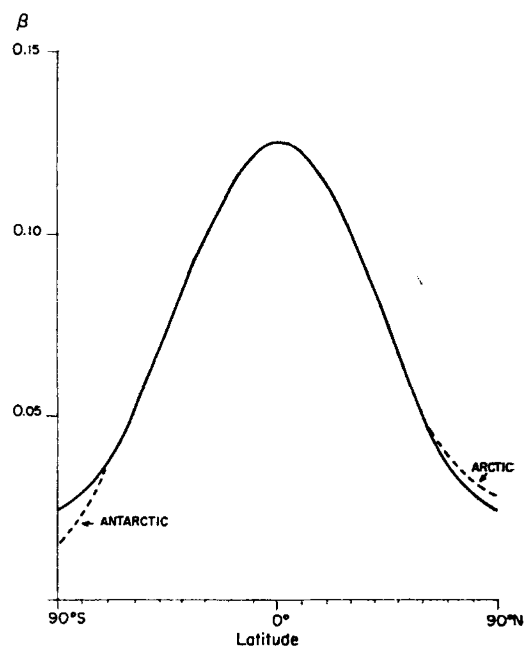

Fig. 3. Dependence of turbidity on latitude. Annual mean values smoothed according to equation: $\beta = [0.025 + 0.100 \cos^2\varphi]\, e^{-0.7h}$, ÅNGSTRÖM (1930), with constants modified to agree with recent values at Antarctic and Arctic stations (h = height above sea level).

within the Artic and the Antarctic (Liljequist 1956).

The annual variation of β at Maudheim (71° 03′ S, 10° 56′ W) is given in Fig. 4 according to Liljequist (1956). The maximum with a value of about 0.025 occurs in summer and coincides very closely with the summer solstice; the minimum can be expected to occur somewhere in the winter. The general trend is thus in general agreement with what we know from temperate latitudes. The explanation of the annual variation at these latitudes, as due to the variation of the convection from the ground, is, according to the view expressed by Liljequist, not applicable to the conditions at Maudheim, because of the great distance from dust producing continental centers. The validity of this conclusion seems, however, doubtful to me. An advection of turbid air from lower latitudes, giving rise to an annual variation of similar character but occurring with some postponement at high latitudes, seems not to be out of the question.

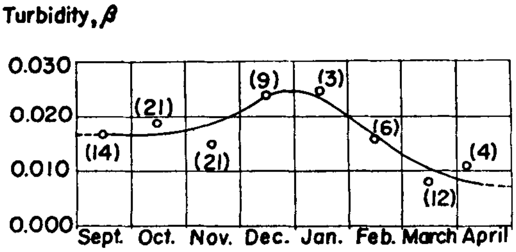

Fig. 4. Monthly means of turbidity β at Maudheim according to the Ångström-Hoelper method. The figures within brackets give the number of days upon which the means are computed. From Liljequist (1956).

It is of interest to notice that the minimum values of β referred to individual days at the Antartic station come very close to zero, which implies that the atmosphere on these occasions is almost perfectly free from scattering particles. This can be expected for the Arctic and Antarctic, and it then forms an additional experimental confirmation of the theory of the Rayleigh scattering. From the paper of Liljequist, in which an abundance of very valuable data are given on the optical conditions of the atmosphere in the Antarctic we may finally add Fig. 5, which shows the diffuse sky radiation at Maudheim in its dependence on β and solar altitude.

III. After this short survey on recent contributions to the problem of the turbidity of the atmosphere, as expressed by the coefficient β, I will finally discuss a rather simple method of determining β as well as α, a method made possible through the more detailed knowledge of the transmission of cutoff glass filters now

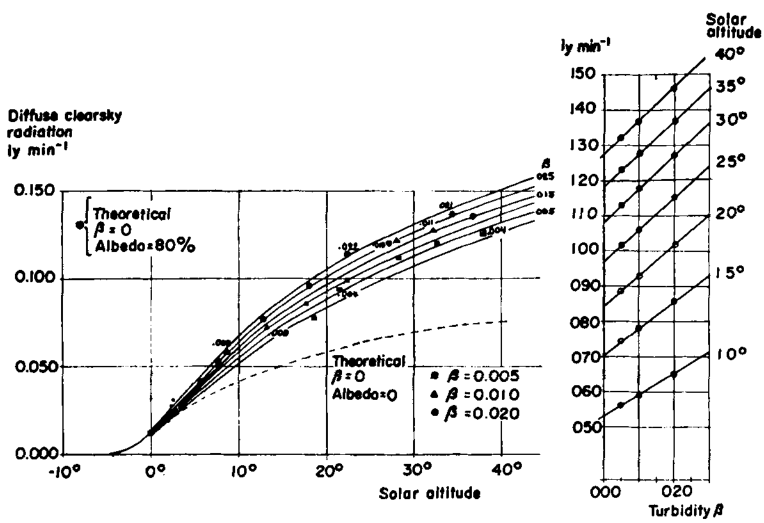

Fig. 5. The diffuse clear-sky radiation as a function of the solar altitude (not corrected for refraction) and the turbidity. The diagram to the right is used for interpolating and extrapolating. "Theoretical" refers to values from Deirmendjian and Sekera (1954). From Liljequist (1956).

in our possession (ÅNGSTRÖM and DRUMMOND 1959, 1960, 1961).

Suppose the solar energy to be measured at two known wavelengths λ_1 and λ_2, where no selective absorption occurs. We compute in the usual way, after elimination of the molecular extinction, the transmission through the turbid medium alone, and obtain the transmission factors p_{λ_1} and p_{λ_2} respectively. Assuming the equation (5) to hold for the wavelength dependence, we get

$$p_{\lambda_1} = e^{-\frac{\beta}{\lambda_1^{\alpha}} \cdot m} \text{ and } p_{\lambda_2} = e^{-\frac{\beta}{\lambda_2^{\alpha}} \cdot m} \quad (10)$$

Here the only unknowns are β and α both of which evidently may becomputed from the two equations.

Through the use of the filters RG8 and RG2 we are able to isolate with application of due filter correction the energy integral:

$$I_r = \int_{0.630}^{0.710} F(\lambda)\, d\lambda \quad (11)$$

and through measurements of the total radiation and of the radiation behind the yellow filter OGl, the integral:

$$I_k = \int_{0}^{0.530} F(\lambda)\, d\lambda \quad (12)$$

If these integrals are plotted against air mass, as shown in Fig. 6, the logarithms of the radiation values fall very closely along straight lines, and the same is the case if, at constant air mass, they are plotted against the turbidity β. This shows that the radiation for air masses between 0.5 and 6.0 and for turbidity coefficients between 0.00 and 0.20 can be regarded practically as homogeneous. If we ask for the effective wavelength, which, assigned to the whole wavelength interval covered by the integral, gives the slope previously derived, this wavelength may be computed from the equation:

$$p_\lambda = e^{-\frac{\beta m}{\lambda^{1.3}}} \quad (13)$$

where p_λ can be obtained from the diagram as the ratio between the radiation integral at fixed values of β and m, and the corresponding radiation at the same air mass for $\beta = 0$.

We thus find:

$$\int_{0.630}^{0.710} F(\lambda)\, d\lambda = 0.183 \cdot e^{-\frac{\beta \cdot m}{0.669^{1.3}}}$$

$$\int_{0}^{530} F(\lambda)\, d\lambda = 0.450 \cdot e^{-\frac{\beta \cdot m}{0.454^{1.3}}}$$

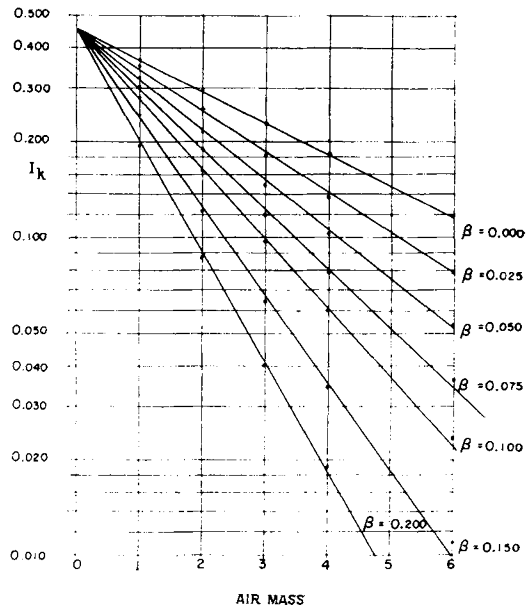

Fig. 6b. Values of I_k, the solar radiation in cal cm^{-2} min^{-1} for $0 < \lambda < 0.530$ according to eq. (12), plotted in logarithmic scale against air mass. Solar constant = 1.98.

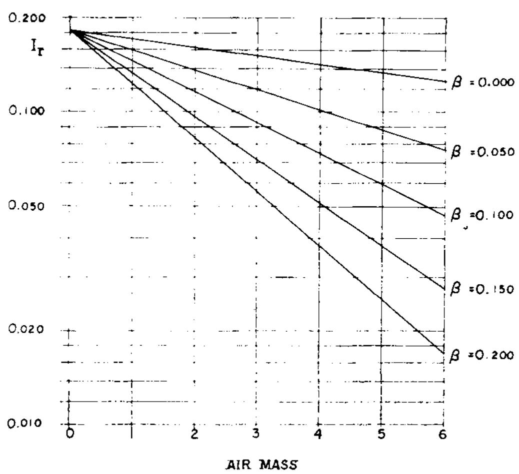

Fig. 6a. Values of I_r, the solar radiation in cal cm^{-2} min^{-1} for $0.630 < \lambda < 0.710\,\mu$ according to eq. (11) plotted in logarithmic scale against air mass. Solar constant = 1.98.

with a remarkable constancy within the whole range. Evidently λ_1 very closely coincides with the mean value between the limits of the integral, whereas λ_2 is only slightly greater than the wavelength ($\lambda = 0.445$) at which the center of gravity of the integral $\int_0^{530} F(\lambda)d\lambda$, corresponding to the extraterrestrial spectrum, occurs.

We may thus for air masses between 1.0 and 6.0 treat the integrals as representing monochromatic radiations and derive β and α from equations (10). Diagrams for this purpose may be easily constructed. As however diagrams and tables for deriving β from direct observations are already in general use, it seems more practical to proceed in a way where use is made of these facilities.

From the diagrams of Fig. 6 values of β may be derived for values of the two integrals $\int_0^{530} F(\lambda)d\lambda$ and $\int_{630}^{710} F(\lambda)d\lambda$, respectively, quite independently of one another. In the case where $\alpha = 1.3$, the two values β_1 and β_2 thus derived coincide, (adopting a common value β_0). When β_1 and β_2 do not coincide, this shows that the assumption $\alpha = 1.3$ is not fulfilled. We will show that the real value $\alpha = \alpha_0$ as well as the true turbidity coefficient β_0 may in a simple way be derived from the values of β_1 and β_2. We have:

$$p_{\lambda 1} = e^{-\frac{\beta_1}{\lambda_1^{\alpha}} \cdot m} ; \; p_{\lambda 1} = e^{-\frac{\beta_0}{\lambda_1^{\alpha_0}} \cdot m} \qquad (14)$$

and

$$p_{\lambda 2} = e^{-\frac{\beta_2}{\lambda_2^{\alpha}} \cdot m} ; \; p_{\lambda 1} = e^{-\frac{\beta_0'}{\lambda_1^{\alpha_0}} \cdot m} \qquad (15)$$

From which we get:

$$\frac{\beta_1}{\lambda_1^{\alpha}} = \frac{\beta_0}{\lambda_1^{\alpha_0}} ; \; \frac{\beta_2}{\lambda_2^{\alpha}} = \frac{\beta_0}{\lambda_1^{\alpha_0}} \qquad (16)$$

which gives:

$$\alpha_0 = \alpha - \frac{1}{\log \lambda_1 - \log \lambda_2} \cdot \log \frac{\beta_1}{\beta_2} \qquad (17)$$

or introducing:

$$\alpha = 1.3 ; \; \lambda_1 = 0.669 ; \; \lambda_2 = 0.454$$

we obtain:

$$\alpha_0 = 1.3 - 5.94 \log \frac{\beta_1}{\beta_2} \qquad (18)$$

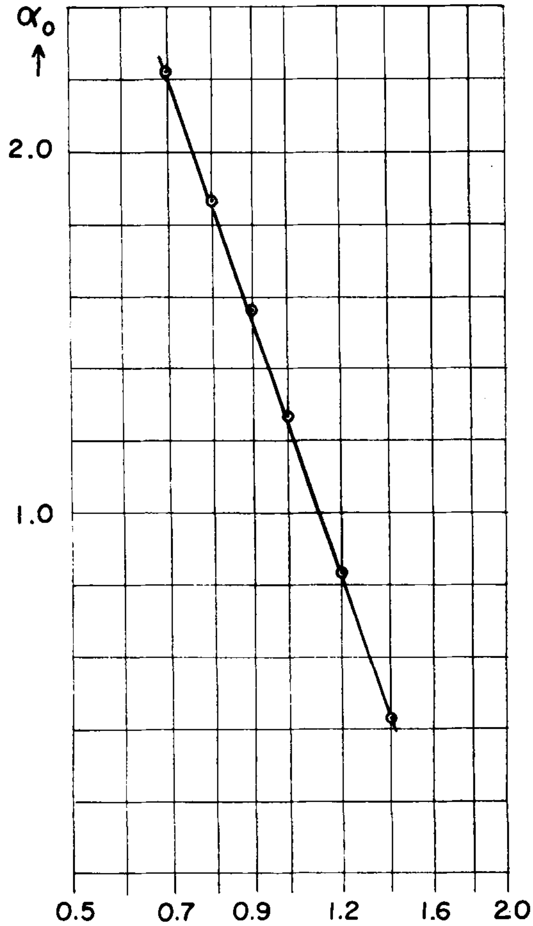

Fig. 7. α_0 as a function of β/β_2 (logarithmic scale); β_1 = coefficient of turbidity $0.630 < \lambda < 0.710$ and β_2 = coefficient of turibdity $\lambda < 0.530$ and assuming $\alpha = 1.3$.

From a diagram where $\frac{\beta_1}{\beta_2}$ is plotted in a logarithmic scale against α_0 we readily obtain α_0 from the linear relationship, as shown in Fig. 7.

We further obtain from (16):

$$\log \frac{\beta_0}{\beta_1} = (\alpha_0 - \alpha) \log \lambda_1 \qquad (19)$$

Or from (17), and introducing known values for λ_1, and λ_2:

$$\log \frac{\beta_0}{\beta_1} = - 1.04 \log \frac{\beta_2}{\beta_1}$$

which gives:

$$\frac{\beta_0}{\beta_1} = \left[\frac{\beta_1}{\beta_2}\right]^{1.04} \qquad (20)$$

As the difference between β_1 and β_2 is in general small, we get with some approximation:

$$\frac{\beta_0}{\beta_1}=\frac{\beta_1}{\beta_2} \text{ or } \beta_0=\frac{\beta_1^2}{\beta_2} \qquad (21)$$

The following table gives α_0 in its dependence on $\frac{\beta_1}{\beta_2}$:

$\frac{\beta_1}{\beta_2}=$	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
$\alpha_0=$	2.6	2.3	1.9	1.6	1.3	1.1	0.8	0.6	0.4	0.3

IV. From equations, tables and diagrams given above it is evident that moderately accurate determination of α_0 demands a rather high accuracy in the determination of β_1 and β_2.

Now, in general, in order to determine β accurately within, let us say, $\pm$ 5 per cent, we must measure the radiation behind the filters with an accuracy not less than 2.5 per cent. This is an accuracy which puts rather high demands on the quality of the filter and on our knowledge of the filter constants (transmittance within main transmission band and position of center of cutoff). Very seldom, up to now, filter constants have been determined accurately enough as to allow such a precision. A systematic error of $\pm$ 5 per cent in the determination of β must be reckoned with as possible in most determinations made hitherto. The influence on the error in α_0 may be derived from:

$$d\alpha_0=\frac{\delta\alpha_0}{\delta\beta_1}\,d\beta_1+\frac{\delta\alpha_0}{\delta\beta_2}\cdot d\beta_2 \qquad (22)$$

which applied to equation (18) gives us:

$$d\alpha_0=\pm 2.6\left[\frac{d\beta_1}{\beta_1}+\frac{d\beta_2}{\beta_2}\right] \qquad (23)$$

Putting

$$\frac{d\beta_1}{\beta_1}=\frac{d\beta_2}{\beta_2}=0.05$$

we get

$$d\alpha_0=\pm 0.3 \qquad (24)$$

Very reasonable assumptions with regard to the accuracy of our measurements, lead us thus to consider an error of $\pm$ 0.6 in the value of α_0 as highly possible. This means that α_0 may be wrong by about 50 per cent. It is to be especially emphasized that this lack of accuracy in the determination of α, is not to in any appreciable degree influenced by the choice of the apparent turbidity coefficients as parameters. The result would have been the same if we had made the computation directly from the transmission values according to the equation (10). An increased accuracy in the filter measurements is the fundamental condition for a higher accuracy in the determination of α. In view of what has been shown on the basis of equations (22) to (24) we must conclude that a rather moderate value ought to be attached to the determinations of α, made up to now on the basis of filter measurements. To deviations less than $\pm$ 0.5 from the standard value 1.3 derived from spectrometric investigations, no safe reality can be ascribed. What has been said here applies equally or in still higher degree to the method proposed by Schüepp for such determinations, a fact which he himself has pointed out on the basis of somewhat different considerations from ours.

The conclusion to be drawn is this: attempts to determine the wavelength exponent α from filter measurements are only justified in cases where the filter constants are so well known as to allow an accuracy of about $\pm$ 1.0 per cent in the measurements of the transmitted radiation. If this condition is not fulfilled, rather erroneous values may be obtained and virtual variations, lacking all reality, may appear.

In a following paper I am planning to treat, according to methods suggested above, observational material obtained with filters, whose filter constants have been determined with a high degree of accuracy and from which comparatively accurate values of α can be derived.

REFERENCES

BAUR, F., 1953: *Linke's Meteorologisches Taschenbuch*. Neue Ausg., II Bd., Akad. Verlagsges. Leipzig.

FOITZIK, L., 1938: Ueber die Lichtdurchlässigkeit der stark getrübten Atmosphäre im sichtbaren Spektralgebiet. *Wiss. Abh. d. Reichsamt f. Wetterdienst* **4**, Nr. 5.

FOITZIK, L., and HINZPETER H., 1958: *Sonnenstrahlung und Lufttübung*. Akad. Verlagsges. Leipzig.

HAURWITZ, B., 1934: *Daytime radiation at Blue Hill Observatory in 1933 with application to turbidity in American air masses*. Harvard Meteorol. Studies, No. 1.

HOELPER, O., 1935: *Der atmosphärische Trübungszustand über Aachen nach kalorimetrischen Strahlungsmessungen in Polarjahr.* Deutsch. Meteor. Jahrbuch. Aachen 1935.

HOELPER, O., 1939: *Atmosphärische Trübungs- und Wasserdampfbestimmungen nach Filtermessungen der Sonnenstrahlung.* Wissenschaftl. Abh. d. Reichsamt f. Wetterdienst.

HOLL, H., 1946: Lichtzerstreuung an die elektrischen Kugeln. *Optik* **1**.

HOLL, H., 1948: Lichzerstreuung an die elektrischen Kugeln. *Optik* **4**.

IGY INSTRUCTION MANUAL, 1958: Radiation, Instruments and Measurements. Part VI.

JUNGE, C., 1952: Die Konstitution des atmosphärischen Dunstes. *Annal. d. Meteor. Beiheft.*

JUNGE, C., 1952: Gesetzmässigkeiten in der Grössenverteilung atmophärischer Aerosole über den Kontinent. *Ber. d. Deutsch. Wetterd. in der U.S.-Zone Nr.* **35**.

JUNGE, C., 1953: Die Rolle der Aerosole und der gasförmigen Beimengungen der Luft im Spurenhaushalt der Troposhäre, *Tellus* **5**.

KEMPE, J., 1947: Die Wellenlängenabhängigkeit der atmosphärischen Extinktion. *Astronom. Nachr.* **275**.

LILJEQVIST, G. H., 1956: Energy Exchange of an Antarctic Snow-field. Short wave radiation. Norwegian-British-Swedish Antarctic Exped. 1949—1952. *Scientific Results* Vol. **2**.

LA MER, V. K. and SINCLAIR, D., 1943: *Verification of the Mie theory.* Rep. No. 1857, Washington, D.C.

MIE, G., 1908: Beiträge zur Optik trüber Medien, speziell kolloidaler Metallösungen. *Ann. d. Phys.* **25**.

SCHULMAN, E., 1943: *Filter measurements of solar radiation at Blue Hill Observatory.* Harvard Meteor. Studies, No. 7.

SCHÜEPP, W., 1949: *Die Bestimmung der Komponenten der atmosphärischen Trübung aus Aktinometermessungen.* Arch. f. Meteor., Geophys. Bioklim. (B).

STRATTON, J. A. and HOUGHTON, H. G., 1931: A theoretical investigation of the transmission of light through fog. *Phys. Rev.* **38**.

TRYSELIUS, O., 1936: *On the turbidity of Polar air.* Statens Meteorol. Hydrogr. Anst., Communications. Series of Papers No.7

ÅNGSTRÖM, A. K., 1929: On the atmospheric transmission of sun radiation and on dust in the air. *Geogr. Ann.* H. **2**.

ÅNGSTRÖM, A. K., 1930: On the atmospheric transmission of sun radiation. II. *Geogr. Ann.* H **2** and **3**.

ÅNGSTRÖM, A. K. and DRUMMOND, A. J., 1959: Transmission of "Cut-off" Glass filters employed in solar Radiation research. *Journ. of the Opt. Soc. Amer.* **49**.

ÅNGSTRÖM, A. K. and DRUMMOND, A. J., 1960: Transmission of "Cut-off" Glass filters employed in solar Radiation research. *Journ. of the Opt. Soc. Amer.* **50**.

ÅNGSTRÖM, A. K. and DRUMMOND, A. J., 1959: On the influence of thickness and temperature on the "cut-off" characteristics of glass filters. *Quart. J. of the Roy. Met. Soc.*, **85**.

ÅNGSTRÖM, A. K. and DRUMMOND, A. J., 1961: Basic concepts concerning cut-off glass filters used in Radiation measurements. *Journ. of Meteorology.* **18**.