Temperature and Steady State Vertical Heat Flux in the Ocean Surface Layers

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Abstract

The occurrence of a quasi-isothermal surface layer is shown to be a necessary consequence of the vertical distribution of heat sources and sinks, which requires an upward transport of heat. The depth of the convective layer depends essentially on a balance between the absorption of visible radiation within the layer and the loss of heat at the surface. Variations in the profile below the isothermal layer are primarily caused by vertical motion. Numerical examples illustrate the effects of meteorological conditions on the heat loss and hence on the surface temperature and thermal structure below.

I. Introduction

Fig. 1 shows two sets of temperature soundings. The left hand side represents the vertical temperature distribution in the sea at 16° 25 N, 23° 09′ W and the potential temperature in the air over Sal (16° 44′ N, 22° 57′ W) in the Cape Verde Islands on the same day. The right hand side represents simultaneous soundings at 17° 33′ N, 81° 11′ W in the Caribbean and over Grand Cayman Island (19° 18′ N, 81° 22′ W). Why have these curves this particular shape, which is fairly typical for conditions in the eastern and western margins of the sub-tropical Atlantic? Why is the surface temperature about 26° C and not 22° or 30°.

A full answer to these questions requires a knowledge of the three dimensional distribution of temperature and velocity in the atmosphere and the oceans. It involves therefore a knowledge of the whole general circulation. The aim of the present discussion is more modest. It intends to show the dependency of the surface temperature and of the profile below upon the rate of upwelling and upon the climatic conditions above.

The sea is cooled by evaporation, conduction and infra-red radiation. These processes are concentrated on the surface. On the other hand, heating by the absorption of visible light from the sun and sky affects an appreciable depth, particularly in biologically unproductive offshore waters. There must be therefore a layer through which heat is transported upwards. As a result, the ocean surface should be somewhat cooler than the water below, an effect observed recently by Ewing and McAlister (1960).

In low latitudes where the sea gains more heat than it loses, the upward heat flux affects only a layer of limited depth. Absorption of solar energy in this layer must be sufficient to compensate for the heat loss from the surface. We may hence expect to find a level where the vertical heat flux changes direction. On the average, between 30° N and 30° S, the amount of energy lost from the surface is about 5—10 times as large as the amount which is being retained and becomes available for export by ocean currents to extra-tropical latitudes.

A turbulent upward flux of heat will be

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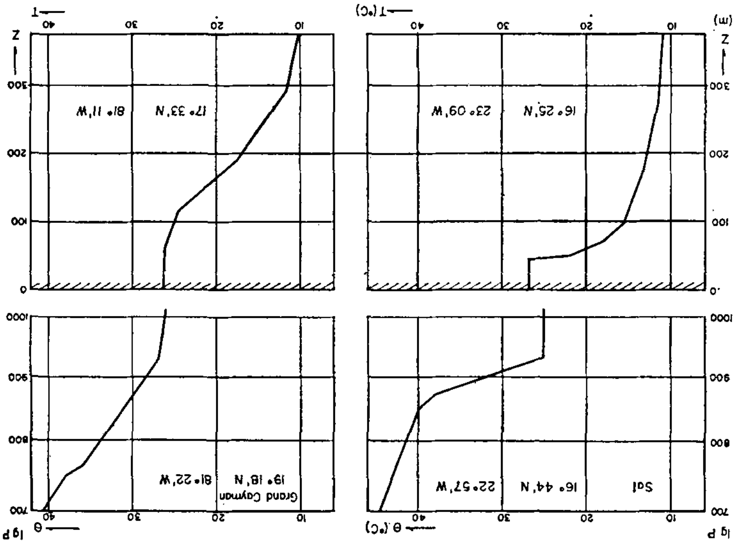

Fig. 1. Characteristic distribution of potential temperature in air and water.

associated with free convection. It generates kinetic energy and tends to produce a layer of nearly constant potential temperature. On the other hand, kinetic energy will be destroyed in the region of downward eddy flux. The stratification there would tend to be stable.

The argument does not indicate that the stirring of the sea surface layers is due to free convection alone. In fact, the kinetic energy created by the buoyancy forces is small compared to that transferred by wind stresses. Particularly in regions of fairly turbid water, and hence of only limited light penetration, wind mixing is likely to overshadow the effects of the upward heat flux. We do feel, however, that over a larger part of the subtropical oceans, the variation of the steady state vertical heat flux with depth and hence indirectly the meteorological conditions pertaining above, exerts a dominating influence on the thermal structure in the uppermost layers.

2. The Absorption of Radiant Energy in the Sea

Radiation from the sun and sky which penetrates the sea surface is mostly absorbed either directly or after multiple scattering. Only a small fraction of the incident energy is scattered back into the atmosphere. Both absorption and scattering vary with the wavelength. Pure ocean water is most transparent to blue light, it absorbs red light more rapidly and is practically opaque to infra-red radiation. The scattering is affected not only by water molecules but also by suspended particles, organisms and dissolved coloured substances. The transparency of sea water in nature is therefore affected by its biological productivity.

Although Beer's law is not strictly applicable in these circumstances, it is closely approximated below the top few metres, at least in the blue and green part of the spectrum. In the present study it will be assumed that the radiation flux in the sea can be represented by the function ($Se^{-\alpha z}$), where S is the flux per unit area across the sea surface and α is the extinction coefficient. (The vertical coordinate z is throughout considered to be 0 at the sea surface and counted positive downwards.)

A brief survey of radiation measurements up to the second world war can be found in SVERDRUP, JOHNSON and FLEMING (1942). Most of these measurements were carried out in comparatively turbid and biologically productive waters of the northern temperate zone. Measurements carried out later in sub-tropical regions showed smaller extinction coefficients.

With the sun at the zenith, and for very clear ocean water, JERLOV (1951) found that ten per cent of the incident vertical beam penetrated to a depth of about 35 metres and one per cent to 85 metres. These figures apply to the vertical component only and, according to Jerlov, have to be increased by about twenty per cent to obtain the total energy.

Jerlov's figures for clear ocean water may be compared with a series of twenty-seven measurements obtained in the Sargasso Sea 15 miles S. E. of Bermuda at 32° 10′ N, 64° 30′ W between 1957 and 1960, BERMUDA BIOLOGICAL STATION (1960). These measurements were carried out with a Weston 856 RR photo-electric cell. Incident solar and sky radiation was measured simultaneously with an Epley pyrheliometer. The extinction coefficients α varied in this series between 0.035 m^{-1} and 0.068 m^{-1}, with a mean of 0.048. This corresponds to an energy penetration of ten per cent to the 48 m level and of 1 per cent to about one hundred metres. Measurements described by CLARKE (1936) give even greater penetrations.

An accurate computation would have to include the spectral variation in α. For the present, fairly simple, model $\alpha = 0.05$ m^{-1} will be assumed for the total extinction coefficient in the trade wind region. In biologically more productive and therefore more turbid tropical waters a value of 0.10 would be more appropriate. In temperate oceanic water Jerlov's figures suggest extinction coefficients approaching 0.2 m^{-1}.

Following BUDYKO (1956), the short wave radiation which penetrates the surface will be approximated by

$$S = 0.94\ (1 - 0.68\ n)\ S_0 \quad \text{cal cm}^2\ \text{day}^{-1} \qquad (1)$$

where S_0 is the solar and short wave sky radiation which reaches the ground, the factor 0.94 is due to an assumed reflection of 6 per cent (BUDYKO 1956, NEIBURGER 1954) and n is the cloud amount in tenths.

3. The Heat Loss from the Sea Surface

The sea surface loses heat by infra-red radiation, turbulent conduction and evaporation.

Following BRUNT (1944) and DORSEY (1940) the mean infra-red radiation balance at the sea surface can be approximated by:

$$Q_a = 0.985\,\sigma\, T_0^4\, (0.39 - 5.04 \,.\, 10^{-2} \sqrt{re_w}) \qquad (2)$$

where σ is Boltzman's constant, T_0 absolute temperature of the sea surface, e_w the corresponding saturation vapour pressure in mbs;

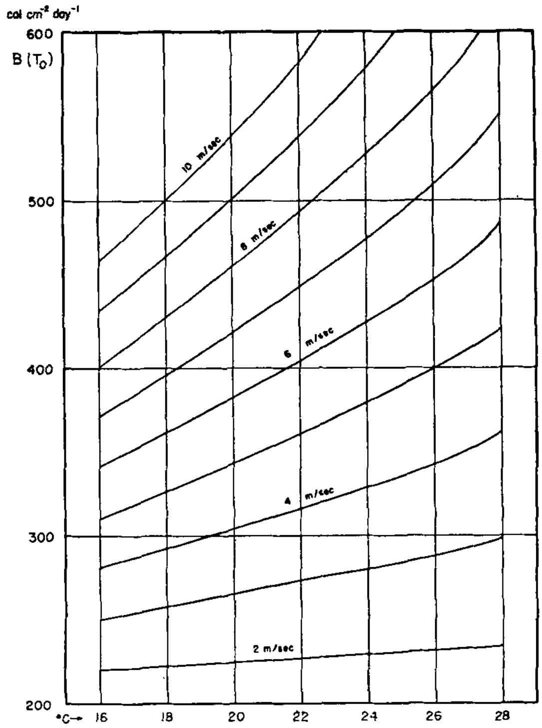

Fig. 2. The rate of heat loss, B, at the sea surface as a function of surface water temperature T_0 and wind speed ($n = 0$, $r = 0.8$, $R = 0.1$).

the quantity r – the ratio of the vapour pressure in the air (at ship's level) to e_w – is closely related to the relative humidity.

When clouds reduce the effective outgoing radiation, the expression (2) is multiplied by a correction factor

$$1 - c\, n^2 \qquad (3)$$

where c, in the sub-tropics, is assumed equal to 0.6. A correction factor of this type is given by BUDYKO (1956), who also lists values of c, for other latitudes.

The conductive and evaporative heat losses can be expressed by

$$Q_h + Q_e = (1 + R)\ LE = (1 + R)\ LK^\star u\ (1 - r)\ e_w \qquad (4)$$

with R denoting Bowen's ratio, L the latent heat of evaporation ($L = 585$ cal g^{-1}) and u the wind velocity. The mean value of the proportionality constant $K^\star = 1.5 \times 10^{-9}$ cm^{-2} sec^2; in broad agreement with the finding of JACOBS (1951), MARCIANO and HARBEK (1954), BUDYKO (1956), KRAUS (1959) and other investigators.

From (2), (3) and (4) with u in m/sec:

$$B = Q_a + Q_h + Q_e = \quad (5)$$

$$0.985\, T_0^4\, (0.39 - 0.05\, re_w)\,(1 - 0.6\, n^2) + (1 + R)\,(1 - r)\, 5.26 \times 10^{-3}\, u\, e_w$$

$$(\text{cal cm}^{-2}\ \text{min}^{-1})$$

The value of e_w is fully determined by T_0. The heat loss B is therefore a function of T_0, the velocity u and the relative humidity r (relative to e_w). It has been plotted in Figure 2 for the special case $n=0$, $R=0.1$, $r=0.8$. It should be noted that variations in u and r tend to affect B more strongly than the variation in T_0 likely to be encountered in the subtropical sea surface.

4. A steady state model; general discussion

To demonstrate the effects of the energy fluxes discussed above, the following simple model is considered.

A water volume is subject to warming in depth and cooling at the surface by radiation, convection and evaporation. At some depth D, not at the bottom but much deeper than the characteristic penetration depth α^{-1} of the visible light, the temperature T_0 and its vertical gradient are prescribed. Effects of horizontal advection are not considered, except for the inclusion of a divergence in the horizontal flow at the surface, compensated for, together with the evaporation, by a prescribed vertical velocity w at depth D. It is assumed that this divergence is confined to the well mixed surface layer, which we anticipate to find. Hence, the temperature of the outflowing water is equal to the surface temperature T_0 and w is constant below the surface layer at least to the depth D. Variations in salinity are not considered and the surface boundary layer described by Ewing is neglected.

Our problem is to determine the structure of the temperature field from the surface to the depth D, and its dependence upon the boundary conditions above and below.

The surface temperature T_0 is readily obtained from a consideration of the energy balance in the column between the surface and the depth D. In the notation of the previous sections we have:

$$S(1\text{-}e^{-\alpha D}) = B\,(T_0) + cpw\,(T_0 - T_D) + K_D\gamma \quad (6)$$

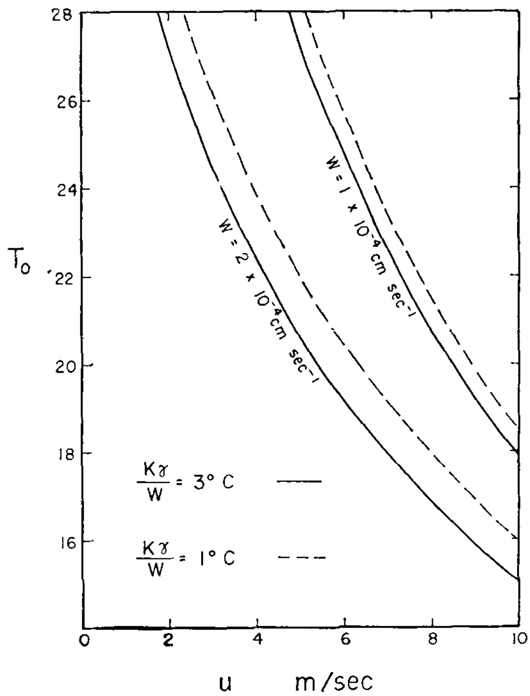

Fig. 3. Surface temperature according to equation (6), with $Kw^{-1} = 250$ m and other parameters as indicated on the diagram.

In this relation exp. $(-\alpha D)$ is negligibly small according to our basic assumptions. Some evidence, discussed below, suggests that the ratio K/w is a scale length which tends to be constant in large parts of the tropical oceans with a value of approximately 250 m. Figure 3 illustrates the relation between T_0 and the wind speed u for some values of w and $\left|\frac{K\gamma}{w}\right|$. D was taken as 500 m and values for T_0 and γ were chosen characteristic of data for low latitudes in the Atlantic ocean (FUGLISTER, 1960).

We shall now proceed to consider the temperature distribution for $0 < z < D$. Our equation (6) implies an upward transport of heat in some layer of depth of order α^{-1}. In these layers any motions induced by wind driven turbulence will tend to be amplified, and a convective regime with nearly isothermal conditions may be expected. Dependent upon the intensity of the convective and wind driven turbulence, this convective regime may penetrate more or less deeply beyond the level

$h^\star$, where balance is struck between the heat loss at the surface, and the total radiation energy absorbed in the layer above.

Since a treatment of the dynamics of this convective layer lies outside the scope of the present study, the following approximation is introduced:

An isothermal layer of temperature T_0 according to (6) corresponding to the entire convective layer described above, reaches down to some depth $h > h^\star$ where $\exp(-\alpha h^\star) = (S-B)/S$. For $z > h$ a constant coefficient K of vertical eddy transfer of heat is assumed to apply. At $z = D$, $T = T_D$ and $\frac{dT}{dz} = \gamma$, in accordance with the conditions applied to equation (6).

With these assumptions, together with the previously stated conditions of constant w below the isothermal layer the thermal energy equation can be integrated upwards from $z = D$, and h is determined by the requirement of temperature continuity: $T(h) = T_0$.

The appropriate equation is:

$$w\frac{dT}{dz} = K\frac{d^2T}{dz^2} + \frac{\alpha S}{\varrho c}e^{-\alpha z} \qquad (7)$$

and the solution:

$$T = T_D - \frac{S_0}{\varrho c(\alpha K + w)}\left(e^{-\alpha z} - e^{-\alpha D}\right) + \frac{K}{w}\left(\gamma - \frac{\alpha_0 S}{\varrho c(\alpha K + w)}e^{-\alpha D}\right)\left(e^{\frac{w}{K}(z-D)} - 1\right) \qquad (8)$$

The heat conduction term $K\frac{dT}{dz}$ at h is balanced by the excess radiation absorbed in the mixed layer, $S_0[\exp(-\alpha h^\star) - \exp(-\alpha h)]$. This balance is automatically secured by the determination of T_0 from the total energy budget for $0 \le z < D$, equation (6).

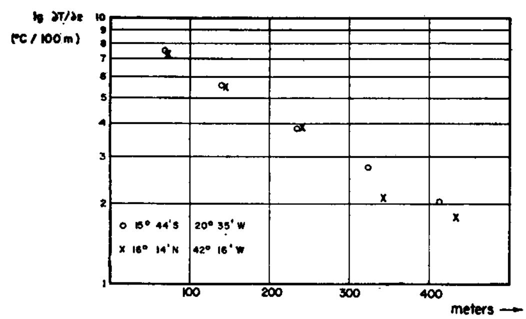

Fig. 4. Variation of $\frac{\partial T}{\partial z}$ with depth at two oceanographic stations.

It is evident from equation (7) that since the short wave radiation term is always positive and $\frac{\partial T}{\partial z}$ outside the convective layer is generally negative, $\frac{\partial^2 T}{\partial z^2}$ can be positive only in the presence of a sufficiently strong upwelling ($w < 0$). Dependent upon the value of the nondimensional numbers $N_1 = \alpha h^\star$ and $N_2 = \alpha K w^{-1}$, inflection points may or may not occur in the profile. The magnitude of N_1 indicates the proportion of the available radiation energy which is consumed for evaporation and N_2 shows the relative importance of the radiation and the heat conduction in the lower layers. A large value of $|N_2|$ indicates that the radiation term is of little importance in the interior part of the region, but for any $N_2 < 0$ a sufficiently small N_1 will introduce an inflection point in the profile, and a layer where the last term in (7) is not negligible.

It is also of interest to note, that if any single term in (7) is negligible, the solution for T according to the reduced equation is a simple exponential, with a characteristic scale length dependent upon which term is negligible. This fact affords us an opportunity to use observations for guidance as to the pertinent values of the scale lengths by simply making plots on a logarithmic scale of $\frac{\partial T}{\partial z}$ versus z on a linear scale.

Figure 4 shows such a plot for two oceanographic stations in the North and South Atlantic. The values fall approximately on a straight line indicating that a reduced form of (7) would indeed apply. Furthermore, since α^{-1} is at most about 30 m (cf. section 2 above), the scale length involved must be identified with Kw^{-1}. Similar plots for a number of oceanographic stations along 16° N and 15° S (FUGLISTER 1960) suggest that a value of about 250 m for this length is characteristic of about two thirds of the sub-tropical Atlantic. The adaptation of a constant Kw^{-1} is not justifiable for data along the margins of the ocean; in particular it is not applicable for the surroundings in figure (1).

5. Numerical Examples

Equations (6) and (8) were used to show how the surface temperature T_0 and the depth h of the isothermal layer are influenced by the meteorological conditions. The following parameters are common to all the examples:

$$D = 500\,\mathrm{m},\ T_D = 9^\circ\,\mathrm{C}, \gamma = 0.8 \times 10^{-4\circ}\,\mathrm{C/cm}.$$

$$w = 1.5 \times 10^{-4}\ \ \mathrm{cm/sec}\ \ Kw^{-1} = 2.5 \times 10^4\,\mathrm{cm}$$

The values of w and of $K\gamma w^{-1}$, (2° C), are seen to lie in between the parameters chosen for the curves in Fig. 3. The results are quite sensitive to the choice of parameters, a fact which will be further commented upon below. Tables 1—3 illustrated the effects of varying wind speed, relative humidity and cloudiness.

Table 1. The surface temperature T_0 and convective layer depth h as functions of the mean wind velocity *ce*. ($n = 0$, $r = 0.8$)

U (m/sec)	3	4	5	6	7	8	9	10
T_0 (°C)	28.8	25.9	23.4	21.5	19.9	18.5	17.3	16.2
h (m)		10	57	87	116	143	169	(200)

Table 2. T_0 and h as functions of the mean ratio r between the vapour pressures in the air, *ea*, and at the sea surface, *ew*. ($u = 5$, $n = 0$)

r	0.5	0.6	0.7	0.8	0.9
T_0		15.9	18.8	23.4	(32)
h		(200)	135	57	

Table 3. T_0 and h as functions of the mean cloudiness n (in tenths). ($u = 5$, $r = 0.8$)

n	0	2	4	6	8	10
T_0	23.4	20.2	16.9	(14)		
h	57	110	178			

It is evident that each of the parameters strongly influences the solution in this model. The same is true for the choice of w and T_0. The range of values of the various parameters, that produce reasonable results is quite limited, and the theory may therefore be said to have quite high information content within its range of validity.

6. Transients

We have limited our discussion to the steady state and our method of treatment cannot immediately be extended to the transient case.

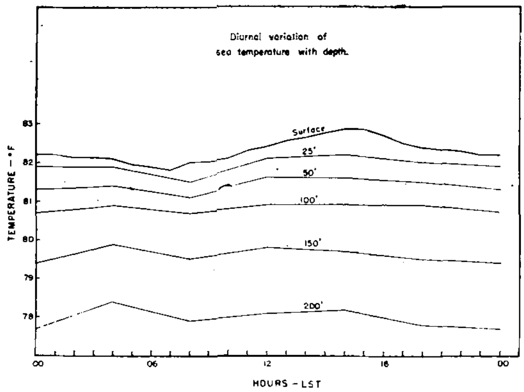

Fig. 5. Temperature variations in the surface layer of the sea, from Garstang (1958).

A brief discussion of some cases of transient development will however be given here, as it is felt that they shed more light on the role of the vertical heat flux.

We can distinguish between three major types of transients, according to the mechanisms and characteristic time scales involved.

The seasonal warming or cooling will cause changes in the total heat storages which are appreciable even compared with the total seasonal insolation (Bryan and Schroeder, 1960).

Weather disturbances of various time scales and dimensions may provide intense wind mixing as well as changes in evaporation conditions, etc. Finally, the diurnal changes in insolation and wind and cloud patterns even on undisturbed days will introduce fluctuations from the idealized steady state with which we have been concerned in our specific model.

The diurnal change of the surface temperature profile in the tropical Atlantic 11° 11′ N, 52° 25′ W has been observed by Garstang (1958). Series of hourly or three hourly bathythermograph soundings are available also from Weathership station Echo (35° N, 48° W). Both series of data tend to show a slight increase in stability during day and a decrease during night.

In explanation it should be considered that during the day, absorption of radiation in a relatively shallow layer will suffice to balance the heat loss from the surface. The lower part of the quasi-isothermal layer gains heat and may become slightly more stable during this

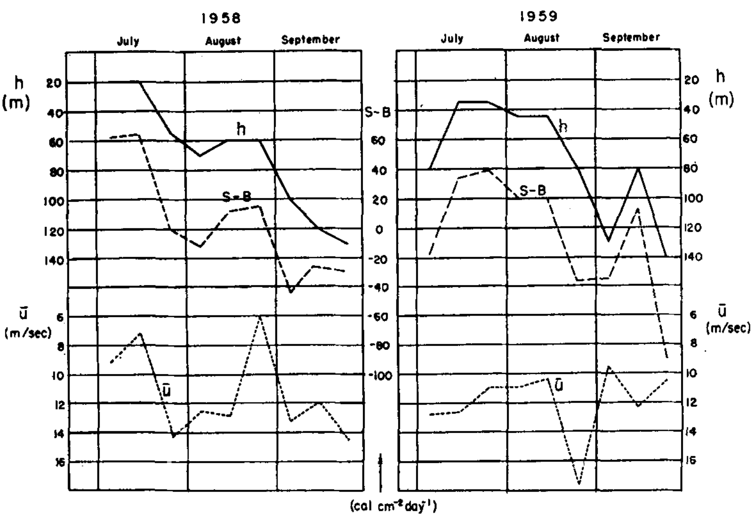

Fig. 6. Wind speed, u, rate of heat loss, $S-B$, and depth, h, of the "wind mixed layer" at weather ship station Echo.

period. The gain of heat is drawn upon during the night when the continuing surface heat loss causes convection to extend through the whole depth of the layer. The diurnal rhythm of convection below the cooled sea surface is therefore opposite to the diurnal rhythm of convection in the air over heated land.

In Garstang's records, as shown in Fig. 5, the mean temperature amplitude at 50 feet is about 0.12° C. The diurnal change in convection may conceivably affect the vertical movement of plankton in the tropics. In extra-tropical regions the diurnal change is likely to be masked by convection changes caused by synoptic disturbances.

Changes on a larger time scale outside the tropics in the northern Sargasso Sea, are illustrated by Fig. 6 which is based also on observations from Weather Ship station Echo (34° N, 48° W). The analysed periods covered late summer and early autumn in 1958 and 1959.

In view of NAMIAS' (1959) findings in the Pacific, it is of some general interest that the sea temperature at this station during summer 1958 was substantially higher than in 1959. This difference was characteristic for every calendar day between the first of July and the last of September. The mean value of T_0 was 76.8° F for August, 1959 against 80.5° F for August 1958. The corresponding July figures were 73.7° F and 77.3° F. The associated difference in the heat storage, that is the enthalpy integral between the surface and 200 metres amounted to about 30,000 cal./cm^2 equivalent to two months solar energy input.

Weather and bathythermograph observations were carried out during the observation periods usually at 3 hourly intervals. The heat loss B was computed from these observations by means of equation (5). The insolation S was estimated on the basis of table 135 in the Smithsonian Meteorological Tables (1951) with an assumed transmission coefficient of 0.9 and with the corrections for albedo and cloudiness given by equation (1).

All estimates were then averaged over ten day periods.

The exact individual results of the computations are doubtful because of unreliable and missing observations. However, the resulting uncertainties have less effect on the ten day means and figure (6) is believed to represent the character of the time change reasonably well.

It can be seen that changes in the mean depth h of the isothermal layer reflect changes in the heat balance $S-B$. There is no obvious correlation with the mean wind velocity as such—in keeping with the present argument.

7. Concluding Remarks

Within the framework of a very simple model we have endeavored to demonstrate that by way of affecting the heat losses from the ocean surface, in particular through changes in evaporation, the meteorological conditions over the ocean may excert a profound influence on the temperature T_0 and depth h of the so-called wind mixed surface layer. We realize that since a radiation excess or deficit of 100 cal./cm^2 day would change the temperature of a 50 m layer by only .02° C/day, advective effects as well as seasonal changes in the total heat content of a water column may be extremely important.

However, a greater challenge is offered by the lack of an adequate theory for the penetration of the convection beyond the depth h^* where the vertical heat flux changes direction. In a sense the classical treatment of the mixed boundary layer by Rossby and Montgomery (1935) represents a limiting case of negligible effects of the heating in depth. The same is true of a recent study by Kitaigorodsky (1960) who considers a case of finite heat supply $= S_0 - B$ in our notation, completely concentrated at the surface, which makes his mixed layer a true layer of forced convection, the depth of which is determined by a kind of Richardson number. The Rossby-Montgomery theory is included as a special case in Kitaigorodsky's work. Attempts at application of the formulae given by Kitaigorodsky did not seem to yield very satisfactory values of h for the cases tried. On the other hand, the procedure outlined in section 4, for finding h on the basis of T_D and γ at $D = 500$ m cannot possibly go far wrong if these values together with T_0 are reasonable. On these grounds it is felt that a true test of a model like this can be made only when it is complete in the sense of containing the dynamics of the combined forced and free convection associated with the mixed surface layer.

Finally, we wish to acknowledge the constructive criticism of our colleagues at the Woods Hole Oceanographic Institution, and the fact that one of us, C. Rooth, is supported under NSF grant No. G 7366.

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